EARTHQUAKE WAVES AND THE MECHANICAL PROPERTIES OF THE EARTH'S INTERIOR

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It has come to be realized in the present century that the science of Seismology, in addition to providing information on the nature and characteristics of earthquakes, is a central source of knowledge of the mechanics and layering of the Earth's deep interior.

An earthquake is associated with the sudden release of a large quantity of energy inside a confined focal region below the Earth's surface. With the largest earthquakes, those of magnitude near 8½, the energy reaches the order of 10^{24} to 10^{25} ergs. The energy arises from the accumulation of strain inside the focal region, and spends itself in the form of earthquake, or seismic, waves which travel out in all directions from the focus.

There are two types of bodily seismic waves which can penetrate the deepest interior. These are the primary or *P* waves, which are compressional or dilational, and the secondary or *S* waves which are rotational or transverse. Their velocities α and β are given to good accuracy by the formulae

$$\alpha^2 = (\kappa + 4\mu/3)/\rho, \qquad (1)$$
$$\beta^2 = \mu/\rho, \qquad (2)$$

respectively. In (1) and (2), ρ denotes the density, κ the incompressibility (or bulk-modulus), and μ the rigidity.

The incompressibility is a measure of the resistance of a material to symmetrical pressure, and the rigidity a measure of the resistance to distortional stress. On the theory of perfect elasticity, the rigidity of a fluid is zero, and therefore the *S* velocity is zero. In ordinary

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solids, including rocks of the Earth, *S* waves travel at about two-thirds the speed of *P* waves. When the terms 'solid' and 'fluid' are used in respect of the interior of the Earth, they relate to the assumption of perfect elasticity, and this assumption is shown from measurements of seismic waves to be adequate for stress variations with periods of the order of those involved in earthquake wave transmission.

One of the notable seismological results of the present century has been the evolution of reliable travel-time tables for seismic waves. These tables give the times of travel of *P* and *S* waves from an earthquake focus to points of the Earth's surface. The precision reached by 1940 enabled the values of the *P* velocity α to be derived to close accuracy throughout the whole Earth, and of the *S* velocity β down to a depth of 2.900 km.

In addition, it is known that the rigidity between depths of 2.900 and 5.000 km is very small compared with the incompressibility. This result comes from seismic evidence, combined with evidence on the precession of equinoxes and the observed tidal yielding of the solid Earth. It is therefore legitimate to treat the *S* velocity as zero in this part of the Earth.

It follows from equations (1) and (2) that observations of records of earthquake waves lead to fairly well determined values of the expressions κ/ρ and μ/ρ down to a depth of nearly 5.000 km, i. e. throughout nearly 99% of the Earth's volume. (The mean radius of the Earth is 6.371 km). Thus seismology supplies, fairly directly, basic information on three important quantities (ρ, κ and μ) concerned with the mechanics of the Earth's interior.

Further, by noting the depths at which the *P* and *S* velocities, or their gradients, change suddenly, it is possible to divide the Earth up into a series of concentric regions. The values in the table below are based on a formal solution derived by Jeffreys in 1940. The nomenclature *A, B, . . ., G* was introduced by the writer in 1942, and has come to be widely used.

Some of the boundaries indicated in the table are much better determined than others, and the following discussion will contain comments both on the principal historical events connected with the table, and on the degree of reliability of the detail shown.

Region	Name	Range of depth (km)	α (km/sec)	β (km/sec)
A	Crustal layers	0 - 33	Widely variable	Widely variable
B		33 - 410	8.1 - 9.0	4.4 - 5.0
C	Mantle	410 - 1000	9.0 - 11.4	5.0 - 6.4
D′		1000 - 2700	11.4 - 13.6	6.4 - 7.3
D″		2700 - 2900	13.6	7.3
E	Outer core	2900 - 4980	8.1 - 10.4	Assumed zero
F		4980 - 5120	10.4 - 9.5	Not observed
G	Inner core	5120 - 6370	11.2 - 11.3	Not observed

The regions *A, B, C* and *D* (including *D′* and *D″*) together constitute the Earth's *mantle.* Inside the region *A*, the velocities vary more erratically than inside the layers below. The region *A* is commonly referred to nowadays as the *crust,* and the lower boundary is the *Mohorovicic discontinuity,* so-named because of early work on the subject by the Balkan seismologist. A. Mohorovicic in 1909. The thickness of *A* is 30 to 35 km in continental shield areas, somewhat greater in general under larger mountain ranges, but only 5 to 10 km under the principal ocean floors.

The *P* and *S* velocity variations in the regions *B* and *C* are less well determined than in *D,* and the depths of the boundaries between *B, C* and *D* are considerably uncertain (by 100 km or more). The principal uncertainties inside *B* and *C* are in respect of the velocity gradients, the velocities themselves being reliable within about 3%. It is clear that somewhere inside the regions *B* and *C* the gradients are much steeper than the average for the Earth (or possibly the velocities are discontinuous), and work of the writer in 1936 showed that there is significant departure from chemical homogeneity in this part of the Earth (or alternatively phase transitions).

Inside *D,* the *P* and *S* velocities are well determined, and work of Birch indicates that the chemical composition of *D′* is fairly uniform. Calculations by the writer indicate that the density gradient in the region *D″* is about three times the gradient that would be caused by pressure alone, suggesting the accumulation of some denser material near the bottom of the mantle.

The whole mantle is solid (in the sense defined above) —apart from the oceans and isolated pockets of magma— and the rigidity steadily increases downward to the bottom of the mantle, where the value is about four times that of steel at ordinary pressures. The solidity of the mantle is established by the fact that *S* as well as *P* waves are transmitted throughout.

The regions *E*, *F* and *G* together constitute the *central core.* The boundary between *D''* and *E* is sharp. In 1913, Gutenberg calculated its depth as 2.900 km, and Jeffreys showed in 1939 that this value is correct within a few kilometers. There is no evidence of *S* wave transmission below the mantle.

The region *G*, of radius about 1.250 km, is the *inner core.* Its existence was first indicated in 1936 in work by Inge Lehmann of Denmark. The region *E* has come to be called the *outer core.* Work of Jeffreys in 1940 and 1942 pointed to the existence of a transition region *F* between *E* and *G*. The solution of Jeffreys gave a negative *P* velocity gradient (with increase of depth) inside the region *F*.

In 1962, Dr. B. A. Bolt (of Sydney University), using data by the writer and Burke-Gaffney from records of certain hydrogen-bomb explosions and some readings of natural earthquakes by Gutenberg, has produced a new solution for the transition zone between *E* and *G*. According to this solution, the *P* velocity gradient falls to zero between depths of 4.560 and 4.700 km. At 4.700 km, the *P* velocity then suddenly jumps from 10.0 to 10.3 km/sec; and again jumps, from 10.3 to 11.2 km/sec., at 5.150 km depth. From 5.150 km to the center of the Earth, the velocity is fairly constant. A physically important feature of this solution (which is compatible with all the presently available data, but is not necessarily the final solution) is that it removes the need for a negative velocity gradient anywhere in the core. At the same time, the solution requires a more complicated transition region than had been previously envisaged.

The initial work of Lehmann showed that, whatever the fine details of the transition region may turn out to be, there is an effective increase of at least 10% in the value of the *P* velocity α from the outer to the inner core. Inspection of the equation (1) shows that this increase must be due to a fairly sudden increase in either κ or μ, since the density ρ cannot decrease with increase of depth. From a variety of considerations, I have shown that κ is not likely to increase sufficiently to account for the increase in α. Hence it follows that there is likely to be an increase in the rigidity μ between outer and inner core. This carries the implication, which I set down in 1946, that the inner core, unlike the outer core, is solid (in the sense above defined). I have estimated that the rigidity of the inner core probably lies between 2 and 4 times that of steel at ordinary pressures.

As already mentioned, knowledge of the P and S velocity values in the Earth gives knowledge of the values of κ/ρ and μ/ρ. By bringing to bear additional information, including data on the Earth's mass and moment of inertia and various supplementary data, I made a determination in 1936 (which I have refined over the years) of the separate distributions of ρ, κ and μ throughout the Earth, as well as of other mechanical properties such as the pressure p and the gravitational attraction g per unit mass.

Starting from an assumed value of 3.3 g/cm^3 for the density just below the crust, I have calculated that the density increases to a value near 5½ g/cm^3 at the bottom of the mantle, then jumps suddenly to about 9½ g/cm^3 at the top of the central core, and reaches 11½ g/cm^3 at the bottom of the region E. The density in the inner core is less certainly determined, but I have shown that the central density must be at least 12.3 g/cm^3. In 1942, on the basis of these calculations and taking an arbitrary value of 17.3 g/cm^3 for the central density, I constructed a model set of values of density, incompressibility and rigidity for the Earth which is called Model A.

A remarkable feature in the results of the Model A calculation was the fairly smooth variation of the incompressibility κ with the pressure p below a depth of 1.000 km in the Earth. This led me in 1950 to construct a second model, Model B, in which smooth variation of κ with p was taken as a central postulate. The numerical detail in Model B is not greatly different from that in Model A; Model B differs from Model A principally in that κ is entirely continuous at the mantle-core boundary and that the density variation is rather different in the outermost 400 km of the Earth.

An important result of the Model B calculations is that, on the Jeffreys P velocity distribution of 1940, a density of at least 18 g/cm^3 is entailed at the Earth's centre. On the velocity distribution lately derived by Bolt for the core, it becomes possible, however, to lower the Earth's central density to 15 to 16 g/cm^3. The latter figure is more in line with surmises from other types of evidence.

The calculations on the density ρ, in addition to yielding values of κ and μ, also led to determinations of the distribution of the pressure p, and the gravitational attraction g throughout the Earth. At the base of the mantle, the pressure is reliably determined as about 1⅓ million atmospheres, and at the centre as between 3½ and 4 million atmospheres.

The value of g keeps within about 1% of 990 cm/sec^2 between the Earth's surface and a depth of 2.400 km, rises to a maximum of about 1.040 cm/sec^2 at the base of the mantle, and then sinks steadily to zero at the centre of the Earth.

The distributions of density and pressure throughout the Earth are indicated in Fig. 1, and of the incompressibility and rigidity in Fig. 2. Fig. 3 gives an artist's impression of the interior of the Earth as revealed by seismology.

REFERENCES

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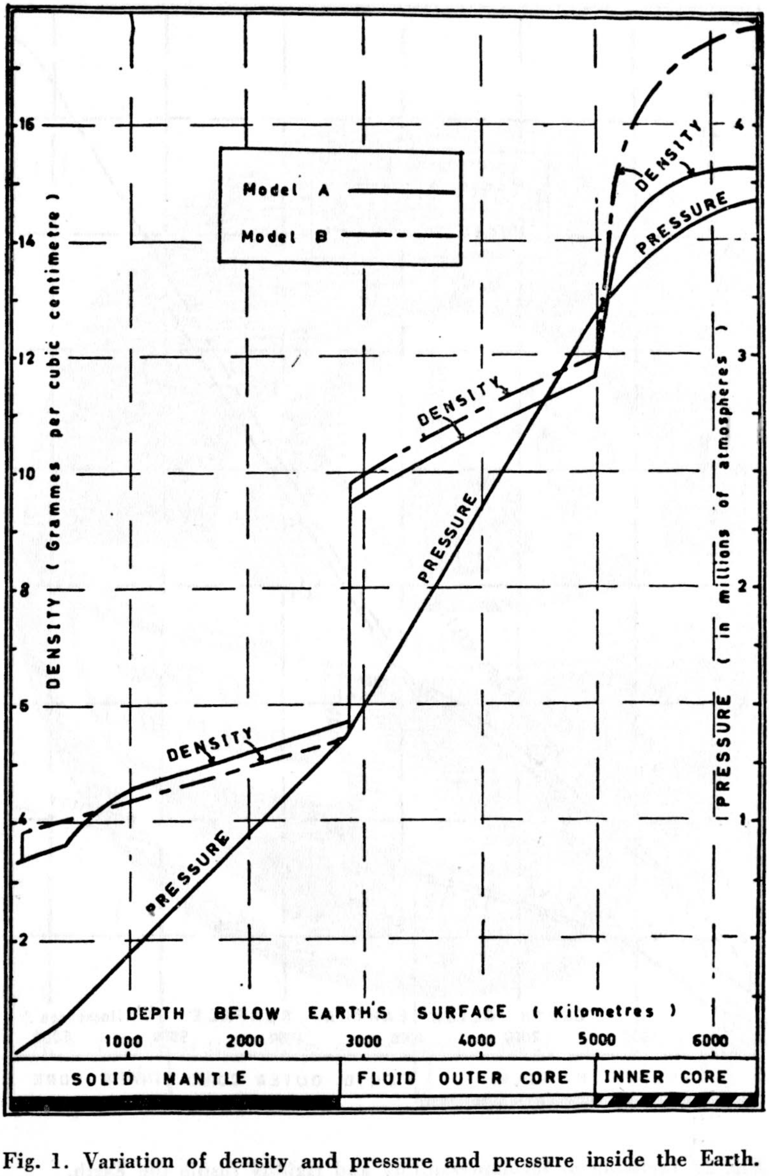

Fig. 1. Variation of density and pressure and pressure inside the Earth.

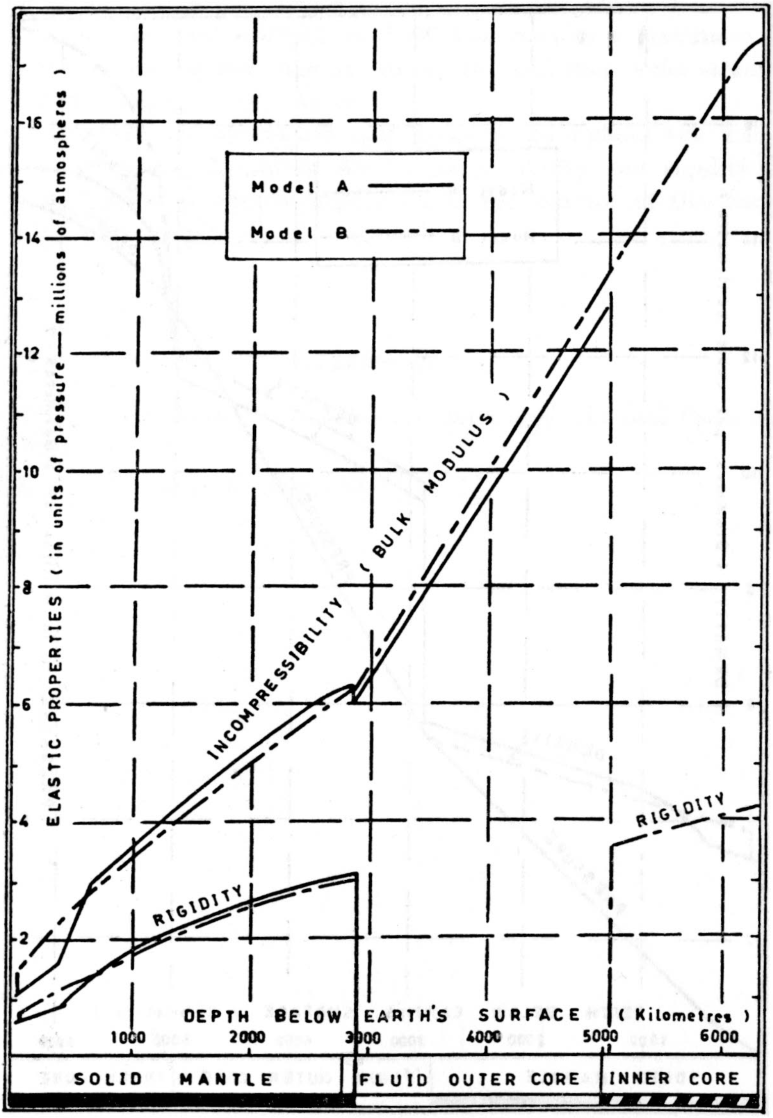

Fig. 2. Variation of incompressibility and rigidity inside the Earth.

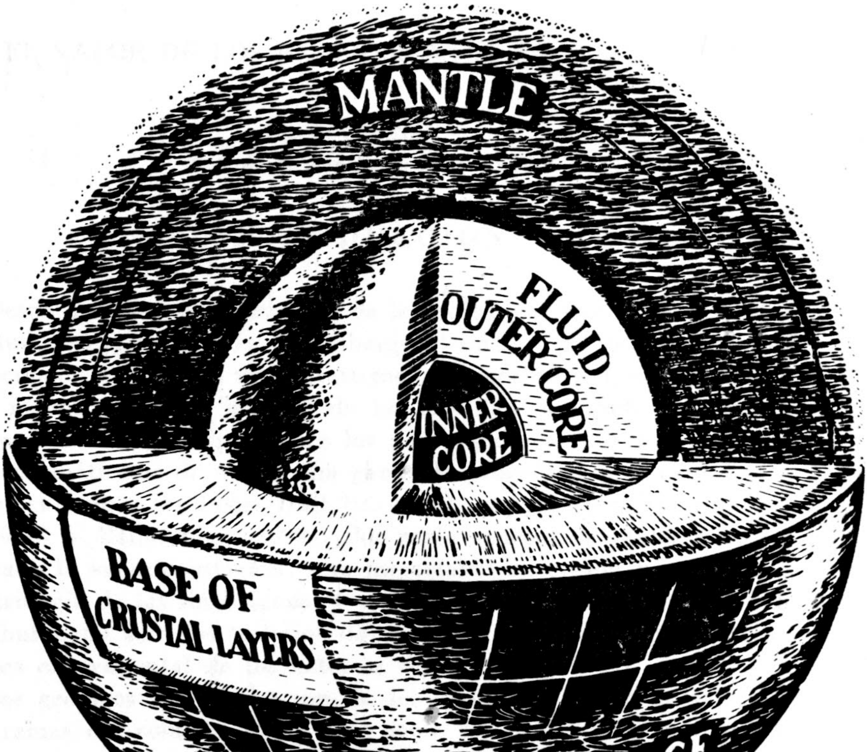

Fig. 3. Picture of the interior of the Earth as broadly revealed by studies of earthquake waves.