Revista Colombiana de Matemáticas
Volumen 45(2011)2, páginas 187-195

Extension of Reverse Hilbert-Type Inequality with a Generalized Homogeneous Kernel

Extensión de la desigualdad tipo Hilbert con núcleo homegéneo

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Abstract. In this paper, by introducing some parameters we establish a new extension of the reverse of Hilbert-type integral inequality with best constant. We also, consider the equivalent inequality.

Key words and phrases. Hilbert-Type Integral Inequality, Reverse Hölder's Inequality.

2000 Mathematics Subject Classification. 26D15.

Resumen. En este artículo, introduciendo algunos parámetros, establecemos una nueva extensión de la desigualdad inversa tipo Hilbert con mejor constante. También consideramos la desigualdad equivalente.

Palabras y frases clave. Desigualdad integral tipo Hilbert, desigualdad inversa de Hölder.

1. Introduction

If $p > 1$, $\frac{1}{p} + \frac{1}{q} = 1$ and $f, g \geq 0$, satisfy $0 < \int_0^\infty f^p(x)\,dx < \infty$ and $0 < \int_0^\infty g^q(x)\,dx < \infty$, then

$$\int_0^\infty \int_0^\infty \frac{f(x)g(y)}{x+y}\,dx\,dy < \frac{\pi}{\sin(\frac{\pi}{p})} \left\{ \int_0^\infty f^p(x)\,dx \right\}^{\frac{1}{p}} \left\{ \int_0^\infty g^q(x)\,dx \right\}^{\frac{1}{q}}, \quad (1)$$

$$\int_0^\infty \int_0^\infty \frac{f(x)g(y)}{\max\{x,y\}}\,dx\,dy < pq\left\{\int_0^\infty f^p(x)\,dx\right\}^{\frac{1}{p}}\left\{\int_0^\infty g^q(x)\,dx\right\}^{\frac{1}{q}}, \qquad (2)$$

where the constant factors $\pi/(\sin\pi/p)$ and pq are the best possible. Inequality (1) and (2) are called Hardy-Hilbert's inequalities (see [1]) and are important in analysis and their applications (cf. Mitrinović et. al. [3]). The corresponding inequalities for series (1) and (2) are

$$\sum_{n=1}^\infty\sum_{m=1}^\infty \frac{a_m b_n}{m+n} < \frac{\pi}{\sin(\frac{\pi}{p})}\left\{\sum_{n=1}^\infty a_n^p\right\}^{\frac{1}{q}}\left\{\sum_{n=1}^\infty b_n^q\right\}^{\frac{1}{q}}, \qquad (3)$$

$$\sum_{n=1}^\infty\sum_{m=1}^\infty \frac{a_m b_n}{\max\{m,n\}} < pq\left\{\sum_{n=1}^\infty a_n^p\right\}^{\frac{1}{q}}\left\{\sum_{n=1}^\infty b_n^q\right\}^{\frac{1}{q}}, \qquad (4)$$

where the sequences $\{a_n\}$ and $\{b_n\}$ satisfies the following condition: $0 < \sum_{n=1}^\infty a_n^p < \infty$, $0 < \sum_{n=1}^\infty b_n^q < \infty$, and the constant factors $\pi/(\sin\pi/p)$ and pq are the best possible.

In 2009, B. Yang (see [4]) obtained the following inequality:

If $p > 1$, $\frac{1}{p}+\frac{1}{q}=1$, $\beta\in\mathbb{R}$, $f,g\geq 0$ such that $0<\int_0^\infty x^{\frac{p}{2}-1}f^p(x)\,dx<\infty$ and $0<\int_0^\infty x^{\frac{q}{2}-1}g^q(x)\,dx<\infty$, then

$$\int_0^\infty\int_0^\infty \frac{\arctan\left(\frac{x}{y}\right)^\beta}{\max\{x,y\}} f(x)\,g(y)\,dx\,dy < \pi\left\{\int_0^\infty x^{\frac{p}{2}-1}f^p(x)\,dx\right\}^{\frac{1}{p}}\left\{\int_0^\infty x^{\frac{q}{2}-1}g^q(x)\,dx\right\}^{\frac{1}{q}}, \qquad (5)$$

where the constant factor π is the best possible.

Recently, Yang (see [5]) obtained the following reverse inequality:

If $0<p<1$, $\frac{1}{p}+\frac{1}{q}=1$, $\alpha>0$, $\beta\in\mathbb{R}$, $f,g\geq 0$ such that $0<\int_0^\infty x^{p-1}f^p(x)\,dx<\infty$ and $0<\int_0^\infty x^{q-1}g^q(x)\,dx<\infty$, then

$$\int_0^\infty\int_0^\infty \left(\frac{\min\{x,y\}}{\max\{x,y\}}\right)^\alpha \arctan\left(\frac{x}{y}\right)^\beta f(x)\,g(y)\,dx\,dy > \frac{\pi}{2\alpha}\left\{\int_0^\infty x^{p-1}f^p(x)\,dx\right\}^{\frac{1}{p}}\left\{\int_0^\infty x^{q-1}g^q(x)\,dx\right\}^{\frac{1}{q}}, \qquad (6)$$

where the constant factor $\frac{\pi}{2\alpha}$ is the best possible.

Recently, B. He (see [2]) gave a reverse inequality as follows:

If $0<p<1$, $\frac{1}{p}+\frac{1}{q}=1$, $\lambda,\mu\in\mathbb{R}$ such that $\lambda+\mu>0$ $f,g\geq 0$ satisfy $0<\int_0^\infty x^{p\left(1+\frac{\lambda-\mu}{2}\right)-1}f^p(x)\,dx<\infty$ and $0<\int_0^\infty x^{q\left(1+\frac{\lambda-\mu}{2}\right)-1}g^q(x)\,dx<\infty$, then

$$\int_0^\infty \int_0^\infty \frac{(\min\{x,y\})^\lambda}{(\max\{x,y\})^\mu} f(x)\, g(y)\, dx\, dy >$$
$$\frac{4}{\lambda+\mu}\left\{\int_0^\infty x^{p\left(1+\frac{\lambda-\mu}{2}\right)-1} f^p(x)\, dx\right\}^{\frac{1}{p}}\left\{\int_0^\infty x^{q\left(1+\frac{\lambda-\mu}{2}\right)-1} g^q(x)\, dx\right\}^{\frac{1}{q}}, \quad (7)$$

where the constant factor $\frac{4}{\lambda+\mu}$ is the best possible.

In this paper, we generalize inequalities (6), (7) and we build reverse inequality (5). The equivalent form is considered.

2. Main Results

Lemma 1. *Let $\alpha, \beta, \gamma \in \mathbb{R}$ such that $\alpha + \beta > 0$. If the weight function $w(\alpha, \beta, \gamma, x)$ is defined as*

$$w(\alpha,\beta,\gamma,x) = \int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta} \arctan\left(\frac{x}{y}\right)^\gamma \cdot \frac{x^{-\frac{\alpha-\beta}{2}}}{y^{1+\frac{\alpha-\beta}{2}}}\, dy, \quad (8)$$

then we obtain

$$w(\alpha,\beta,\gamma,x) = \frac{\pi}{\alpha+\beta}. \quad (9)$$

Proof. Setting $u = y/x$ in (8) and using the fact that $\arctan \frac{1}{x} + \arctan x = \frac{\pi}{2}$ $(x > 0)$, we have

$$w(\alpha,\beta,\gamma,x) = \int_0^\infty \frac{\left(\min\{1,u\}\right)^\alpha}{\left(\max\{1,u\}\right)^\beta} \cdot u^{-1-\frac{\alpha-\beta}{2}} \cdot \arctan\frac{1}{u^\gamma}\, du \quad (10)$$
$$= \int_0^1 u^{-1+\frac{\alpha+\beta}{2}} \cdot \arctan\frac{1}{u^\gamma}\, du + \int_1^\infty u^{-1-\frac{\alpha+\beta}{2}} \cdot \arctan\frac{1}{u^\gamma}\, du \quad (11)$$
$$= \int_0^1 u^{-1+\frac{\alpha+\beta}{2}} \cdot \arctan\frac{1}{u^\gamma}\, du + \int_0^1 u^{-1+\frac{\alpha+\beta}{2}} \cdot \arctan u^\gamma\, du \quad (12)$$
$$= \int_0^1 u^{-1+\frac{\alpha+\beta}{2}} \cdot \left(\arctan\frac{1}{u^\gamma} + \arctan u^\gamma\right) du \quad (13)$$
$$= \frac{\pi}{2}\int_0^1 u^{-1+\frac{\alpha+\beta}{2}}\, du = \frac{\pi}{\alpha+\beta}. \quad (14)$$

Thus the Lemma 1 is proved. ☑

Theorem 2. *If $0 < p < 1$, $\frac{1}{p} + \frac{1}{q} = 1$, $\alpha, \beta, \gamma \in \mathbb{R}$ such that $\alpha + \beta > 0$ and $f, g \geq 0$ satisfy $0 < \int_0^\infty x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1} f^p(x)\, dx < \infty$ and*

$0<\int_0^\infty x^{q\left(1+\frac{\alpha-\beta}{2}\right)-1}g^q(x)\,dx<\infty$, *then*

$$I:=\int_0^\infty\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta}\arctan\left(\frac{x}{y}\right)^\gamma f(x)\,g(y)\,dx\,dy>$$
$$\frac{\pi}{\alpha+\beta}\left\{\int_0^\infty x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1}f^p(x)\,dx\right\}^{\frac{1}{p}}\left\{\int_0^\infty x^{q\left(1+\frac{\alpha-\beta}{2}\right)-1}g^q(x)\,dx\right\}^{\frac{1}{q}}, \quad (15)$$

where the constant factor $\frac{\pi}{\alpha+\beta}$ *is the best possible.*

Proof. By the reverse Hölder's inequality, we have

$$\begin{aligned} I&=\int_0^\infty\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta}\arctan\left(\frac{x}{y}\right)^\gamma f(x)\,g(y)\,dx\,dy\\ &=\int_0^\infty\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta}\arctan\left(\frac{x}{y}\right)^\gamma\\ &\qquad\times\left[\frac{x^{\left(1+\frac{\alpha-\beta}{2}\right)/q}}{y^{\left(1+\frac{\alpha-\beta}{2}\right)/p}}f(x)\right]\left[\frac{y^{\left(1+\frac{\alpha-\beta}{2}\right)/p}}{x^{\left(1+\frac{\alpha-\beta}{2}\right)/q}}g(y)\right]dx\,dy \qquad (16)\\ &\geq\left\{\int_0^\infty\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta}\arctan\left(\frac{x}{y}\right)^\gamma \frac{x^{(p-1)\left(1+\frac{\alpha-\beta}{2}\right)}}{y^{\left(1+\frac{\alpha-\beta}{2}\right)}}f^p(x)\,dx\,dy\right\}^{1/p}\\ &\quad\times\left\{\int_0^\infty\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta}\arctan\left(\frac{x}{y}\right)^\gamma \frac{y^{(q-1)\left(1+\frac{\alpha-\beta}{2}\right)}}{x^{\left(1+\frac{\alpha-\beta}{2}\right)}}g^q(y)\,dx\,dy\right\}^{1/q}. \end{aligned}$$

If (16) takes the form of the equality, then there exist constants M and N which are not all zero such that

$$M\cdot\frac{x^{(p-1)\left(1+\frac{\alpha-\beta}{2}\right)}}{y^{\left(1+\frac{\alpha-\beta}{2}\right)}}f^p(x)=N\cdot\frac{y^{(q-1)\left(1+\frac{\alpha-\beta}{2}\right)}}{x^{\left(1+\frac{\alpha-\beta}{2}\right)}}g^q(y), \quad \text{a.e in } (0,\infty)\times(0,\infty),$$
$$M\cdot x^{p\left(1+\frac{\alpha-\beta}{2}\right)}f^p(x)=N\cdot y^{q\left(1+\frac{\alpha-\beta}{2}\right)}g^q(y), \quad \text{a.e in } (0,\infty)\times(0,\infty).$$

Hence, there exists a constant c such that

$$M\cdot x^{p\left(1+\frac{\alpha-\beta}{2}\right)}f^p(x)=N\cdot y^{q\left(1+\frac{\alpha-\beta}{2}\right)}g^q(y)=c, \quad \text{a.e in } (0,\infty)\times(0,\infty).$$

We claim that $M=0$. In fact, if $M\neq 0$, then

$$x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1}f^p(x)=\frac{c}{M\cdot x}, \quad \text{a.e in}(0,\infty),$$

which contradicts the fact that $0 < \int_0^\infty x^{p(1+\frac{\alpha-\beta}{2})-1} f^p(x)\,dx < \infty$. Hence, by (8), (15) takes a strict inequality as follows:

$$\int_0^\infty \int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta} \arctan\left(\frac{x}{y}\right)^\gamma f(x)\,g(y)\,dx\,dy > \left\{\int_0^\infty w(\alpha,\beta,\gamma,x) x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1} f^p(x)\,dx\right\}^{\frac{1}{p}} \times \left\{\int_0^\infty w(\alpha,\beta,\gamma,y) x^{q\left(1+\frac{\alpha-\beta}{2}\right)-1} g^q(x)\,dx\right\}^{\frac{1}{q}}.$$

In view of (9), we have (15).

Assume that the constant factor $\frac{\pi}{\alpha+\beta}$ in (15) is not the best possible, then there exists a positive number k with $k > \frac{\pi}{\alpha+\beta}$ such that

$$\int_0^\infty \int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta} \arctan\left(\frac{x}{y}\right)^\gamma f(x)\,g(y)\,dx\,dy > k\left\{\int_0^\infty x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1} f^p(x)\,dx\right\}^{\frac{1}{p}} \left\{\int_0^\infty x^{q\left(1+\frac{\alpha-\beta}{2}\right)-1} g^q(x)\,dx\right\}^{\frac{1}{q}}. \quad (17)$$

For $0 < \varepsilon < \frac{(\alpha+\beta)|q|}{2}$, setting

$$\widetilde{f}(x) = \begin{cases} 0, & x \in (0,1); \\ x^{-\left(1+\frac{\alpha-\beta}{2}\right)-\frac{\varepsilon}{p}}, & x \in [1,\infty). \end{cases}$$

$$\widetilde{g}(y) = \begin{cases} 0, & y \in (0,1); \\ y^{-\left(1+\frac{\alpha-\beta}{2}\right)-\frac{\varepsilon}{q}}, & y \in [1,\infty). \end{cases}$$

We have that

$$\left\{\int_0^\infty x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1} \widetilde{f}^p(x)\,dx\right\}^{\frac{1}{p}} \left\{\int_0^\infty x^{q\left(1+\frac{\alpha-\beta}{2}\right)-1} \widetilde{g}^q(x)\,dx\right\}^{\frac{1}{q}} = \frac{1}{\varepsilon}, \quad (18)$$

and taking $u = \frac{y}{x}$, by Fubini's theorem, we obtain

$$\widetilde{I} = \int_0^\infty \int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta} \arctan\left(\frac{x}{y}\right)^\gamma \widetilde{f}(x)\,\widetilde{g}(y)\,dx\,dy$$
$$= \int_1^\infty \left[\int_1^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta} \arctan\left(\frac{x}{y}\right)^\gamma x^{-\left(1+\frac{\alpha-\beta}{2}\right)-\frac{\varepsilon}{p}} y^{-\left(1+\frac{\alpha-\beta}{2}\right)-\frac{\varepsilon}{q}}\,dy\right] dx$$

$$
\begin{aligned}
&= \int_1^\infty x^{-1-\varepsilon}\left[\int_{1/x}^1 u^{-1+\frac{\alpha+\beta}{2}-\frac{\varepsilon}{q}} \arctan\frac{1}{u^\gamma}\,du\right]dx + \\
&\qquad\qquad\qquad\qquad \frac{1}{\varepsilon}\int_1^\infty u^{-1-\frac{\alpha+\beta}{2}-\frac{\varepsilon}{q}}\arctan\frac{1}{u^\gamma}\,du \\
&= \int_0^1 \left(\int_{1/u}^\infty x^{-1-\varepsilon}\,dx\right) u^{-1+\frac{\alpha+\beta}{2}-\frac{\varepsilon}{q}}\arctan\frac{1}{u^\gamma}\,du + \\
&\qquad\qquad\qquad\qquad \frac{1}{\varepsilon}\int_1^\infty u^{-1-\frac{\alpha+\beta}{2}-\frac{\varepsilon}{q}}\arctan\frac{1}{u^\gamma}\,du \\
&= \frac{1}{\varepsilon}\int_0^1 \left(u^{-1+\frac{\alpha+\beta}{2}+\frac{\varepsilon}{p}}\arctan\frac{1}{u^\gamma} + u^{-1+\frac{\alpha+\beta}{2}+\frac{\varepsilon}{q}}\arctan u^\gamma\right)du \\
&\le \frac{\pi}{2\varepsilon\left(\frac{\alpha+\beta}{2}+\frac{\varepsilon}{q}\right)}.
\end{aligned}
$$

Multiplying both sides of (17) by ε, the above inequality and (18), become

$$
\begin{aligned}
\frac{\pi}{2\left(\frac{\alpha+\beta}{2}+\frac{\varepsilon}{q}\right)} &\ge \varepsilon\int_0^\infty\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta}\arctan\left(\frac{x}{y}\right)^\gamma \widetilde{f}(x)\,\widetilde{g}(y)\,dx\,dy > \\
&\varepsilon k\left\{\int_0^\infty x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1}\widetilde{f}^p(x)\,dx\right\}^{\frac{1}{p}}\left\{\int_0^\infty x^{q\left(1+\frac{\alpha-\beta}{2}\right)-1}\widetilde{g}^q(x)\,dx\right\}^{\frac{1}{q}} = k.
\end{aligned}
$$

It follows that $\frac{\pi}{\alpha+\beta}\ge k$, which contradicts the hypothesis. Hence the constant factor $\frac{\pi}{\alpha+\beta}$ in (15) is the best possible. The theorem is proved. ☑

Theorem 3. *Let $f\ge 0$ such that $\int_0^\infty x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1}f^p(x)\,dx<\infty$. Then we have the following inequality, which is equivalent to* (15)*:*

$$
\begin{aligned}
J := \int_0^\infty y^{-\frac{p(\alpha-\beta)}{2}-1}\left[\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta}\arctan\left(\frac{x}{y}\right)^\gamma f(x)\,dx\right]^p dy > \\
\left(\frac{\pi}{\alpha+\beta}\right)^p\int_0^\infty x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1}f^p(x)\,dx, \quad (19)
\end{aligned}
$$

where the constant factor $\left(\frac{\pi}{\alpha+\beta}\right)^p$ is the best possible.

Proof. Let us define

$$
g(y) = y^{-\frac{p(\alpha-\beta)}{2}-1}\left[\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta}\arctan\left(\frac{x}{y}\right)^\gamma f(x)\,dx\right]^{p-1},
$$

with $y\in(0,\infty)$. Applying (15) (see Theorem 2), we have

$$
\begin{aligned}
J = I &= \int_0^\infty \int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta} \arctan\left(\frac{x}{y}\right)^\gamma f(x)\, g(y)\, dx\, dy \\
&> \frac{\pi}{\alpha+\beta} \left\{ \int_0^\infty x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1} f^p(x)\, dx \right\}^{\frac{1}{p}} \left\{ \int_0^\infty y^{q\left(1+\frac{\alpha-\beta}{2}\right)-1} g^q(y)\, dy \right\}^{\frac{1}{q}} \\
&= \frac{\pi}{\alpha+\beta} \left\{ \int_0^\infty x^{p\left(1+\frac{\alpha-\beta}{2}\right)-1} f^p(x)\, dx \right\}^{\frac{1}{p}} \times \\
&\qquad \left\{ \int_0^\infty y^{-\frac{p(\alpha-\beta)}{2}-1} \left[\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta} \arctan\left(\frac{x}{y}\right)^\gamma f(x)\, dx \right]^p dy \right\}^{\frac{1}{q}}.
\end{aligned}
$$

Finally

$$
\begin{aligned}
&\left\{ \int_0^\infty y^{-\frac{p(\alpha-\beta)}{2}-1} \left[\int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta} \arctan\left(\frac{x}{y}\right)^\gamma f(x) dx \right]^p dy \right\}^{\frac{1}{p}} \\
&\qquad > \left(\frac{\pi}{\alpha+\beta}\right) \left\{ \int_0^\infty x^{p(1+\frac{\alpha-\beta}{2})-1} f^p(x) dx \right\}^{\frac{1}{p}}.
\end{aligned}
$$

Hence (19) is valid.

On the other hand, suppose that (19) is valid. By the reverse Hölder's inequality with weight, we find

$$
\begin{aligned}
I &= \int_0^\infty \left[y^{-\frac{(\alpha-\beta)}{2}-\frac{1}{p}} \int_0^\infty \frac{(\min\{x,y\})^\alpha}{(\max\{x,y\})^\beta} \arctan\left(\frac{x}{y}\right)^\gamma f(x)\, dx \right] \times \\
&\qquad\qquad \left[y^{\frac{(\alpha-\beta)}{2}+\frac{1}{p}} g(y) \right] dy \qquad (20) \\
&\geq J^{\frac{1}{p}} \left\{ \int_0^\infty y^{q\left(1+\frac{\alpha-\beta}{2}\right)-1} g^q(x)\, dx \right\}^{\frac{1}{q}}.
\end{aligned}
$$

Then by (19), we have (15). Thus (15) and (19) are equivalent. It is clear that the constant factor in (19) is the best possible. Otherwise, by (20), we may get a contradiction that the constant factor in (15) is not the best possible. This completes the proof of Theorem 3. ☑

***Remark* 4.**

i) For $\alpha = 0$, $\beta = 1$ in Theorem 2, we have the reverse of (5) as

$$\int_0^\infty \int_0^\infty \frac{\arctan\left(\frac{x}{y}\right)^\gamma}{\max\{x,y\}} f(x)\, g(y)\, dx\, dy > \pi \left\{ \int_0^\infty x^{\frac{p}{2}-1} f^p(x)\, dx \right\}^{\frac{1}{p}} \left\{ \int_0^\infty x^{\frac{q}{2}-1} g^q(x)\, dx \right\}^{\frac{1}{q}}.$$

ii) For $\alpha = \beta$ in Theorem 2, we have (6).

iii) For $\gamma = 0$ in Theorem 2, we have (7).

Acknowledgement. The authors wish to thank the anonymous referee for through review and insightful suggestions.

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(Recibido en abril de 2011. Aceptado en septiembre de 2011)

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