Matching between supersymmetric effective theory of inflation and pure de Sitter SUGRA

Matching between supersymmetric effective theory of inflation and pure de Sitter SUGRA

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Contents

To Nature, for being so confusing.

Preface

Symmetries are important to understand Nature. More specifically, the study of symmetries and symmetry breaking are topics of considerable relevance among physicist and astronomers who do research on fundamental physics. Gauge symmetries, in particular, are at the heart of contemporary physics and they play a crucial role on most of the successful theories that we use to describe Nature. For instance, the strong interaction is properly described by Quantum Chromodynamics where it is supposed that an inner symmetry of Nature is represented by the symmetry group $SU(3)$. Another successful theory portrays the electromagnetic and weak interactions as different manifestations of a unified electroweak interaction with its symmetry group being $SU(2) \times U(1)$. Together, they constitute the Standard Model of particle physics, several predictions of this model have been exhaustively tested in contemporary experiments, nowadays we talk of irresistible rise of the Standard Model, even if this could mean a big desert in a very large interval of energies, referring to the fact that no other theory allows to explain phenomenology there, even if the theory require an ultraviolet completion.

Massless vector theories require invariance under space dependent transformations belonging to an infinite dimensional Lie group because Poincaré invariance is tightly related to gauge invariance. For instance, under a general Lorentz transformation $\Lambda^\mu{}_\nu$ the polarization vectors of momentum $\mathbf{p}$ transform as [1]

$$\varepsilon^\mu_{\pm 1}(\mathbf{p}_\Lambda)e^{\pm i\theta(\mathbf{p}_\Lambda)} = \Lambda^\mu{}_\nu \varepsilon^\nu_{\pm 1}(\mathbf{p}) + p^\mu \Omega_{\pm 1}(\mathbf{p}_\Lambda) \tag{0.1}$$

where ± 1 denote the helicity state and the angle θ is related to the Wigner's little group for massless particles which is $SO(2)$.

There is, however, one important issue about gauge symmetries that makes it difficult to understand the notion of gauge theories. It turns out that gauge symmetries are not symmetries in the formal sense or, more precisely, we regard them as redundancies of the theoretical formalism. We interpret gauge symmetries not as empirical features of Nature but as purely formal properties of gauge theories; by most accounts, we say that gauge symmetries exist because different configurations of the fields involved in such theories represent identical physical situations. The notion of gauge redundancies may seem confusing at first glance, for it seems straightforward to ask what may be the meaning of a symmetry that exists only at the level of our formal description of physical reality but not at the level of reality itself.

For example, at the level of the Feynman path integral quantization, a gauge symmetry provide an ill definition of the functional integral due to the introduced over-counting of the fields configurations on which we need to integrate. This redundancy is fixed by the DeWitt-Faddeev-Popov method, where a gauge choice is introduced and the quantized gauge theory provide gauge invariant results in a perturbative formulation with many cancellations that strongly slow the reaching of predictions.

In this sense, the idea of gauge in Nature is a nuisance since it is a symmetry which has nothing to do with the organization in multiplets of the spectrum and, more generally, it does not directly address to physical states, except for separating them by non physical excitation that the process of quantization introduce. The organization of the spectrum in multiplets is a duty of the global symmetries by means of the Noether theorem. That is why global symmetries actually have empirical meaning. In this line of thought gauge symmetries are not strictly symmetries of quantum mechanics realized by a unitary or an anti-unitary operator (Wigner theorem), which has the role of preserving the transition probability.

A gauge theory is regarded as any theory whose physical content is preserved by a transformation represented by an infinite dimensional Lie group[1] and whose parameter of transformation is local, i.e. it is space-time dependent. This last point is of significant relevance in the context of gauge theories that describe fundamental interactions, it is also the reason of why gauge symmetries, which we understand as redundancies of the theoretical formalism, are tied with physical reality, for example interactions, and therefore they are not merely theoretical scrap.

To further illustrate this point, consider a theory for a field ϕ that is invariant under a global symmetry transformation that acts on the field as

$$\phi \longrightarrow \exp(i\Lambda)\phi, \tag{0.2}$$

and whose Lagrangian density $\mathcal{L}_\phi$ is left invariant under such transformation. The parameter Λ is the generator of the transformation, which does not depend on the specific space-time point where ϕ is evaluated. One can weaken this condition and promote the parameter Λ to a local parameter $\Lambda(x)$, evidently the derivative $\partial_\mu\Lambda(x)$ is no longer zero and therefore the Lagrangian $\mathcal{L}_\phi$ does not manifest invariance anymore:

$$\mathcal{L}_\phi \longrightarrow \mathcal{L}_\phi + \text{terms with } \partial_\mu\Lambda(x). \tag{0.3}$$

However, we know that there are physical systems which are invariant under a symmetry transformation whose parameter is

[1] Roughly speaking, a gauge symmetry.

space-time dependent[2], thus there must be a method to formulate theories to describe such systems. One way is to restore by hand the invariance of the action under the transformation (0.3) by introducing a new vector field A with the transformation property

$$A \longrightarrow A + \partial_\mu \Lambda(x). \tag{0.4}$$

This property is convenient for two reasons: it allows us to find a Lagrangian $\mathcal{L}_A$ for the field A that is manifestly invariant under the local transformation (0.4) by itself. Moreover, the field A will be useful to cancel out the unwanted terms that are present on the transformation (0.3). The composed action for ϕ and A has the Lagrangian density

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_{\mathrm{A}}, \tag{0.5}$$

which is invariant only under the global symmetry transformation and not under the local one. However, by including an interaction term $\mathcal{L}_{\text{int.}}$ that couples ϕ with A, with a specific form such that its variation satisfies

$$\delta\mathcal{L}_{\text{int.}} = -\text{terms with } \partial_\mu \Lambda(x), \tag{0.6}$$

then, under this assumption, the complete Lagrangian

$$\mathcal{L}_{\text{gauge}} = \mathcal{L}_\phi + \mathcal{L}_A + \mathcal{L}_{\text{int.}}, \tag{0.7}$$

will be invariant under the local gauge transformation. The action now describes a model for a particle ϕ that interacts with the boson A, the latter was introduced only because we wanted

[2] For example, Maxwell's equations are invariant under the local transformation of the vector potential $\mathbf{A} \longrightarrow \mathbf{A} + \nabla\boldsymbol{\Lambda}$. Since the magnetic field is $\mathbf{B} = \nabla \times \mathbf{A}$, then the equation $\nabla \times \mathbf{B} = 0$ is still valid after the transformation of the vector potential no matter if $\nabla\boldsymbol{\Lambda}$ depends on the coordinates or not.

to restore the gauge symmetry, for this reason we call this particle gauge boson. From this analysis, we feel that interactions are a physical implication of promoting a global symmetry, represented by a Lie group, to a local symmetry; in this sense gauge redundancies display physical implications.

Despite the philosophical and empirical discussions about the reality of gauge theories [2, 3], many predictions have been derived from the formalism behind these theories. Take as an example Quantum Chromodynamics (QCD), which is a renormalizable non-Abelian gauge theory for quark and gluon fields that are invariant under color-$SU(3)$ transformations. The theory provides a compelling explanation for the basic features of hadronic physics such as the meson and baryon spectra, the quark statistics and color confinement at large distances, all of these are essential in the description of atomic nuclei [4]. Treating QCD perturbatively is particularly relevant to obtain predictions for large momentum transferred photoproduction, see [5] for a short review.

Other important aspect of gauge theories that involves empirical implications, is the mechanism of how a gauge symmetry brakes at some energy scale. We formulate models of this kind using the formalism of effective field theory.

The present work focuses on a specific process of symmetry breaking: the theory of inflation. Inflation is the period of time where the Universe went through a phase of exponential expansion that stopped. The end of inflation is a signal of a symmetry breaking since it designates a cosmological clock to the Universe. By most accounts, we say that inflation is the theory where time diffeomorphisms are broken while time-dependent spatial diffeomorphisms are preserved. One consequence of particular relevance is the presence of a Goldstone boson, referred to as π, that non-linearly realizes the symmetry. The last point is of great importance when constructing effective theories. Here we provide a general introduction.

We stated that inflationary cosmology is the theory where time diffeomorphisms behaved as a broken symmetry at the beginning of the history of the Universe. To clarify the underlying idea of this concept, let us first comment briefly about the aspects of the symmetry breaking mechanism behind inflation. In the context of quantum mechanics, a symmetry is an automorphism[3] of the observables, or operators, which preserves all algebraic structures, so every possible state of a system is individuated in terms of the expectation value ascribed to some algebraic function of any operator. In the Heisenberg picture, time evolution counts as an algebraic operation, therefore the invariance of all the algebraic relations under an assumed symmetry in this picture implies that the equations for the dynamics of the system are also invariant under the symmetry. More precisely, if a system were to be invariant under a given symmetry, then the equations of motion that describe the evolution of such system must manifest the same invariance.

But Nature does not need to be always symmetric. When one describes a phenomenon by means of an effective theory, the symmetries that were once manifest tend to shatter. There are different ways of how a symmetry breaks, think for example a theory which is described by an action that is invariant under a given symmetry. If, for effective purposes, an additional operator is introduced in the theory, then the action will not necessarily be invariant anymore. In this case we say that the symmetry has been explicitly broken at the action level. If the scale of the operator that breaks the symmetry is sufficiently small compared with the remaining operators, then the symmetry is said to be an approximate symmetry.

[3]In mathematics, an automorphism is a function that maps a mathematical object into itself while preserving all of its structures. However, not all the elements of the mathematical object transform to themselves.

This is not the case for inflation. There is another way to break a symmetry that plays an important role in various thermodynamical phenomena, condensed matter physics, elementary particle physics and, of our particular interest, inflationary cosmology: we are talking about spontaneous symmetry breaking. The idea is that the laws that describe the behavior of a system may be invariant under a symmetry even though the system itself is not. Put differently, this means that a state of a physical system not necessarily have all the symmetries which the laws of motion governing its dynamics have. As anticipated, the theory of inflation falls into this category.

Typically, in a theory of spontaneously symmetry breaking, the fields may not transform linearly under a representation of the symmetry. However, it is possible to perform a field transformation defined in terms of a new parameter with a certain transformation property so that linearity is restored. Consider, for example, a field A that transforms in a nonlinear way as

$$g : A \longrightarrow A' = D(g) \circ A + \mathcal{O}(A^2), \tag{0.8}$$

where $D(g)$ is a representation of the group element g that acts on the vector space spanned by A. If the symmetry group is broken to the subgroup H, then we can redefine A as $\tilde{A}$ so the new field transforms linearly, say

$$g : \tilde{A} \longrightarrow \tilde{A}' = D(g) \circ \tilde{A}, \tag{0.9}$$

finding such redefinition is essential to construct general gauge invariant actions.

Theories of broken gauge symmetry have played a crucial role in the development of physics. Let us recall, for example, the description of a massive W-boson at energies bellow the Higgs mass. A simplified action, that is not gauge invariant, for such model is written as

$$S[A] = \int \mathrm{d}^4x \left[F_a^{\mu\nu} F^a_{\mu\nu} + m^2 A^a_\mu A^\mu_a\right] + \dots, \tag{0.10}$$

where the index a indicates the number of generators of the symmetry group. Notice that here the mass term explicitly breaks gauge invariance and the scale of the symmetry breaking is given by m. If it happens that the parameter m is small compared with the energy scale, then we say that gauge invariance is softly broken[4]. In fact, one can find that at energies above the mass of the states, the spectrum effectively splits into a helicity 1 particle and one additional particle of lower helicity which we identify with the gauge boson of a spontaneously broken symmetry. In this case, introducing the gauge invariance at the Lagrangian level significantly simplifies the description of the theory around a vacuum which is not fully symmetric.

The central idea behind this argument is that when a softly broken gauge symmetry is linearly realized, we say then that the non-linearly realized gauge symmetry is spontaneously broken. Therefore, we can use the tools of spontaneously broken symmetries to describe the physics of an effective theory which is not necessarily around the maximally symmetric vacuum.

In the context of the model for the W-bosons, gauge invariance is restored by the introduction of a set of new fields $\pi = \pi^a t_a$ that we understand as the Goldstone bosons[5], there is one field for each broken generator. We can replace the field A with a new field $\tilde{A}$ defined by

$$\tilde{A}_\mu = e^{-\pi}\left(A_\mu + \frac{i}{g}\partial_\mu\right) e^{\pi}, \tag{0.11}$$

[4]In this case, gauge invariance is an approximate symmetry.

[5]In the global limit, these fields are equivalent with the Goldstone bosons related with the symmetry breaking.

that transforms linearly under $e^{-\pi}$. This field is known as the Stückelberg field and it plays a fundamental role in the analysis of nonlinear realizations. The most important feature, for instance, is that the action written in terms of the Stückelberg field $\tilde{A}$ will be automatically gauge invariant. This can be seen because an additional degree of freedom coming from the gauge boson has been introduced: the action written in terms of $\tilde{A}$ is equivalent to an action for A and π with mixing terms (interactions), we have

$$S[\tilde{A}] = S[A, \pi], \tag{0.12}$$

and π represents the gauge liberty. Notice that the action (0.10) is equivalent to $S[\tilde{A}]$ evaluated in $\pi = 0$, this case is known as the unitary gauge. This prescription will be fundamental during the development of an effective theory of inflation, which is one of the central topics of this work.

We stated that nonlinear realizations and the Stückelberg fields are the main tools to construct effective theories of broken symmetries, for example the theory of Inflation. Of particular interest are the models that describe the primordial fluctuations around a vacuum that is highly constrained by the symmetries of space-time. In this scenario, the Lagrangian describing these fluctuations can be constructed without having information about the mechanism that spontaneously breaks the symmetry. Throughout this thesis we will present a derivation of a Lagrangian of this type, which is of great value for cosmological applications. Furthermore, we focus our work on including an additional ingredient to the core of space-time: Supersymmetry.

For many years, Supersymmetry has been a relevant subject in different branches of physics. Recent interest for this topic has emerge in the context of inflation. Such interest is motivated for, among others, three principal reasons: first, we know that the only non-trivial extension of the Poincaré group corresponds to the addition of a new set of generators that satisfy a graded Lie algebra which, in turn, are required to couple fermionic states with gravity. Second, as we will see later, imposing an equivalence between the bosonic and fermionic degrees of freedom, correctly solves the hierarchy problem of the Higgs mass. One last argument is that the Standard Model does not include any field capable to produce inflation, i.e. dark energy is not incorporated in the Standard Model. With Supersymmetry, and more specifically with Supergravity, dark energy fields emerge naturally to the spectrum of the theory.

If Supersymmetry is a symmetry of Nature, then it must be realized only at high energies. The explanation to this is rather simple: there is no experimental evidence of a symmetry between fermions and bosons at the probed energy scale. In addition, we know that during inflation the invariance under time diffeomorphisms broke down and, consequently, so did Supersymmetry. In this order of ideas, the description of inflation reduces to the description of a symmetry breaking of both time diffeomorphisms and Supersymmetry.

So far, we stressed about how to construct an effective model to describe the primordial fluctuations during inflation, later in this thesis we will develop, in more detail, a model of this kind which, in turn, corresponds with the most general effective Lagrangian describing the density perturbations. A further analysis comes from the fact that we want to incorporate Supersymmetry. In the presence of gravity Supersymmetry is itself gauged, thus leading to Supergravity. One important topic treated during this work is the description of the energy perturbations in the presence of Supergravity. We will show that all the information needed to model the primordial fluctuations is contained in the gauge fields produced by the symmetry breaking: the bosonic field π for the case of time diffeomorphisms breaking and the fermionic field λ for the case of Supersymmetry breaking.

There is, however, a sharp subtlety about every effective theory of broken Supersymmetry. Auxiliary fields must be included in any supersymmetric description of Nature, they are used to equate the on-shell degrees of freedom with the off-shell ones. This presupposes a problem since most of these degrees of freedom are not physical and they must be removed. One standard way to integrate out the auxiliary fields is by taking its equations of motion in the large mass limit, however this is not always a convenient method to eliminate the heavy fields.

In the language of Supergravity, non-physical degrees of freedom are removed by imposing invariant constraints directly on the superfields. The study of such constraints is known as constrained superfields. The main goal of this work is to demonstrate that the formalism used to obtain effective actions based on nonlinear realizations and Stückelberg fields is equivalent to the formalism of constrained superfields. Both approaches consist on an effective description of a symmetry breaking, the difference between them is, by most accounts, conceptual.

Nonlinear realizations describe an effective model by imposing *a priori* the symmetries involved. The result is an effective action within a range of validity, however there is not any information about the UV-complete theory it comes from. On the other hand, one could start from a fundamental theory and then restrict the fields on it to describe the relevant physics at some energy range. This is performed by the formalism of constrained superfield, here the resulting effective theory comes from the elimination of nonrelevant degrees of freedom. We will show that both approaches are, under several considerations, equivalent.

This thesis is organized as follows: in Chapter I we introduce the current state of art concerning inflationary theories in cosmology. There, we will review the effective field theory of inflation and some of its phenomenological implications. In Chapter II we summarize the elements of Supergravity needed in the development of a supersymmetric effective field theory of inflation which will be presented on Chapter III. Chapter IV is devoted to the study of an effective theory of inflation based on the study of constrained superfields. At the end we present the main conclusions and comments about the results.

Inflation

1.1 Starting the Universe

The preferred theory to describe the origin and dynamics of the primordial fluctuations is the theory of inflation [6, 7, 8]. It is proposed that the Universe went through a period of expansion at nearly constant rate given by the Hubble parameter $H \equiv \dot{a}/a$ associated with the Friedmann-Robertson-Walker metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\boldsymbol{x}^2. \tag{1.1}$$

To study the propagation of light in this space-time it is convenient to introduce the *conformal time* as $\mathrm{d}t/a(t)$, so that equation (1.1) becomes

$$\mathrm{d}s^2 = a(\tau)\left[-\mathrm{d}\tau^2 + \mathrm{d}\boldsymbol{x}^2\right], \tag{1.2}$$

so it can be seen as a conformal transformation of the Minkowski metric produced by the scale factor $a(\tau)$.

The scale factor evolves exponentially as $a(t) \propto e^{Ht}$ when the rate of expansion H is constant in time. This exponential behavior stretches the homogeneous initial configuration of the energy density from subhorizon scales to apparently non causally connected superhorizon scales, thus solving the horizon problem. Inflation also predicts the presence of primordial quantum inhomogeneities in the Cosmic Microwave Background (CMB) and their relationship with large-scale structures. In other words, inflation is essentially the mechanism of how the expansion rate $H(t)$ experiences zero-point energy fluctuations, say $\delta H(t)$, leading to spatial variations in the energy density after inflation, namely $\delta\rho(x)$.

It is mandatory to ask what causes inflation. Imagine something with energy density ρ and pressure P that filled the Universe in its initial stages. Assuming that the Universe is both homogeneous and isotropic, then the energy-stress tensor is diagonal and has components $T^0_{\ 0} = \rho$ and $T^i_{\ j} = -P\delta^i_j$. Such assumption is widely accepted in the scientific community and it is known as the cosmological principle. We can, therefore, use the conservation of energy-momentum

$$\nabla_\mu T^{\mu 0} = 0, \tag{1.3}$$

to derive the continuity equation of ρ:

$$\dot{\rho} + 3H(\rho + P) = 0. \tag{1.4}$$

In general, the Hubble parameter H plays the role of a friction and can be a function dependent on time. However, we are interested in the case where the scale of the Universe increases exponentially, therefore we require H to be a constant. From the first Friedmann equation

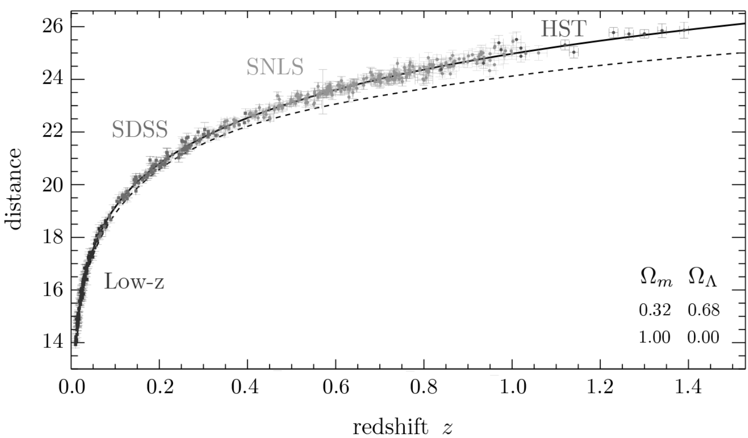
$$3M_{\rm pl}^2 H^2 = \rho, \tag{1.5}$$

derived in Appendix A, it is deduced that the energy density of this substance must be constant, meaning that $\dot{\rho} = 0$. Curiously enough, under these considerations the continuity equation (1.4) implies the atypical equation of state[1]

$$P = -\rho. \tag{1.6}$$

This is not a trivial result in any sense; it means that if we want to include an accelerate phase of expansion at the beginning of time, then the Universe must have been filled with a strange kind of energy that produces negative pressure [9, 10]. Something with that property has not been detected in any current experiment.

In the following plot, taken from [11], it is shown the distance of deep field galaxies observed using type Ia supernovae compared with the redshift $z = (1-a)/a$ they present:



According to the data observed by the Supernova Legacy Survey (SLS) [12], the Hubble Space Telescope (HST) and the Sloan

[1] We are also taking into account the strong energy condition and that the Hubble parameter is non-zero.

Digital Sky Survey (SDSS) [13], the Universe is currently entering through a phase of accelerated expansion with constant Hubble constant. Such strange behavior can be explained by the presence of dark energy, which is the name given to a substance with negative pressure.

In the figure, two different theoretical predictions are plotted: one considers that the ratio of matter energy and total energy is $\Omega_m = 1.00$ and the other considers that the same ratio is $\Omega_m = 0.32$. The difference between a Universe filled with ordinary matter with one filled, in part, with dark energy is clearly visible. Observations of this kind are considered as indirect evidence of the presence of energy with the property mentioned in equation (1.6). The interpretation of this result as indirect evidence of dark energy was first suggested in [14, 15] using less accurate measurements.

The current stage of experimental observations supports the Standard Model of Cosmology, namely the ΛCDM model, where it is proposed that the Universe is filled with 68% dark energy, 27% dark matter and 5% ordinary atoms[2]. In addition, it is believed that the origin of the Universe and, in particular, the presence of energy inhomogeneities is explained by the theory of inflation.

Inflation cosmology is based on the presence of a fundamental real scalar field ϕ, called the *inflaton*, that is coupled with gravity by the geometry (1.1) and whose dynamics are regulated by some potential $V(\phi)$ that satisfies the energy condition (1.6).

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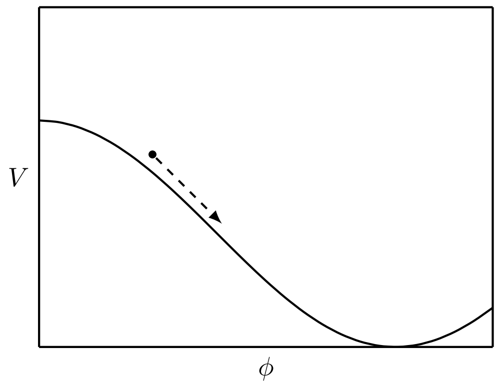
[2]We include on this percentage the energy density coming from radiation, which corresponds to only $10^{-4}\%$ of the content of the Universe.



The dynamics of the scalar field slow-rolling downhill the potential is the cause of inflation. The simplest scenario is given by the action

$$S_\phi = \int \mathrm{d}^4x \sqrt{-g}\mathcal{L}_\phi, \tag{1.7}$$

where the Lagrangian consists only of the kinetic term and the potential V, which we write here as

$$\mathcal{L}_\phi = \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi). \tag{1.8}$$

The action gives us the essential information about the energy density and pressure of the scalar field. According to the principle of stationary action, after making a small change $\delta g^{\mu\nu}$ in the metric produced by a change in the coordinate system, the action has a variation equivalent to[3]

$$\delta S_\phi = \int \mathrm{d}^4x \frac{\sqrt{-g}}{2}\left[2\frac{\partial\mathcal{L}_\phi}{\partial g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}_\phi\right]\delta g^{\mu\nu}, \tag{1.9}$$

[3]To be clear, equation (1.9) was obtained using the relation $\delta g/g = g^{\mu\nu}\delta g_{\mu\nu} = -g_{\mu\nu}\delta g^{\mu\nu}$.

and we expect this quantity to be invariant. The conserved current related with the required invariance can be obtained directly from Noether's theorem, this current is the stress tensor

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}(\partial\phi)^2 - V(\phi)\right), \tag{1.10}$$

which, in the isotropic case, satisfies the relations $T^0_{\ 0} = \rho$ and $T^i_{\ j} = -P\delta^i_j$. A careful replacement of these relations in equation (1.10) leads to an energy density which is simply the sum of the kinetic and potential energy densities

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{1.11}$$

and a pressure density which corresponds to the difference between the kinetic and potential energy densities

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{1.12}$$

We infer that a field configuration leads to inflation if the potential energy dominates over the kinetic energy and therefore $P/\rho \longrightarrow -1$, that is why this is called slow-roll inflation.

Now that the mechanism of inflation has been studied, we want to quantify the amount of expansion produced by the inflation. For that, we introduce a quantity N known as the number of *e-folds*; it is the natural logarithm of the fraction between the size of the Universe when inflation ended and its size when inflation started, it is simply

$$N = \int_{t_{\text{start}}}^{t_{\text{end}}} H\text{d}t = \ln\frac{a(t_{\text{end}})}{a(t_{\text{start}})}. \tag{1.13}$$

The largest scales of the CMB suggest that inflation ended after approximately 50 e-folds of expansion.

Since inflation actually stopped[4], then the Universe is not perfectly described by a de Sitter background and therefore $\dot{H}$ cannot be exactly zero, instead it must satisfy

$$-\dot{H} \ll H^2, \tag{1.14}$$

so, the de Sitter approximation is still valid, thus keeping the accelerated expansion, but also designating an end of the inflationary period. The former condition produces one restriction on the field that can be interpreted using the parameter ε defined as the dimensionless ratio

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \ll 1. \tag{1.15}$$

From the continuity equation (1.4) and the Friedmann equation (1.5) it is possible to show that ε is in fact the ratio between the kinetic energy of the scalar field and one third of the total energy

$$\varepsilon = \frac{\dot{\phi}/2}{M_{\mathrm{pl}}^2 H^2}, \tag{1.16}$$

this constraint also verifies the condition of slow-rolling. Inflation is over when the kinetic energy of the field exceeds the potential energy and $\varepsilon \sim 1$, at this point the background is not de Sitter anymore.

One other condition comes from the fact that inflation must last enough time to expand the Universe to at least its observable size, if the quantity

$$\eta \equiv \frac{\dot{\varepsilon}}{H\varepsilon} \tag{1.17}$$

satisfies $|\eta| \ll 1$, the change of ε per Hubble time will be small and inflation will persist. Once we assume the slow-roll solution,

[4] At least as far as we observe.

it is possible to translate the previous inequalities in terms of the potential, these are the renown slow-roll conditions:

$$\frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \qquad \text{and} \qquad M_{\rm pl}^2\frac{|V''|}{V} \ll 1. \tag{1.18}$$

Different kinds of potentials that satisfy (1.18) have been proposed, thus establishing a broad spectrum of inflationary theories from which we highlight the chaotic inflation [6], the natural inflation [16] and the hilltop inflation [17].

– **Chaotic inflation.** An important class of inflationary models arise when the potential is simply a power of ϕ multiplied by a mass-scale parameter μ

$$V(\phi) = \mu^{4-p}\phi^p. \tag{1.19}$$

– **Natural inflation.** It has been considered that the field responsible of inflation is a pseudoscalar field called *axion*. The action is endowed with a decay rate f. From a top-down perspective, the action should arise from string theory and the decay rate must exceed the Planck scale. The potential is

$$V(\phi) = \frac{V_0}{2}\left[1 - \cos\left(\frac{\phi}{f}\right)\right]. \tag{1.20}$$

– **Hilltop inflation.** Consider that inflation occurs near a point in field space with $V' = 0$. Expanding around this point, the potential takes the form

$$V(\phi) = V_0\left[1 + \frac{1}{2}\frac{m^2}{V_0}\phi^2 + \ldots\right], \tag{1.21}$$

and the symmetry around V_0' is spontaneously broken when m^2 is negative and the higher order terms in the expansion become relevant for large ϕ.

Different interpretations have been made regarding the scalar field ϕ. Of particular interest, for example, is the model developed by F. Bezrukov and M. Shaposhnikov in [18], where it is argued that the Higgs boson of the Standard Model can lead to inflation and produce cosmological perturbations if it is minimally coupled with gravity.

Sometimes, one field sliding down a smooth potential is not a complete description of inflation. Some authors have inquired in the use of multiple fields of inflation [19] based on the idea of assisted inflation developed in [20]. This is essentially a dimensional extension of the problem leading to a more complex description of the dynamics. An additional consideration that extend these models is to assume a non-minimal coupling with gravity. Later on this Chapter we will explain how the potential of the real scalar field ϕ and its coupling with the metric can be related with observable quantities.

Leaving the potential aside we address a different issue: gravity. The scalar fields ϕ gravitates and therefore a suitable description of gravity at low energies is necessary to completely recast the physics of inflation.

1.2 Gravity

In inflationary theories gravity is evidently very important, its presence indicates that the Poincaré group is gauged so that the gravitational interaction comes from the gauge redundancy, which is not formally a symmetry. The degrees of freedom coming from this redundancy parametrize the gravitational field, however there is not a UV complete theory of gravity and therefore we treat it as an effective quantum theory.

What we know about gravity is that its low energy degrees of freedom and interactions are those described by general relativity, however we do not need to know what is the complete form of the full theory, instead we assume that this theory has a low energy limit that looks like the world around us.

One clever observation about gravity is that, in essence, it is one of the four fundamental interactions of nature, and being that way, we feel that its physical description has a resemblance with a gauge theory. In [21] it was proposed that general relativity can be seen as a gauge theory based on the local Lorentz group in the same manner Yang-Mills theories were formulated from the gauging of the internal isospin symmetry group $SU(2)$. In this prescription, the Yang-Mills gauge fields are analogous to the Riemannian connection in gravity and in this sense the curvature of space-time is the source of the gravitational potential.

Lorentz invariance is a general coordinate transformation which leaves the Minkowski metric invariant, the transformation is defined using the Λ matrices such that

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu}. \tag{1.22}$$

The set of all orthogonal Λ matrices satisfying

$$\det\Lambda = \pm 1 \tag{1.23}$$

$$\Lambda^{0}{}_{0} = \pm 1, \tag{1.24}$$

form the not connected Lorentz group L, it consists of four disjoint subgroups depending on the signs designation of the relations (1.23) and (1.24)[5].

If we intend to promote Lorentz invariance to a local symmetry, we better recall Einstein's equivalence principle in which local space-time structure is identified with Minkowski space possessing Lorentz symmetry. The local transformation is

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu}(x) x^{\nu}, \tag{1.25}$$

[5]More precisely, the physically relevant subgroup is the proper orthochronus Lorentz group $SO(3,1)$ in which we fix $\det\Lambda = \Lambda^{0}{}_{0} = +1$.

consequently, the metric is now a coordinate dependent 2-form with components $g_{\mu\nu}(x)$.

As in Yang-Mills gauge theories, here the locality of the symmetry induces an interaction. The explanation is rather geometrical: since for every space-time point there is a Lie group associated, we can define a principal L-bundle with the space-time manifold being its base. The general result is that we induce a connection in the cotangent space which has an associated curvature. In turn, the regular space-time derivatives are no longer covariant, this is not a real issue since a covariant derivative with the right transformation properties can be defined. We interpret the curvature manifested in the new derivative as the source of the gravitational interaction.

To describe in more detail the effects of having a local metric field $g_{\mu\nu}(x)$, consider the infinitesimal coordinate transformation $x'^{\mu} = x^{\mu} - \xi(x)$ where $\xi(x)$ is an arbitrary infinitesimal vector function. The transformation of the gravitational field is

$$g'_{\mu\nu}(x') = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta}(x), \tag{1.26}$$

and this allows us to determine the infinitesimal variation under general coordinate transformation (G) of the metric, namely $\delta_G g_{\mu\nu}(x)$, evaluated on the same space-time point. The variation is equivalent to

$$\begin{aligned} \delta_G g_{\mu\nu}(x) &= g'_{\mu\nu}(x) - g_{\mu\nu}(x) \\ &= \xi^{\alpha}\partial_{\alpha} g_{\mu\nu} + \partial_{\mu}\xi^{\alpha} g_{\alpha\nu} + \partial_{\nu}\xi^{\alpha} g_{\mu\alpha} \\ &= D_{\mu}\xi_{\nu} + D_{\nu}\xi_{\mu}, \end{aligned} \tag{1.27}$$

where D_{μ} represents the covariant derivative that satisfies $D_{\lambda} g_{\mu\nu} = 0$.

In order to relate the curved space-time with the local Lorentz symmetry, we need to weld the external curved space with local space. We introduce the vierbein field ${e_a}^\mu(x)$ – with inverse ${e_\mu}^a$ – that constitutes a transformation that maps local Lorentz (nonholonomic coordinates x^a) to external space-time (holonomic coordinates x^μ). We are assigning D independent vectors[6] at each space-time point that satisfy

$$ {e_a}^\mu(x){e_b}^\nu(x)g_{\mu\nu}(x) = \eta_{ab}, \qquad \eta_{ab} = \mathrm{diag}(-1,1,1,1). \tag{1.28} $$

The Greek $\mu,\ \nu,\ \ldots$ indices are called the world indices while the Latin indices $a,\ b,\ \ldots$ are called the local Lorentz indices.

Tensor fields with local Lorentz indices transform under local Lorentz transformation through the matrices ${\Lambda^a}_b$, which in infinitesimal form are expressed as ${\Lambda^a}_b(x) = \delta^a_b + {\lambda^a}_b(x)$, with ${\lambda^a}_b(x)$ being an infinitesimal transformation parameter. For instance, the vierbein transforms under general coordinate (G) and local Lorentz (L) as

$$ \delta_G {e_\mu}^a = \xi^\nu \partial_\nu {e_\mu}^a + \partial_\mu \xi^\nu {e_\nu}^a \tag{1.29a} $$
$$ \delta_L {e_\mu}^a = -{\lambda^a}_b {e_\mu}^b. \tag{1.29b} $$

The statement that follows from the former discussion is: if the Lorentz group is gauged only by space-time diffeomorphisms (G), then the degrees of freedom of the graviton are encoded in the metric $g_{\mu\nu}(x)$, whereas the vierbein $e^a_\mu(x)$ parametrizes the degrees of freedom in the case where the Lorentz group is gauged by the tensor product of space-time diffeomorphisms and local Lorentz invariance ($G \times L$).

So far, we have avoided, on purpose, how to obtain an expression of the covariant derivative directly from the imposed symmetry. We did this simply because we want to emphasize in the differences of the covariant derivative in both the G and $G \times L$ cases.

---

[6] D is the number of dimensions. In particular, for physical purposes we choose $D = 4$.

We argued that the gauge redundancy induces a connection that must be included in the way we take derivatives in order to hold covariance. For instance, the covariant derivative of a vector field V^μ that transforms properly under general coordinate transformation (G) is

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\alpha} V^\alpha, \tag{1.30}$$

where Γ designates the torsion-less connection that restores the covariance in the derivative $D_\mu V^\nu$. With this requirement one can find the connection to be

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\alpha} \left(\partial_\mu g_{\nu\alpha} + \partial_\nu g_{\mu\alpha} - \partial_\alpha g_{\mu\nu} \right). \tag{1.31}$$

The curvature associated with Γ is defined from the Riemann tensor

$$R_{\mu\nu}{}^{\rho}{}_{\sigma} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}, \tag{1.32}$$

and corresponds to the Ricci scalar

$$R = R_{\mu\nu}{}^{\mu\nu}. \tag{1.33}$$

The vierbein $e_\mu{}^a$ behaves in an analogous way. In this case, the gauge field of local Lorentz transformations is the spin connection $\omega_\mu{}^a{}_b(x)$ from which we define the covariant derivative of a vector V^a to be

$$D_\mu V^a = \partial_\mu V^a + \omega_\mu{}^a{}_b V^b, \tag{1.34}$$

this quantity transforms in the same way as V^a under Local Lorentz transformation. This covariant derivative can be also applied to a spinor χ in the Weyl representation, in this case the connection is contracted with the Lorentz group generators σ^{ab}

defined in (B.12a) and (B.12b):

$$D_\mu \chi = \partial_\mu \chi + \frac{1}{2}\omega_\mu{}^{ab}\sigma_{ab}\chi \tag{1.35a}$$

$$D_\mu \bar{\chi} = \partial_\mu \bar{\chi} + \frac{1}{2}\omega_\mu{}^{ab}\bar{\sigma}_{ab}\bar{\chi}. \tag{1.35b}$$

The torsion-less spin connection induces a field strength $R_{\mu\nu}{}^a{}_b$ that is related with the Riemann tensor by

$$R_{\mu\nu}{}^\rho{}_\sigma = R_{\mu\nu}{}^a{}_b e_a{}^\rho e_\sigma{}^b. \tag{1.36}$$

In this scenario, the tensors with holonomic indices denoted as $\mu,\ \nu,\ \ldots$ are derived using the connection Γ whereas the tensors with non-holonomic indices $a,\ b,\ \ldots$ are derived using the connection ω.

So far, we have neglected the torsion in the connection for no particular reason. The inclusion of torsion is important and will be presented in Chapter II, in fact in [22] it was recognized that space-time should be endowed with torsion to couple gravity with spinor fields, this means that the gravitational interaction for spinning particles has to be non-Riemannian. The space-time with torsion we need is not realized by gauging the Lorentz group, instead in [23] it was shown that the gauging of the Poincaré group can generate space with torsion as well as curvature.

The core of the argument constructed above is that gravity can be treated as a gauge theory. If we impose G as the gauge redundancy, then the gauge field comes from the connection Γ. When the gauge redundancy is $G \times L$ the gauge field comes from ω instead. Either connection adds new degrees of freedom that we interpret as the action of gravity.

The Lagrangian that describes the Universe must include these gravitational degrees of freedom. However, one must determine a range of validity of the effective theory. For example, below the Planck scale, graviton-graviton scattering violates unitarity and therefore one expects that some other degrees of freedom become relevant.

Gravity is understood as an effective quantum field theory with cutoff M_{pl}. To further understand this, consider the low-energy case where all the degrees of freedom of gravity are in the metric $g_{\mu\nu}$ and the interactions are settle by the Einstein-Hilbert action

$$S_{\mathrm{EH}} = \frac{M_{\mathrm{pl}}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} R. \tag{1.37}$$

If we fix the metric as a flat background and perturb it, so $g_{\mu\nu} = \eta_{\mu\nu} + \alpha h_{\mu\nu}$, the action becomes

$$S_{\mathrm{EH}} = M_{\mathrm{pl}}^2 \int \mathrm{d}^4 x \left[\alpha^2 (\partial h)^2 + \alpha^3 h (\partial h)^2 + \alpha^4 h^2 (\partial h)^2 + \ldots\right], \tag{1.38}$$

where the ellipses denote higher orders in the perturbation. The Hilbert-Einstein action contains an infinite number of terms, the leading ones correspond to the lower powers in α, which we assume is $\alpha \sim M_{\mathrm{pl}}^{-1}$.

If the energy of some process we are interested in is E, then the dimensionless ratio E/M_{pl} is the coefficient of the perturbative expansion, therefore the effective theory breaks down when the energy exceeds the Planck scale. At this point new physics must emerge and, in the absence of detailed information about the UV behavior of gravity, the simplest assumption is that the effective action contains all terms invariant under coordinate transformations.

The most general Lagrangian constructed by the effective theory of gravity serves as a starting point to completely describe the phenomenology of inflation. The coefficients of its operators characterize different models, each one of those covers a specific range of predictions; accurate measurements of the CMB relics will eventually reduce the wide spectrum of theories. How the anisotropies of the CMB are produced is determined by the dynamics of the primordial perturbations.

1.3 Primordial Perturbations

Once inflation is over, the inflaton field will take a vacuum expectation value $\langle\phi\rangle$ in all space-time points x^μ. This is a symmetry breaking where the gauge group related with the space-time diffeomorphisms spontaneously breaks to the group of time dependent spatial diffeomorphisms. Inflation is, in this sense, the theory of broken time diffeomorphism [24, 25].

A consequence of the symmetry breaking is that the hypersurfaces of constant field $\phi(x) = \langle\phi\rangle$ will not necessarily match the hypersurfaces of constant time; inflation lasts longer in some regions, thus creating local fluctuations in the curvature of the Universe which lead to energy density fluctuations $\delta\rho(x)$ [26, 27, 28, 29].

One way to introduce the quantum effects is by considering the inflaton field as the sum of a classical background and a quantum perturbation

$$\phi = \phi_0 + \delta\phi. \tag{1.39}$$

The first term determines the evolution of the Universe background $H(t)$, the second term is the reason of the symmetry breaking. Coordinate invariance of general relativity ensures that there is a coordinate system where these fluctuations are zero, we can perform the transformation $t' = t + \delta t(x)$ under which the perturbation is $\delta'\phi = 0$ and hence

$$\delta'\phi(x) = \delta\phi(x) - \dot{\phi}_0\delta t(x) = 0, \tag{1.40}$$

therefore the time delay of inflation is

$$\delta t(x) = \frac{\delta\phi(x)}{\dot{\phi}_0}, \tag{1.41}$$

which is space-time dependent. The physical implication of δt is that the hypersurfaces of constant field do not necessary match the hypersurfaces of equal time, this causes anisotropies all over the Universe because there are places where the potential energy of the field ϕ is different from other places. One way to interpret what has been said is that inflation last longer in some regions of the Universe and the difference in time is $\delta t(x)$.

The time delay also triggers curvature perturbations that we can parametrize by writing the metric as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \tag{1.42}$$

where $\bar{g}$ is the solution of the de Sitter background and δg encodes the curvature perturbations which can be scalar, vector or tensorial perturbations. The perturbed de Sitter metric in conformal time is

$$\mathrm{d}s^2 = a^2\left[(1+2A)\mathrm{d}\tau^2 - 2B_i\mathrm{d}x^i\mathrm{d}\tau - (\delta_{ij}+h_{ij})\mathrm{d}x^i\mathrm{d}x^j\right], \tag{1.43}$$

where the degrees of freedom of the perturbations A, B_i and h_{ij} can be sorted as:

		scalar		vector		tensor
A	$=$	s				
B_i	$=$	$\partial_i b$	$+$	b_i		
h_{ij}	$=$	$2c\delta_{ij} + 2\partial_{\langle i}\partial_{j\rangle}e$	$+$	$2\partial_{(i}e_{j)}$	$+$	$2e_{ij}$
d.o.f.		4		4		2

In the scalar sector we used the index notation $\langle i \ldots j\rangle$ to simplify the trace-less second derivative

$$\partial_{\langle i}\partial_{j\rangle} = \partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2. \tag{1.44}$$

Expanding the metric in this way, one can note that the scalar, the vector and tensor modes are decoupled. However, not all the degrees of freedom coming from the perturbation have physical meaning, in fact some of them come from the gauge invariance. Only the modes that are invariant under general coordinate transformation (G) are physical modes. The tensor mode alone is invariant under G and therefore one smart manner to fix the gauge is

$$g_{ij} = a^2(t)e^{2\zeta}[\exp(2e_{ij})] \tag{1.45a}$$
$$e_{ii} = 0 \tag{1.45b}$$
$$\partial_i e_{ij} = 0. \tag{1.45c}$$

In this case, e_{ij} can be interpreted as the amplitude of a gravitational wave while $\zeta(x)$ is a scalar field that somehow is conserved during inflation. This conservation is important[7], it allows us to trust the predictions of inflation when some inputs observed in later processes (say, for example, the Big Bang nucleosynthesis) are used.

The reason of why the scalar field $\zeta(x)$ is constant is not simple and it is still under debate, however we can give a picture of the argument. This property has been used in various special cases, see for example [30, 31].

Following the argument of [32], considering that the scalar field represents the perturbation of the scale factor and also considering that the evolution of the scale factor does not depend on the perturbations[8], therefore one expects $\zeta(x)$ to be constant.

[7] Though we have not argued yet why this is in fact a conserved quantity.

[8] We are considering the case where all modes are longer than the Hubble scale H^{-1}.

As a matter of fact, the change of the scale factor is not time dependent so neither is $\zeta(x)$.

The excess in the expansion is $\delta a(t)$, and it is related to the curvature perturbation by

$$\zeta \sim \frac{\delta a}{a} = H(t)\delta t, \tag{1.46}$$

and therefore, it is also related with the inflaton field. The correlation function of the quantum perturbations $\delta\phi$ of the inflaton can be computed using the curvature perturbation, in some sense the curvature field ζ and the scalar field $\delta\phi$ have the same physical relevance.

So far, we have assumed that the field responsible of inflation is a scalar, however this has not been justified and, as a matter of fact, it was not necessary. In spite of the Goldstone equivalence theorem, at high energies it is possible to group all the degrees of freedom of scalar fluctuations in the Goldstone bosons coming from the time diffeomorphisms symmetry breaking. In this mechanism the time delay becomes a field:

$$\delta t \to \pi(x), \tag{1.47}$$

and this field non-linearly realizes the broken symmetry. The overall effect is that our initial scalar field is replaced by π, we can link both fields by making the replacement

$$\frac{\delta\phi}{\dot{\phi}_0}(x) \longrightarrow \pi(x). \tag{1.48}$$

The last expression suggests that the Goldstone boson π and the curvature perturbation ζ are effectively (but not conceptually) the same.

Having said this, the physical description of the primordial perturbations of energy density are obtained using correlation functions that involve the Goldstone boson π coming from the symmetry breaking of time diffeomorphisms

$$\langle \delta\phi(x)\delta\phi(x')\rangle \longrightarrow \langle \pi(x)\pi(x')\rangle. \tag{1.49}$$

In every model that describes inflation we see that, at small scales, the correlation functions of the density fluctuations exhibit an oscillatory behaviour. Inflation forces the field ζ to behave like $\delta t/t$ which is constant on large scales.

The relationship between the Goldstone boson π and the curvature perturbation ζ in the comoving gauge can be expanded as

$$\zeta = -H\pi + \dots \tag{1.50}$$

where the ellipses denote higher orders in π. During inflation, curvature perturbations were generated through the fluctuations at the end of the accelerated expansion phase, the amplitude of such perturbations in the phase space is of the form [30]

$$\langle \zeta^2\rangle(k) \simeq \frac{1}{24\pi^2}\frac{V(k)}{\varepsilon}, \tag{1.51}$$

and from here we can define the spectral index parameter η_s as

$$\eta_s - 1 \equiv \frac{\mathrm{d}\ln\langle\zeta^2\rangle}{\mathrm{d}\ln k} \simeq -6\varepsilon + 2\eta. \tag{1.52}$$

This parameter encodes the information of the scalar perturbations on the curvature. From here we deduce that scale-invariant perturbations of the curvature correspond with a spectral index $\eta_s = 1$, any deviation from this value indicates the level of scale-dependence of the power spectrum.

Other important parameter is the one related with the tensor perturbations: gravitational waves. The quantization of the tensor fluctuations is almost the same as for the scalar fluctuations except that the latter involves additional polarization states. A.

A. Starobinsky found in [33] that the power spectrum of the tensor fluctuations in the phase space has the form

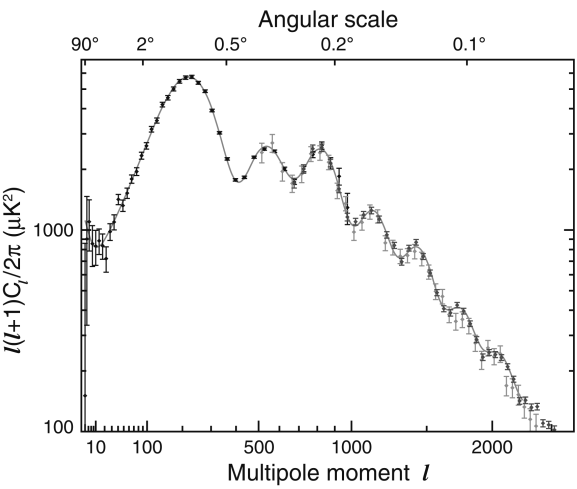
$$\langle e_{ij}^2 \rangle(k) \simeq \frac{2V(k)}{3\pi^2}. \tag{1.53}$$

This finding, together with the one showed in equation (1.51) for the scalar perturbation, allows us to define the tensor to scalar ratio as

$$r \equiv \frac{\langle e_{ij}^2 \rangle}{\langle \zeta^2 \rangle} \simeq 16\varepsilon, \tag{1.54}$$

which quantifies the size of the tensor fluctuations. A detailed derivation of these quantities can be found in [34].

For instance, in [35] the authors combined the result of several CMB experiments such as WMAP, SPT [36] and Planck [37] to produce the following power spectrum for the CMB fluctuations:



where oscillations are clearly manifest. We stated that the signatures of inflation are encoded in the spectrum of the primordial perturbations and, at the same time, the information of these perturbations is summarized in three parameters: the curvature perturbation spectrum $\langle\zeta^2\rangle$, the spectral index η_s and the tensor to scalar ratio r. The presence of fluctuations in the metric during inflation is what leads to acoustic oscillations in the CMB. The greatest verification of inflation, which is only qualitative, is that the CMB shows such behavior.

These observations can strongly constraint the observable quantities, the currently experimental bounds for inflation are

$$\langle\zeta^2\rangle(k_0) = 2.4 \pm 0.01 \times 10^{-9} \tag{1.55a}$$

$$\eta_s = 0.963 \pm 0.012 \tag{1.55b}$$

$$r < 0.24, \tag{1.55c}$$

where k_0 is the value of the scale at the horizon crossing which is equivalent to $k_0 = 0.002$ Mpc^{-1} [38].

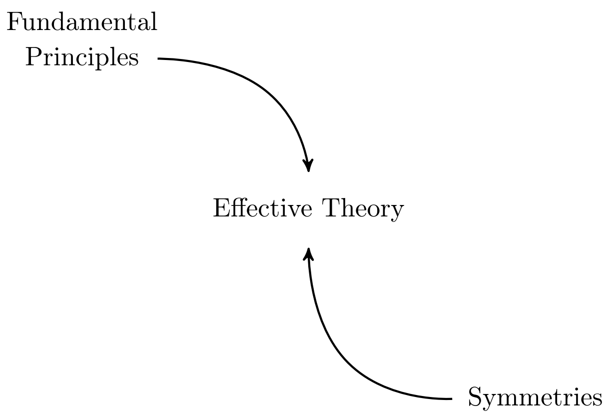
So far, we placed particular interest in arguing why different potentials $V(\phi)$ produced different observable parameters. An important inflationary model developed by A. A. Starobinsky in [39] is constructed by considering quantum corrections to general relativity. This accounts to examine terms in the action with higher order in the curvature R. So far, the Starobinsky inflationary model is the one that best fits the predictions with the measured observable parameters.

To conclude this section, we want to highlight that the only way to describe the dynamics of the perturbation is by constructing an effective Lagrangian. The fact that the space-time translations symmetry is broken to time dependent space translations implies that this general Lagrangian depends only on operators that are invariant under spatial Lorentz transformations that can be time dependent. The preferred method to construct such a Lagrangian is using an effective field theory.

1.4 Effective Field Theory of Inflation

It has been stated that the best way to describe the dynamics of the primordial fluctuations produced after inflation by quantum effects is by an effective field theory. Here is how such a theory can be constructed.

There are two approaches to the physics of inflation: starting from fundamental principles to build the theory or constructing the more general Lagrangian not knowing the small-scale (or large-scale) details of the fundamental theory behind inflation. The first approach is known as top down effective theory and the second one is the bottom up effective theory. In the diagram below both the top down and bottom up prescriptions are represented schematically.



Effective theories are particularly useful when there is not a known or complete description of the phenomenon, it also includes the case in which the theory is specified but not computable, in simpler words, effective theories synthesize the relevant physics at the energy scale of interest.

Recent developments, summarized in [40, 25, 24, 41], of the effective field theory of inflation had led to predictions of the scale dependence of the density fluctuations, represented by the power spectrum. The fluctuation predicted by these models are meant to be tested within the next generation of CMB experiments.

In the EFT, inflation is considered as a period of time at the beginning of the Universe where there is a preferred time-slicing, in that case it is said that the time diffeomorphisms are spontaneously broken, while the (time dependent) spatial diffeomorphisms are not. The principal implication of the symmetry breaking is the presence of a Goldstone boson, that we have called $\pi(x)$, which non-linearly realizes the broken time diffeomorphism symmetry. The new degree of freedom is related with the curvature perturbation $\zeta(x)$, therefore the dynamics of π determines the primordial curvature perturbations scale dependence. A particularly important aspect of this development is that the Lagrangian can be constructed without any knowledge of the details of the symmetry breaking mechanism.

The principal distinction between the bottom-up build and the top-down derived effective Lagrangians leans on the presence or absence of the scalar field ϕ. In the first case the scalar field is no longer present in the Lagrangian; as it was discussed above, the symmetry breaking introduces the Goldstone boson π which represents all the degrees of freedom of the system: the scalar field has been *eaten* by π[9]. On the other hand, it is possible to assume that we know the full Lagrangian of the UV theory[10], and then we can integrate out the heavier fields to obtain an

[9]The Goldstone boson does not actually eat anything, what we meant is that the scalar field is no longer present in the Lagrangian and the degree of freedom it represents is in π.

[10]In the case of inflation, the full Lagrangian must couple with gravity, therefore the kind of Lagrangians used are derived from quantum gravity theories such as Supergravity.

effective theory at the desired energy scale which corresponds to the theory of the dynamics of ϕ.

The previously mentioned considerations allow us to work out the structure of an effective action of a field ϕ coupled to gravity. The line of thought goes like this:

- Consider a background inflationary solution $H(t)$ produced by a field slow-rolling down a potential.

- Assume that during the epoch of inflation the gauge redundancy related with the group of spacetime diffeomorphism is spontaneously broken to the subgroup of the time dependent spatial diffeomorphism.

- Introduce the symmetry breaking by dividing the field ϕ into one field ϕ_0 that transform as a scalar under all diffeomorphism and is itself a solution of the background, and a perturbation $\delta\phi$ that is a scalar only under spatial diffeomorphism while it transforms non-linearly with respect of time translations.

- Fix the gauge. One convenient option is the unitary gauge in which all the degrees of freedom are in the metric $g_{\mu\nu}$. There is also the π-gauge, in this case the scalar field ϕ is, in the sense mentioned before, *eaten* by the field π.

- Write all the operators that are invariant under the residual symmetry. Since there is no time translation symmetry, one is allowed to write any term that respects spatial diffeomorphism.

When one talks about physical systems with spontaneously broken symmetries, what we mean is that a symmetry of the action is not a symmetry of the ground state of the system. Some of the principal characteristics of such systems are synthesized in the low-energy effective theory which involves the Goldstone boson associated with the breaking mechanism. Since the symmetry of the system is reduced, now there are many extra terms allowed besides the usual Riemann term. These new terms describe the additional degree of freedom[11]. The breaking of the time diffeomorphism defines a preferred slicing of space-time, therefore the action can contain geometric objects that describe the slicing, the extrinsic curvature $K_{\mu\nu}$ of a slice of constant time is a good example, for instance it is possible to show that all other geometric objects can be expressed in terms of $K_{\mu\nu}$.

It is useful to define inflation as a de Sitter background with constant rate of expansion. However, the metric is not exactly de Sitter since inflation actually ends, meaning that the time translation invariance is a spontaneously broken symmetry and we treat inflation as an effective model of symmetry breaking.

The effective theory of inflation was developed in [42, 25], here we review the structure of the principal steps to construct such a theory.

Let us start by specifying a unitary transformation that shifts time as

$$t \longrightarrow t + \pi, \tag{1.56}$$

promoting the parameter π to a field allows us to construct the unitary operator U that acts on the fields of the theory. The operator has the form

$$U \equiv e^{i\pi(x)/f}, \tag{1.57}$$

note that it is space-time dependent. A Lagrangian containing the term $UU^\dagger$ will not be in general gauge invariant, however

[11]For example, the time component g_{00} of the metric tensor is invariant under spatial diffeomorphism but not under time diffeomorphism, therefore it can appear in the unitary gauge Lagrangian.

the invariance can be *restored* if the field $\pi(x)$ transforms non-linearly. If we require that—under a time translation with parameter $\xi(x)$—the Goldstone field $\pi(x)$ transforms as

$$\pi \to \pi(x) - \xi(x), \tag{1.58}$$

all the terms in the Lagriangian involving the product of U and $U^\dagger$ will be invariant. This is why we say that the Goldstone boson non-linearly realizes the symmetry.

To construct the effective Lagrangian we use the shifted-field U because it preserves the symmetry we want to impose in our model. This general Lagrangian is therefore expressed in terms of U and its derivatives

$$\mathcal{L} = \mathcal{L}[U, (\partial_\mu U)^2, \Box U, \ldots]. \tag{1.59}$$

At leading order, the quadratic Lagrangian has the form

$$\mathcal{L} = \Lambda^4(U) - f^4(U) g^{\mu\nu} \partial_\mu U \partial_\nu U, \tag{1.60}$$

where the functions $\Lambda(U)$ and $f^4(U)$ are determined by the Friedmann equations. One can show that the constants Λ^4 and f^4 take the values

$$\Lambda^4 = -M_{\rm pl}^2(3H^2 + \dot{H}) \qquad \text{and} \qquad f^4 = M_{\rm pl}^2 \dot{H}.$$

The Lagrangian presents a coupling between the Goldstone π and the metric $g_{\mu\nu}$ as expected, since it represents a gauge theory Lagrangian. It is possible to define a decoupling limit in which π alone controls the dynamics, i.e. we use the Goldstone equivalence theorem [43]. In this case, when $M_{\rm pl} \to \infty$ and $\dot{H} \to 0$, keeping $M_{\rm pl}^2 \dot{H}$ fixed, we can ignore the mixing between π and the metric perturbations $\delta g_{\mu\nu}$, therefore we evaluate the Lagrangian taking the metric simply as the de Sitter background space, this allows us to write the metric perturbation as

$$
\begin{aligned}
g^{00} &\longrightarrow \partial_\mu(t+\pi)\partial_\nu(t+\pi) \\
&\longrightarrow g^{00} - 2\dot{\pi} + (\partial_\mu \pi)^2.
\end{aligned} \tag{1.61}
$$

Replacing the transformed metric into equation (1.60) we obtain the effective Lagrangian written in terms of π which is

$$
\mathcal{L}_{\text{eff.}} = -M_{\text{pl}}^2(3H^2 + \dot{H}) + M_{\text{pl}}^2 \dot{H}(-1 - 2\dot{\pi} + (\partial_\mu \pi)^2). \tag{1.62}
$$

This prescription can be taken further by including higher orders in derivatives. The trick is to write all the terms that are invariant under time dependent space translations. It may seem that the allowed terms may be more than what can be controlled, however the authors of [24] argued that the most general Lagrangian in the unitary gauge is only a function of the Riemann Tensor $R_{\mu\nu\rho\sigma}$, the zero-zero component of the metric g^{00}, the extrinsic curvature $K_{\mu\nu}$, the covariant derivative ∇_μ and time t:

$$
S_{\text{eff.}} = \int \mathrm{d}^4x \sqrt{-g} F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t). \tag{1.63}
$$

The effective action turns out to be an expansion around the fluctuations δg^{00}

$$
\begin{aligned}
S_{\text{eff.}} = \int \mathrm{d}^4x \sqrt{-g} \Big[&\frac{1}{2} M_{\text{pl}}^2 R + M_{\text{pl}}^2 \dot{H} g^{00} - M_{\text{pl}}^2(3H^2 + \dot{H}) + \\
&+ \frac{1}{2!} M_2^4(t)(g^{00}+1)^2 + \frac{1}{3!} M_3^4(t)(g^{00}+1)^3 + \\
&- \frac{1}{2} \bar{M}_1^3(t)(g^{00}+1)\delta K^\mu{}_\mu - \frac{1}{2} \bar{M}_2^2(t) \delta K^\mu{}_\mu{}^2 + \\
&- \frac{1}{3} \bar{M}_3^2(t) \delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots \Big],
\end{aligned} \tag{1.64}
$$

where the dots represent terms with a higher order in the fluctuation or with more derivatives, and the coefficients $M(t)$ are in general functions of time.

To write a Lagrangian in terms of the Goldstone π, we make the replacement of the metric component g^{00} shown explicitly in equation (1.61). Up to second order, the terms that involve the extrinsic curvature change to

$$\delta g^{0}0 \delta K^{\mu}{}_{\mu} \longrightarrow -6\dot{H}\dot{\pi}^2 - \frac{2\dot{\pi}}{a^2}\partial^2\pi \tag{1.65a}$$

$$\delta K^{\mu}{}_{\mu}{}^2 \longrightarrow \left(\frac{\partial^2\pi}{a^2}\right)^2 \tag{1.65b}$$

$$\delta K^{\mu}{}_{\nu}\delta K^{\nu}{}_{\mu} \longrightarrow \left(\frac{\partial^2\pi}{a^2}\right)^2, \tag{1.65c}$$

these terms are meant to be replaced in the effective action, thus leading to the simplified final expression

$$S_{\text{eff.}} = \int \mathrm{d}^4x \left[a^3\dot{\pi}^2 - a\alpha\left(\nabla\pi\right)^2 - \frac{\beta}{aH^2}\left(\partial^2\pi\right)^2 + \ldots\right], \tag{1.66}$$

where we have introduced the coefficients α and β defined by

$$\alpha = \frac{-M_{\text{pl}}^2\dot{H} - H\bar{M}_1^3/2}{-M_{\text{pl}}^2\dot{H} + 2M_2^2} \quad \text{and} \quad \beta = \frac{\bar{M}_{\text{b}}^2 H^2}{2\left(-M_{\text{pl}}^2\dot{H} + 2M_2^4\right)} \tag{1.67}$$

and where $\bar{M}_{\text{b}}^2 = \bar{M}_2^2 + \bar{M}_3^2$.

This effective action written in the unitary gauge represents the more general action, up to second order, of an inflationary model based on the breaking of the time diffeomorphism symmetry. The primordial density perturbations can be obtained using the equation of motion of π, and this allows us to determine phenomenological observations that can be tested in the CMB.

We derived an effective model to describe the primordial density perturbations produced at the last stages of inflation. For simplicity, we restricted our analysis to a single field inflationary model where the inflaton ϕ is coupled with gravity. Since we do not have access to a UV-complete theory of gravity we treat inflation as an effective theory. In this scenario, inflation is understood as the breaking mechanism of the time diffeomorphisms symmetry, thus giving the Universe a preferred time slicing.

Using the idea that inflation can be thought as a symmetry breaking, we constructed a general Lagrangian that is only invariant under time dependent spatial diffeomorphisms. In order to restore the symmetry, we applied to the fields[12] a nonlinear transformation under time diffeomorphisms with parameter π, which is also a field. The introduction of π indicates the presence of density perturbations, in fact the correlation function of the curvature field ζ is equivalent to the correlation function of the field π. The final result was the action (1.66) that describes the interactions of the Goldstone particle π in the low energy regime.

[12]Here we only consider the metric $g_{\mu\nu}$.

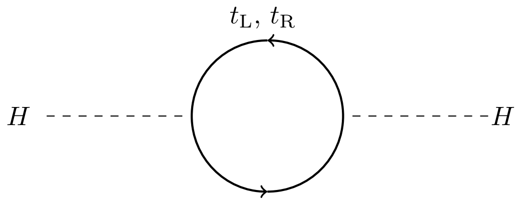
Supergravity

2.1 Supersymmetry

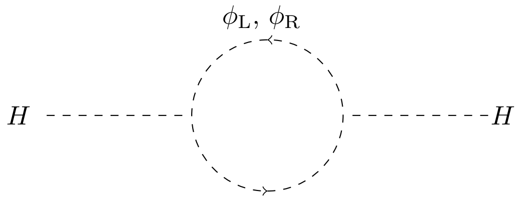
In particle physics, the content of the Universe and the way particles interact are predicted by the Standard Model. Previously, we stated that the initial phase in the history of the Universe is best described by the theory of inflation, in which the expansion of space is taken to be an accelerated expansion, and we called this kind of solution de Sitter space. However, such effect can only be produced by a field of dark energy that gravitates: neither dark energy or gravity are regarded in the Standard Model.

This make us feel that the Standard Model has to be originated from a more complete theory were gravity and dark energy emerge naturally so its presence in the Universe is explained in a fundamental level. Several proposals have been formulated: we refer to them as beyond the Standard Model scenarios.

There is another issue with the Standard Model. The presence of a large gap between the cut-off of the theory, namely M_{pl}, and the breaking scale of the electroweak interaction is not well understood. The value of the electroweak breaking scale is related to the mass of the Higgs boson which has been measured in the LHC experiment and has a value of 125 GeV [44, 45]. This value is highly unstable under quantum loop corrections because these corrections diverge quadratically with the Planck mass. For example the *top quark* contribution to the Higgs mass is given by a diagram with a fermionic loop:



such contribution must be cancelled exactly to reproduce the observed Higgs mass, in fact all the loop contributions must be cancelled in the same way. Those dramatic cancellations indicate the presence of a hidden symmetry. One possibility is the existence of a particle with the same mass as the top quark, but such particle is a boson rather than a fermion. This other particle ϕ will also contribute to the Higgs mass by a loop contribution of the form:



except that, in this case, the bosonic loop diagram has an opposite sign[1]: it cancelled itself with the top quark contribution and

[1] Remember that each fermionic loop brings a -1 factor when computing the diagram.

stabilizes the Higgs mass.

This mechanism works if all particles of the Standard Model have a partner with the opposite statistics. Demanding the existence of a universal correspondence between fermions and boson is one of the most popular beyond-the-Satandard Model scenarios and it can be thought as an additional symmetry which has been called *Supersymmetry*[2] by the scientific community.

For the previously mentioned reasons, it is of great interest to study inflationary scenarios where Supersymmetry is considered as a fundamental symmetry of nature. An additional advantage of including Supersymmetry in the description of the dynamics of the universe is that gravity comes naturally to the theory when Supersymmetry is gauged, the study of gauged Supersymmetry is known as *Supergravity*. In the following section we show some relevant aspects of both Supersymmetry and Supergravity.

Supersymmetry is generated by an operator Q that transforms bosonic states into fermionic states and an operator $\bar{Q}$ that works conversely. One important result obtained in [46] shows that a symmetry group represented by a graded Lie algebra allowed the possibility of having a correspondence between bosons and fermions, so it is postulated that the generators of Supersymmetry satisfy the non-trivial anticommutator relations[3]

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} \propto \sigma_{\alpha\dot{\beta}}{}^{\mu}. \tag{2.1}$$

Back in 1967, S. Coleman and J. Mandula provided a demonstration in [47] of a 'no-go' theorem stating that the only symmetry of the S-matrix that includes the Poincaré symmetry has to be the product between Poincaré and an internal symmetry group, hence if we want $\mathcal{N} = 1$ Supersymmetry to be compatible with a quantum field theory as an extension of Poincaré, it has

[2]SUSY for short.

[3]For simplicity we are considering only the $\mathcal{N} = 1$ supersymmetrical case.

to be represented by the Lie algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma_{\alpha\dot{\beta}}{}^{\mu} P_\mu \tag{2.2a}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \tag{2.2b}$$

$$[Q_\alpha, P_\mu] = [\bar{Q}_{\dot{\alpha}}, P_\mu] = 0 \tag{2.2c}$$

$$[P_\mu, P_\nu] = 0. \tag{2.2d}$$

The indices denoted by the Greek letters from the beginning of the alphabet (α, β, ..., $\dot{\alpha}$, $\dot{\beta}$, ...) run from one to two and correspond to two-component Weyl spinors, whereas the indices from the middle of the Greek alphabet (μ, ν, ...) identify Lorentz four-vectors.

With the Supersymmetry algebra defined, it is forthright to construct the corresponding representations. Start by calling *supermultiplet* one irreducible representation of the Supersymmetry algebra. Since the Poincaré algebra is a subgroup of the Supersymmetry algebra it follows that any irrep. of Supersymmetry is a representation of Poincaré that, in general, can be reducible. The real implication of this feature is simply that one supermultiplet coincides with a collection of particles. For instance, in the top quark example mentioned above, the multiplet would be $\{t, \phi\}$, with both particles having the same mass but the first one being a boson and the second one being a fermion.

This is a good time to point out an important characteristic of the Supersymmetry algebra. We know that the quantity

$$P^2 = P^\mu P_\mu, \tag{2.3}$$

is a Casimir operator and therefore the mass of each particle in the multiplet is the same. Contrary to the pure Poincaré algebra, in Supersymmetry the operator W^2 is not a Casimir operator anymore and therefore the particles in the multiplet, thought they have the same mass, differ in the value of the spin[4].

[4]Take a look at Appendix C for instructive examples.

In the general picture, a multiplet is a set of fields $\{A, \psi, \ldots\}$ that represent the Supersymmetry algebra and are not restricted by any mass-shell condition. To guarantee that the on-shell degrees of freedom are the same as the off-shell degrees of freedom some of the fields in the multiplet do not propagate; we regard these fields as auxiliary fields and they do not have physical meaning.

On each field in the multiplet we define an infinitesimal Supersymmetry transformation with anticommuting parameters ϵ and $\bar{\epsilon}$ as

$$\delta_\epsilon A = (\epsilon Q + \bar{\epsilon}\bar{Q}) \times A \tag{2.4a}$$
$$\delta_\epsilon \psi = (\epsilon Q + \bar{\epsilon}\bar{Q}) \times \psi \tag{2.4b}$$
$$\ldots$$

The algebra is preserved only if the transformations satisfy the identities

$$(\delta_{\epsilon'}\delta_\epsilon - \delta_\epsilon\delta_{\epsilon'})A = 2(\epsilon'\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\epsilon}')P_\mu A \tag{2.5}$$
$$\ldots$$

which are a direct consequence of the algebra itself. According to the definition of Supersymmetry, the transformation δ_ξ maps tensor fields into spinor fields. To be more specific, we say that the transformation increases the value of the spin in one half.

One important example is the chiral multiplet, which consists on one vector field A, a spinor field ψ and an auxiliary field F that carries the non-physical degrees of freedom. These fields transform as

$$\delta_\epsilon A = \sqrt{2}\epsilon\psi \tag{2.6a}$$
$$\delta_\epsilon \psi = i\sqrt{2}\sigma^\mu\bar{\epsilon}\partial_\mu A + \sqrt{2}\epsilon F \tag{2.6b}$$
$$\delta_\epsilon F = i\sqrt{2}\bar{\epsilon}\bar{\sigma}^\mu\partial_\mu\psi, \tag{2.6c}$$

and it can be proved that these transformation laws verify the relations (2.5).

Having analyzed the basic structure of Supersymmetry, we now present an elegant mathematical device that encapsulates the relevant properties of the former representations of Supersymmetry. Such tool is the key to construct Lagrangians with a specific field content.

2.2 Superspace

There is a simple way to manipulate the fields in Supersymmetry theories that was first introduced by A. Salam and J. Strathdee in [48] for the particular case of $\mathcal{N} = 1$. The idea is to define a *superfield* that collects all the fields in a multiplet in a single mathematical object. This object is merely a mathematical device and it does not gives us any new information, however the interpretation of where the superfield lives on is more involved.

Think that Supersymmetry is generated by translations along a new set of coordinates θ and $\bar{\theta}$ that are Grassmann variables[5]. The manifold assembled by these new coordinates and the usual space-time coordinates is called *superspace*.

All the elements of superspace are labelled by the more general coordinate $z = (x, \theta, \bar{\theta})$. In this sense, Supersymmetry is generated by translations inside a Lie group which is associated to the Supersymmetry algebra, we parametrize these group elements as

$$G(x, \theta, \bar{\theta}) = \exp\left[i(-x \cdot P + \theta Q + \bar{\theta}\bar{Q})\right], \tag{2.7}$$

and naturally, the composition of two group elements induces a motion in the parameter space. The right-action of the group

[5] Variables that anticommute: $\{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\} = 0$.

element $G(0,\epsilon,\bar{\epsilon})$[6] acting on $G(x,\theta,\bar{\theta})$ is a map g defined in the parameter space so that

$$g(\epsilon,\bar{\epsilon}) : (x,\theta,\bar{\theta}) \longrightarrow (x^\mu - i\theta\sigma^\mu\bar{\epsilon} + i\epsilon\sigma^\mu\bar{\theta}, \theta+\epsilon, \bar{\theta}+\bar{\epsilon}). \quad (2.8)$$

This transformation is what defines a translation along the Grassmann variables of superspace.

As usual, if we go to the infinitesimal case, we can define a differential operator D that acts on z such that the change produced by the map g is written as

$$\begin{aligned} &\epsilon D + \bar{\epsilon}\bar{D} = \\ &\epsilon^\alpha\left(\frac{\partial}{\partial\theta^\alpha} + i\sigma_{\alpha\dot{\alpha}}{}^\mu\bar{\theta}^{\dot{\alpha}}\partial_\mu\right) + \bar{\epsilon}^{\dot{\alpha}}\left(-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma_{\alpha\dot{\alpha}}{}^\mu\partial_\mu\right), \end{aligned} \quad (2.9)$$

and therefore the induced motion is generated by the differential operators

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma_{\alpha\dot{\alpha}}{}^\mu\bar{\theta}^{\dot{\alpha}}\partial_\mu \quad (2.10a)$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma_{\alpha\dot{\alpha}}{}^\mu\partial_\mu, \quad (2.10b)$$

that satisfy the following anticommutation relations

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i\sigma_{\alpha\dot{\alpha}}{}^\mu\partial_\mu \quad (2.11a)$$

$$\{D_\alpha, D_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0. \quad (2.11b)$$

In this sense, superspace is understood as a generalization of space-time and Supersymmetry is the invariance under the change generated by D in such space. As a consequence, all the

[6] This group element is what we identify with a pure supersymmetric transformation with parameter ϵ.

fields in a multiplet can be assembled in a function $F(z)$ that, in general, can be Taylor-expanded as

$$\begin{aligned} F(x,\theta,\bar{\theta}) =& f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \\ & + \theta^2 m(x) + \bar{\theta}^2 n(x) + \theta\sigma^\mu\bar{\theta} v_\mu(x) + \\ & + \theta^2\bar{\theta}\bar{\lambda} + \bar{\theta}^2\theta\psi(x) + \theta^2\bar{\theta}^2 p(x). \end{aligned} \tag{2.12}$$

Note that higher powers in θ and $\bar{\theta}$ vanish due to the anticommutation condition, meaning that the components of the superfield are always finite in number. The transformation of F under Supersymmetry is obtained from

$$\delta_\epsilon F = (\epsilon D + \bar{\epsilon}\bar{D})F, \tag{2.13}$$

where each of its components—in spite of equation (2.11)—satisfy the condition (2.5). The consequence is that these fields form a linear representation of the Supersymmetry algebra: the superfield gathers in its components all the fields of a reducible representation. This is a nice feature.

There is a problem though. The representation is highly reducible and some of the components may be eliminated. To reduce the number of components in a superfield we impose covariant constraints, here we highlight two important examples:

$$\begin{array}{lcl} \bar{D}F = 0 & \longrightarrow & \text{Chiral superfield} \\ F = F^\dagger & \longrightarrow & \text{Vector superfield} \end{array}$$

When the first constraint is solved for F we recover the chiral multiplet of equations (2.6).

One can also build a superfield starting from one of the components of the multiplet, take A as an example. The superfield is generated by the operator $\exp(\theta D + \bar{\theta}\bar{D})$ acting on A:

$$F(x,\theta,\bar{\theta}) = \exp(\theta D + \bar{\theta}\bar{D}) \times A = A + \delta_\theta A + \ldots \tag{2.14}$$

Here lies the first advantage of superfields: they shift the problem of finding irrep. of Supersymmetry to that of finding covariant constraints that are appropriate to collect all the fields[7] present in a multiplet

If superfields are important, then constrained superfields are essential[8]. The reason to impose constraints on superfields is to reduce their components, however we do not want to restrict the dependence on x through differential equations. For example, a constraint of the type

$$DDF = \bar{D}F = 0, \tag{2.15}$$

yields to massless field equations. In fact, such differential equations are reducing the degrees of freedom of the multiplet and we interpret them as the equations of motion of the auxiliary fields. We are looking for a superfield that represents Supersymmetry off-shell so we place our interest in constraints that only restricts the θ-space dependence.

Now we focus our attention in one particular superfield that forms the basic structure of every $\mathcal{N} = 1$ Supergravity theory. We are talking about, of course, the Supergravity multiplet.

2.3 Supergravity Multiplet

Supergravity is the theory of local Supersymmetry. If we treat local Supersymmetry as a gauge theory, then we encounter that there is one field realizing the gauging of Supersymmetry transformations and one other field realizing the gauging of translations. We interpret these two fields as the components of a representation of Supergravity.

[7] Not more, nor less.

[8] Though we use constrained superfields to find a proper representation of Supersymmetry, they are also useful to spontaeously break the symmetry. This property will be studied in Section 4.1.

Since the parameter of a supersymmetric transformation is a spinor ϵ then we expect that the gauge field related with local Supersymmetry is an algebraic object consisting of one world index μ and one spinor index α, such an object is the Rarita-Schwinger field ψ_μ representing a fermion with spin 3/2. This field is the gravitino.

In addition, the gauging of Supersymmetry leads to the gauging of translations and we know that local translations are the same as general coordinate transformations[9]. The result is that gravity comes naturally to the theory. We also know that in order to couple gravity with spinor fields, space-time should be equip with torsion[10] and it is convenient to use the vierbein $e_\mu{}^a$ as the gauge field. This bosonic field of spin 2 is the graviton.

So far, we argued that Supergravity comes from the gauging of Supersymmetry and that the multiplet for the super Poincaré algebra consists of two states corresponding to the pair of fields ψ_μ and $e_\mu{}^a$. In 1976 a consistent Supergravity theory was constructed by D. Z. Freedman, P. van Nieuwenhuizen, S. Deser and B. Zumino and it was called $\mathcal{N} = 1$ Poincaré Supergravity [49, 50, 51]. Here we higlight the basic consturction of such theory using the method of superfields.

Start by imposing a symmetry that depends on the coordinates of superspace, so the transformation is supergauged. In our case, the supergauge transformation consists of a general coordinate transformation of superspace

$$z'^M = z^M - \epsilon^M(z), \tag{2.16}$$

and a local Lorentz transformation with parameter Λ that, in general, acts on local Lorentz indices[11]. In this sense, a general

[9] We already discussed this in the analysis of equation (1.26)

[10] This was recognized by D. W. Sciama in [22]

[11] The local Lorentz indices are denote with capital Latin letters from the

tensor field V^A transforms as

$$\delta V^A = -\epsilon^M \partial_M V^A + V^B \Lambda_B{}^A. \tag{2.17}$$

The supergauge changes the way we take derivatives on this manifold because superspace is not rigid anymore. As a matter of fact, the gauge introduces new physical degrees of freedom that we understand as a new interaction: Supergravity. The vierbein superfield E and the connection ϕ will be the dynamical variables of Supergravity.

The role of Supergravity becomes evident when the connection ϕ is completely determined, our purpose here is to explain what is ϕ when we treat Supergravity as a supergauge theory. First remind that we distorted the geometry, so the derivatives are no longer covariant as they should. However, if we replace the derivative in the supergauge transformation of V^A given in (2.17) with a covariant derivative $\hat{D}$ of the form[12]

$$\hat{D}_M V^A = \partial_M V^A + (-)^{mb} V^B \phi_{MB}{}^A, \tag{2.18}$$

then covariance would be restored only if the connection ϕ satisfies the condition

$$\Lambda_B{}^A = -\epsilon^C \phi_{CB}{}^A, \tag{2.19}$$

this condition is what identifies the supergauge transformations, from now on we assume that (2.19) holds for ϕ.

The supergauge transformation consists of a general coordinate transformation with field-dependent parameter ϵ^A followed

beginning of the alphabet A, B, C, ..., these indices include the x-space indices a, b, c, ... and also the θ-space indices α, $\dot{\alpha}$, β, $\dot{\beta}$, ... The world indices are denote by Greek letter from the middle of the alphabet such as μ, ν, ... and they also include the spinor indices.

[12]For simplicity, we are using the same notation between the covariant derivative just introduced and the differential operator of Supersymmetry defined in (2.10), from this point and forward, when we use the symbol D_M we mean the covariant derivative of the supergauge transformation.

by a structure group Lorentz transformation with field-dependent parameter $-\epsilon^C \phi_{CB}{}^A$. It is implicit that all supergauge transformations form a group and that the infinitesimal generators follow the algebra of Supergravity.

The elements of the algebra can be sorted into three cathegories: space-time diffeomorphism δ_G, Lorentz transformations δ_L and Supersymmetry transformations δ. Each of the transformations δ_G and δ_L form independently a subgroup that satisfies the following commutation relations

$$[\delta_G(\xi'), \delta_G(\xi)] = \delta_G(\xi \cdot \partial\xi' - \xi' \cdot \partial\xi) \tag{2.20a}$$
$$[\delta_L(\lambda'), \delta_L(\lambda)] = \delta_L([\lambda', \lambda]). \tag{2.20b}$$

When Supersymmetry is introduced, the additional commutation relations that complete the algebra are

$$[\delta_G(\xi), \delta_L(\lambda)] = \delta_L(-\xi \cdot \partial\lambda) \tag{2.21a}$$
$$[\delta(\epsilon'), \delta(\epsilon)] = \delta_G(y) + \delta_L(\Lambda) + \delta(\hat{\epsilon}) \tag{2.21b}$$
$$[\delta(\epsilon), \delta_G(\xi)] = \delta(\xi \cdot \partial\epsilon) \tag{2.21c}$$
$$[\delta(\epsilon), \delta_L(\lambda)] = \delta(\lambda^{ab}\sigma_{ab}\epsilon/2), \tag{2.21d}$$

here the parameters y, Λ and $\hat{\epsilon}$ are computed in Appendix D, where it is also shown that the algebra is closed[13].

At the beginning of the section we introduced the vierbein and the connection as the only dynamical variables of the theory, however we can recast these variables of Supergravity to express the curvature R and torsion K of superspace, in Appendix E we defined these two superfields in terms of $E_M{}^A$ and ϕ. However,

[13]In the adopted notation the subscript indicates the type of transformation: G for diffeomorhpism L for Lorentz transformations and none for Supersymmetry transformations. The parameter of each transformation is inside the parenthesis. For simplicity, sometimes we will indicate the parameter of a Supersymmetry transformation as a subscript $\delta(\epsilon) \to \delta_\epsilon$.

we still have redundancies that can be gauged away, in fact, only the lowest components of the vierbein and the connection are the physical Supergravity degrees of freedom. It is possible to find a transformation so that the lowest component of ${E_M}^A$ has the form

$$ {E_M}^A(z)\Big|_{\theta=0} = \begin{pmatrix} {e_\mu}^a & \frac{1}{2}{\psi_\mu}^\alpha & \frac{1}{2}\bar{\psi}_{\mu\dot{\alpha}} \\ 0 & {\delta_\beta}^\alpha & 0 \\ 0 & 0 & {\delta^{\dot{\beta}}}_{\dot{\alpha}} \end{pmatrix}, \tag{2.22} $$

where, as stated from the beginning, ${e_\mu}^a$ represents the graviton and ${\psi_\mu}^{\dot{\alpha}}$ represents the gravitino.

Similarly, the highest components of the connection can be gauged away, leaving ${\phi_{MA}}^B|$ as the only component of the connection with degrees of freedom. We write such component as

$$ {\phi_{\mu a}}^b\Big|_{\theta=0} = {\hat{\omega}_{\mu a}}^{\;b}, \tag{2.23} $$

with ${\phi_{\alpha A}}^B| = {{\phi^{\dot{\alpha}}}_A}^B| = 0$. Using the metric tensor η^{ab}, we rewrite the connection as

$$ {\hat{\omega}_\mu}^{\;ab} = \eta^{ac}{\hat{\omega}_{\mu c}}^{\;b}, \tag{2.24} $$

which can be reorganized in order to make explicit the dependence with the torsion:

$$ {\hat{\omega}_\mu}^{\;ab} = {\omega_\mu}^{ab} + {K_\mu}^{ab}, \tag{2.25} $$

with ${\omega_{\mu A}}^B$ being the torsion-free spin connection. This tensor has no dependence on the gravitino, so it is related with the symmetric Christoffel symbols from general relativity, the association is given by

$$ \Gamma^\lambda_{\mu\nu}{e_\lambda}^a = \partial_\mu {e_\nu}^a + {{\omega_\mu}^a}_b {e_\nu}^b. \tag{2.26} $$

Using the results (E.4) and (E.7) of Appendix E, we evaluate explicitly the curvature and torsion of superspace:

$$R_{\mu\nu}{}^{ab} = \partial_\mu \hat{\omega}_\nu{}^{ab} + \hat{\omega}_\mu{}^{a}{}_{c}\hat{\omega}_\nu{}^{cb} - (\mu \leftrightarrow \nu) \tag{2.27}$$

$$K^{ab}{}_{\mu} = -\frac{i}{4}\left(\psi^a\sigma^b\bar{\psi}_\mu + \psi^a\sigma_\mu\bar{\psi}^b + \psi_\mu\sigma^a\bar{\psi}^b\right) - (a \leftrightarrow b). \tag{2.28}$$

Now that we have attributed a curvature to superspace, we can take derivatives. The covariant derivative $\hat{D}$ acts differently depending on which representation it is acting on. For vectors, the covariant derivative is

$$\hat{D}_\mu V^a = \partial_\mu V^a + \hat{\omega}_\mu{}^{a}{}_{b} V^b, \tag{2.29}$$

and for spinors in the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations, it is respectively

$$\hat{D}_\mu \chi = \partial_\mu \chi + \frac{1}{2}\hat{\omega}_\mu{}^{ab}\sigma_{ab}\chi \tag{2.30a}$$

$$\hat{D}_\mu \bar{\chi} = \partial_\mu \bar{\chi} + \frac{1}{2}\hat{\omega}_\mu{}^{ab}\bar{\sigma}_{ab}\bar{\chi}. \tag{2.30b}$$

It is also convenient to define the torsion-free covariant derivatives, which are defined by taking equations (2.29) and (2.30) and removing the hat in $\hat{D}$ and $\hat{\omega}$.

So far, the tensor field $e_\mu{}^a$ and the Rarita-Schwinger spinor ψ_μ of Supergravity have been introduced, however we avoided the discussion about the non-physical fields in the multiplet. Remeber that in order to equate the on-shell degrees of freedom with the off-shell ones, some of the components of the superfield are not meant to propagate. They are not physical degrees of freedom. We sort these new degrees of freedom in two different fields, which are the scalar field m and the vector field b_a. These fields are defined in (E.10) and (E.11) respectively in terms of the curvature superfield.

The independent components of the vierbein superfield form the complete Supergravity multiplet which, as we discussed prevously, is

$$\{e_\mu{}^a, \psi_\mu, m, b_a\}, \tag{2.31}$$

consisting of the graviton $e_\mu{}^a$, the gravitino ψ_μ and the forementioned auxiliary fields m and b_a.

The supegravity transformation acts on the vierbein as

$$\delta E_M{}^A = -D_M \epsilon^A - \epsilon^B T_{BM}{}^A, \tag{2.32}$$

from where we derive, for each field in the multiplet, the correct transformation that realizes Supergravity. A proper and complete analysis is given in [52] and leads to the transformations

$$\delta_\epsilon e_\mu{}^a = i\left(\psi_\mu \sigma^a \bar{\epsilon} - \epsilon \sigma^a \bar{\psi}_\mu\right) \tag{2.33a}$$

$$\delta_\epsilon \psi_\mu = -2\hat{D}_\mu(\epsilon) + i\left[m\sigma_\mu \bar{\epsilon} + b_\mu \epsilon + \frac{1}{3} b^\nu (\sigma_\mu \bar{\sigma}_\nu \epsilon)\right] \tag{2.33b}$$

$$\delta_\epsilon m = -\frac{1}{3}\epsilon(\sigma^\mu \bar{\sigma}^\nu \psi_{\mu\nu} + i b^\mu \psi_\mu - 3i\sigma^\mu \bar{\psi}_\mu m) \tag{2.33c}$$

$$\begin{aligned}\delta_\epsilon b^a = {} & \frac{3}{8}(\bar{\psi}_{\mu\nu}\sigma^a \bar{\sigma}^\mu \sigma^\nu \epsilon) - \frac{1}{8}(\bar{\psi}_{\mu\nu}\bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^a \epsilon) + \\ & - \frac{3i}{2} m^* (\epsilon \psi^a) - \frac{i}{8} b_c (\epsilon \sigma^c \bar{\sigma}^a \sigma^\mu \bar{\psi}_\mu) + \\ & + \frac{i}{4} b^a (\epsilon \sigma^\mu \bar{\psi}_\mu) + \frac{i}{8} b^c (\bar{\psi}_\mu \bar{\sigma}^a \sigma_c \bar{\sigma}^\mu \epsilon) + \text{c.c.},\end{aligned} \tag{2.33d}$$

where it has been defined the antisymmetric tensor $\psi_{\mu\nu}$ as

$$\psi_{\mu\nu} = \hat{D}_\mu \psi_\nu - \hat{D}_\nu \psi_\mu. \tag{2.34}$$

These transformation properties are the starting point to formulate a theory of inflation as a model of broken supersymmetry.

Having discussed the essential developement of Supergravity, we shall construct a Lagrangian that is invariant under this symmetry transformation, this allows us to build an effective theory of inflation taking Supergravity as a broken symmetry.

The following Chapter is devoted to the study of broken Supergravity. There, we will discuss one particular method to construct a Lagrangian where local Supersymmetry is restored by the introduction of a new fermionic field that defines the supersymmetric transformation of the multiplet given in equations (2.33). The following proceedings are framed in the well known technique of nonlinear realizations.

Symmetry Breaking I

3.1 Nonlinear Realizations

Nonlinear realizations constitute a useful mechanism to build theories where a local symmetry group is spontaneously broken down to a subgroup. We learned from perturbation theory that if one writes the most general Lagrangian that includes all terms that are consistent with an assumed symmetry, and then compute the matrix elements, the result will be, at all orders, the most general $S-$matrix that respects perturbative unitarity and that is consistent with the assumed symmetry principles.

As anticipated in Section 1.2, if we were interested in a physical process of energy E, then we expect that the coefficients of the relevant operators in the Lagrangian are of order E/M_{pl} and the operators proportional to a power of this ratio are suppressed. This sets a cutoff for the effective theory, meaning that there is a range of energies where the model effectively describes the dynamics and physical features of inflation.

As we shall see, an action that is invariant under a full local symmetry can be assembled using the transformation properties of the fields at our disposal under the broken symmetry, those transformations are parametrized by a field that is effectively decoupled at a sufficiently large energy scale. The construction of models where a symmetry group is broken requires the study of the transformation properties of the fields involved. The examination of such properties of the fields in a model is done through group theory, this was first noted by C. G. Callan, S. R. Coleman, J. Wess and B. Zumino in [53, 54] and we refer to such prescription as CCWZ construction. If the fields of a particular theory transform linearly, then it is possible to classify all the field transformation laws using only elements of representation theory. Making this classification is essential when we build Lagrangians that describe the symmetry breaking mechanism, however, for nonlinear theories the situation is more complicated: here the fields transform linearly only under the action of a subgroup of the full symmetry group. This is where the nonlinear realizations are useful.

We devote this section to the study of one particular case of symmetry breaking where the Supergravity group is spontaneously broken. The broken generators are those of Supersymmetry, leaving local Lorentz (L) and space-time diffeomorphism (G) unbroken. One can reintroduce the full gauge invariance at the Lagrangian level, however the vacuum state will not necessarily be invariant so neither will be the spectrum of the theory, which, at energies above some mass scale, will split into two particles of helicity 1 and 3/2 (or one particle of helicity 2) and an additional particle λ that preserves the overall number of degrees of freedom. We introduce this new field by making local the parameter of the group elements of the supersymmetric sector of the total symmetry (Supergravity). Furthemore, if we take the global limit of the gauge group, the particle λ coincides with the Goldstone fermion obtained from the spontaneously breaking mechanisms of the global symmetry.

As discussed in the introduction, in a theory of a spontaneously broken symmetry the fields involved may not transform linearly under a representation of the symmetry. Here is where nonlinear realization become useful. A nonlinear realization of symmetry breaking is, by most accounts, a transformation that makes the fields to transform linearly under the unbroken symmetry. A transformation of this type requires the presence of a local parameter, namely λ, that transforms non-linearly under the broken symmetry. The transformed fields serve us to write the terms of an action that is invariant under the complete symmetry, this is one of the main advantages of such formalism.

Other important aspect of nonlinear realizations is that, after performing the transformation, the introduced parameter is coupled with the other fields of the theory and new interactions emerge. Later on this section, we will show that at sufficiently high energies the parameter λ decouples and, in turn, it manifest itself effectively as a new polarization state of the remaining fields.

The introduced parameter λ has an additional role. If one imposes a transformation of λ such that it makes the other fields to transform linearly under the residual gauge symmetry, as we will see, then one can restore the full gauge invariance at the Lagrangian level. In this sense, we say that λ non-linearly realizes Supergravity. The nonlinear realization of Supersymmetry was first developed for the rigid case by D. V. Volkov and V. P. Akulov in [55], here we present the extension to local Supersymmetry.

To clarify how the nonlinear realization works, we start by considering a symmetry group S that acts on the points of a manifold $\mathcal{M}$, that can be understood as space-time. The points in the manifold are denoted by x and the group elements are $s \in S$. In this sense, the transformation of x under the action of some group element s is given by

$$x' = s \cdot x, \tag{3.1}$$

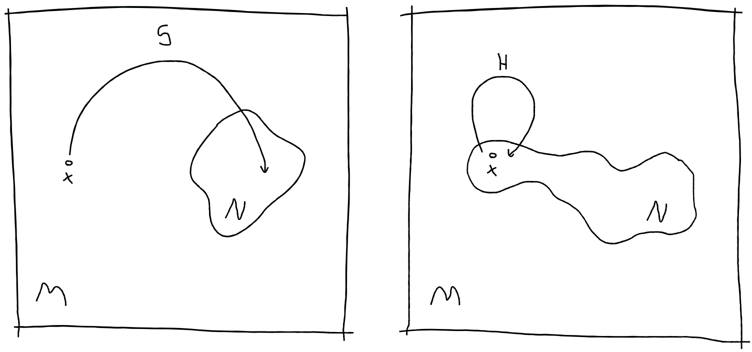
these transformations induce a *realization* of S on some neighborhood of the coordinates of x.

In general, these realizations are not necessarily linear, however it is a well known fact that in a compact, connected and semi-simple Lie group a given realization can be, in fact, linearized if and only if it leaves a point in the coordinate patch of x invariant [56]. The linear representation of S is obtained by a coordinate transformation that will depend on the fields. This condition holds for Supergravity.

The action of S at the point x spans a submanifold $\mathcal{N}$ defined as all the points in $\mathcal{M}$ that can be reached by the transformation (3.1). The elements of S that leave x invariant form a subgroup H, known as the *stability group*, so an element $h \in H$ satisfies

$$h \cdot x = x, \tag{3.2}$$

note that the elements of $\mathcal{N}$ are in one-one correspondence with the elements of the coset space S/H. The following image shows schematically the mentioned algebraic structure:



In the context of Supergravity, in order to develop a theory of inflation that relies on observations, local Supersymmetry must be spontaneously broken. If S is broken down to the subgroup H, we must introduce a fermionic field $\lambda(x)$ that transform under S like the coset space S/H, this field is, in principle, massless. Furthemore, since the breaking mechanism is characterized by a mass scale, say M_{break}, when we build effective theories in the low energy limit ($M_{\text{break}} \to \infty$) and heavy particles are integrated out, the theory is said to be nonlinear. The Goldstone field $\lambda(x)$ manifest the non-linearity of the theory in two ways: the transformation of λ under S will not be linear and the remaining fields will no longer be massless; in our context the gravitino acquires a mass value that we call $m_{3/2}$.

We are interested in the particular case where Supergravity breaks down to local Lorentz invariance and space-time diffeomorphisms, meaning that the broken generators are those of Supersymmetry, say Q. We identify the stability group H with $G \times L$ and let us name its generators as T. Using the group parameters λ and u, we are allowed to write every group element as

$$s = e^{-\lambda \cdot Q} e^{-u \cdot T}, \tag{3.3}$$

this means that the coset space S/H is parametrized by the elements dentoed as $e^{-\lambda \cdot Q}$.

An arbitrary element s that belongs to the total symmetry group can be composed with an element of the coset space by left multiplication:

$$s e^{-\lambda \cdot Q} = s', \tag{3.4}$$

since both s and $e^{-\lambda \cdot Q}$ belong to S, then s' will also belong to S and, therefore, it can be parametrized as

$$s e^{-\lambda \cdot Q} = e^{-\lambda' \cdot Q} e^{-u' \cdot T}, \tag{3.5}$$

where the coordinates of the new parameters λ' and u' are meant to be completely determined in terms of s and λ:

$$s : \lambda \longrightarrow \lambda'(s, \lambda) \tag{3.6a}$$

$$s : u \longrightarrow u'(s, \lambda), \tag{3.6b}$$

note that, in this case, the transformed parameters depend also on the fields contained in the Supergravity multiplet $\{e, \psi, m, b\}$. This happens because the structure constants of the algebra depend on the fields, however, even in this case, the CCWZ construction still works. Equation (3.5) defines what is known as a nonlinear realization of the group S on the multiplet $\{\lambda, e, \psi, m, b\}$, where the transformation under S of the additional field λ is in accordance with equation (3.6a).

This result suggests that any representation of H can be promote to a realization of S with the help of the parameter λ. To show this, consider a vector space spanned by A that linearly transforms under the action of $h \in H$ as

$$h : A \longrightarrow D(h) \circ A, \tag{3.7}$$

with $D(h)$ being a unitary representation of the subgroup H. In spite of equation (3.6b), the transformation of A can be extended to a realization of S:

$$s : A \longrightarrow D(e^{-u' \cdot T}) \circ A, \tag{3.8}$$

where u' parametrizes an element of H and it is a function that depends on both λ and s. The transformation presented in equation (3.8) is indeed a realization of S, this can be seen by the composition of the elements s_1 and s_2 of S, the action of each of these elements leads to

$$s_1 e^{-\lambda \cdot Q} = e^{-\lambda' \cdot Q} e^{-u' \cdot T} \tag{3.9a}$$

$$s_2 e^{-\lambda' \cdot Q} = e^{-\lambda'' \cdot Q} e^{-u'' \cdot T}, \tag{3.9b}$$

and their composition gives

$$s_2 s_1 e^{-\lambda\cdot Q} = e^{-\lambda''\cdot Q} e^{-u''\cdot T} e^{-u'\cdot T}. \tag{3.10}$$

From the last expression, we can make the identification

$$e^{-u'''\cdot T} = e^{-u''\cdot T} e^{-u'\cdot T}, \tag{3.11}$$

and, using the fact that D is a faithful representation of H, we are allowed to imply the closure relation

$$D(e^{-u'''\cdot T}) = D(e^{-u''\cdot T}) D(e^{-u'\cdot T}), \tag{3.12}$$

which means that the group G is realized on the vector space spanned by A.

In a practical way, we are looking for the proper transformation of λ that linearizes the transformation under the unbroken symmetry of the other fields. It is possible to classify all the nonlinear realizations of Supergravity that become linear when the group is restricted to $G \times L$ invariance and build a nonlinear Lagrangian density that, by construction, is invariant under the nonlinear field transformations.

Start by considering the Supergravity multiplet $\{e, \psi, m, b\}$ whose transformation properties are given in equations (2.33):

$$\delta_\epsilon e_\mu{}^a = i\left(\psi_\mu \sigma^a \bar{\epsilon} - \epsilon \sigma^a \bar{\psi}_\mu\right) \tag{3.13a}$$

$$\delta_\epsilon \psi_\mu = -2\hat{D}_\mu(\epsilon) + i\left[m\sigma_\mu\bar{\epsilon} + b_\mu\epsilon + \frac{1}{3} b^\nu(\sigma_\mu\bar{\sigma}_\nu\epsilon)\right] \tag{3.13b}$$

$$\delta_\epsilon m = -\frac{1}{3}\epsilon(\sigma^\mu\bar{\sigma}^\nu\psi_{\mu\nu} + ib^\mu\psi_\mu - 3i\sigma^\mu\bar{\psi}_\mu m) \tag{3.13c}$$

$$\begin{aligned}\delta_\epsilon b^a = {} & \frac{3}{8}(\bar{\psi}_{\mu\nu}\sigma^a\bar{\sigma}^\mu\sigma^\nu\epsilon) - \frac{1}{8}(\bar{\psi}_{\mu\nu}\bar{\sigma}^\mu\sigma^\nu\bar{\sigma}^a\epsilon) + \\ & - \frac{3i}{2} m^*(\epsilon\psi^a) - \frac{i}{8} b_c(\epsilon\sigma^c\bar{\sigma}^a\sigma^\mu\bar{\psi}_\mu) + \\ & + \frac{i}{4} b^a(\epsilon\sigma^\mu\bar{\psi}_\mu) + \frac{i}{8} b^c(\bar{\psi}_\mu\bar{\sigma}^a\sigma_c\bar{\sigma}^\mu\epsilon) + \text{c.c.}\end{aligned} \tag{3.13d}$$

Since Supergravity is broken, we add to this multiplet the Goldstone fermion λ that transforms non-linearly under Supersymmetry. The transformation of λ is essential to understand how the nonlinear realization works. For this reason we will present a method to derive this transformation rule up to first order in the transformation parameter.

We start by rephrasing the general equation (3.5) that defines the nonlinear realization as

$$e^{\lambda'\cdot Q} s e^{-\lambda\cdot Q} = h(\lambda, s), \tag{3.14}$$

in this case, we are calling $h(\lambda, s)$ the element of the stability group that maps the full group S into H. If we take the element s to be a pure supersymmetric transformation, say an element of the coset space given by

$$s \equiv e^{\epsilon\cdot Q}, \tag{3.15}$$

then the transformation of λ can be put into the form

$$\lambda'(x') = \lambda(x) + \eta(\lambda, \epsilon), \tag{3.16}$$

where η is a function that depends on λ, the parameter ϵ and the fields in the multiplet.

Following this argument, it is possible to recast the left hand side of equation (3.16) by considering that λ transforms as a scalar under diffeomorhpisms and as a spinor under Lorentz transformations, therefore

$$\lambda'(x) - \alpha^\mu \partial_\mu \lambda - \frac{1}{2}\beta_{ab}\sigma^{ab}\lambda = \lambda(x) + \eta \tag{3.17}$$

where we associate the parameters α^μ and β_{ab} to the resulting stability group transformation, or, in simplier words; these parameters are the ones obtained by the transformation $h(\lambda, s)$ of equation (3.14), so we have

$$\log h(\lambda, s) = \delta_G(\alpha^\mu) + \delta_L(\beta_{ab}). \tag{3.18}$$

To establish a transformation law for the gravitino λ, we need to compute the quantity $\delta_\epsilon \lambda = \lambda'(x) - \lambda(x)$, this can be done if we find an expression of the parameter η in terms of the parameter ϵ of the transformation. Such parameter ϵ is introduced by considering the element $s = e^{\epsilon \cdot Q}$ of the coset space S/H, in this sense we write the equation (3.14) as

$$e^{(\lambda+\eta)\cdot Q} e^{\epsilon \cdot Q} e^{-\lambda \cdot Q} = h(\lambda, \epsilon), \tag{3.19}$$

where we used the already mentioned definition of the parameter η. Now expand the left-hand side of the former equation using the Baker-Campbell-Hausdorff formula up to order ϵ, and take the natural logarithm in both sides, after some algebraic steps one obtains the simplified expression

$$\delta_\epsilon + \delta_\eta + \frac{1}{2}[\delta_\lambda, \delta_\eta] + \frac{1}{3!}[\delta_\lambda, [\delta_\lambda, \delta_\eta]] + \ldots = \delta_G(\alpha^\mu) + \delta(\beta_{ab}), \tag{3.20}$$

here, many terms in the expansion have been neglected because we know that η starts at order ϵ.

This analysis allows us to compute the parameters α^μ and β_{ab} of the stability group transformation using only the algebra of Supergravity. We rewrite equation (3.17) as

$$\delta_\epsilon \lambda = \eta + \alpha^\mu \partial_\mu \lambda + \frac{1}{2}\beta_{ab}\sigma^{ab}\lambda, \tag{3.21}$$

and then, by replacing the parameters α and β, we obtain the supersymmetric transformation of λ:

$$\begin{aligned} \delta_\epsilon \lambda = -\epsilon + \frac{1}{4} y \cdot \left[(\psi + \delta_\lambda \psi) - \frac{2i}{3} m \sigma \bar{\lambda} \right] + 2m^*(\lambda\epsilon)\lambda + \\ - \frac{4}{3}(\lambda\epsilon)(\bar{\lambda}\not{b}) + \ldots, \end{aligned} \tag{3.22}$$

where the additional parameter y^μ comes from the commutator between two supersymmetric transformations, it has the form

$$y^\mu = 2i(\lambda\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\lambda}). \tag{3.23}$$

This non-trivial transformation of λ was first obtained by A. A. Kapustnikov in [57], we show more relevant steps of this derivation in Appendix F.

Now we shift the discussion to a different point: the CCWZ construction [53, 54]. This is a formalism that allows us to write an action invariant under an imposed broken symmetry in terms of the fields of the theory and the introduced local parameter λ. The presence of λ in the action indicates that the theory contains an additional degree of freedom. However, since λ comes from a gauge redundancy, we do not expect such field to propagate but it represents an additional polarization state of the physical field. At sufficiently high energies λ decouples from the theory and the field behave as massless particle. The advantage of writing the action in this form is that, in the context of supergravity, it makes explicit the presence of the helicity 1/2 component of the gravitino[1] which plays an important role in the dynamics.

Following the prescription stated in [58] by L. V. Delacrétaz, V. Gorbenko and L. Senatore, to apply the CCWZ construction to Supergravity we need to write a Lagrangian in terms of fields, say A, that transform linearly under the action of s but with a transformation parameter that depends on λ. If we refer to the fields in the multiplet as a, then we define these new fields as

$$A = D(e^{\lambda\cdot Q}) \circ a, \tag{3.24}$$

[1]In the bosonic case, the CCWZ constructions allows us to write an action where the non parallel polarization state of the metric is explicit. This state is the Goldstone boson π that at high energies decouples from the theory. Here, in the fermionic case, the component with helicity 1/2 of the gravitino particle turns out to be λ.

so we refer to the fields denoted with capital letter as dressed fields. This is actually the correct way to *dress* the fields if we want to restore the gauge symmetry, note that now they transform linearly as

$$s : A \longrightarrow D(e^{-u'\cdot T}) \circ A, \tag{3.25}$$

and here is why: applying s to equation (3.24) gives

$$s : A \longrightarrow D(e^{\lambda'\cdot Q}) \circ a' = D(e^{\lambda'\cdot Q} s) \circ a, \tag{3.26}$$

on the other hand, we can use equation (3.5) to show that

$$e^{\lambda'\cdot Q} = e^{-u'\cdot T} e^{\lambda\cdot Q} s^{-1}, \tag{3.27}$$

so when we replace this result back into (3.26), we obtain

$$s : A \longrightarrow D(e^{-u'\cdot T} e^{\lambda\cdot Q} s^{-1} s) \circ a = D(e^{-u'\cdot T})A, \tag{3.28}$$

as expected.

We are looking for a Lagrangian written in terms of dressed fields, here denoted as A, that transform linearly under S. We compute these fields right from the Supergravity multiplet by applying the definition (3.24) which is expressed explicitly as

$$A = a + \delta_\lambda a + \frac{1}{2}\delta_\lambda^2 a + \ldots, \tag{3.29}$$

so the dressed fields are obtained by using the transformations given in (2.33), the result is:

$$E_\mu{}^a = e_\mu{}^a + i(\psi_\mu - D_\mu\lambda)\sigma^a\bar{\lambda} + \text{c.c.} + \ldots \tag{3.30a}$$

$$\Psi_\mu = \psi_\mu - 2D_\mu\lambda + i\left[m\sigma_\mu\bar{\lambda} + b_\mu\lambda + \frac{1}{3}\sigma_\mu\not{b}\lambda\right] + \ldots \tag{3.30b}$$

$$M = m - \frac{2}{3}\lambda\sigma^{\mu\nu}\psi_{\mu\nu} + \ldots \tag{3.30c}$$

$$B^a = b^a + \bar{\psi}_{\mu\nu}\left(\bar{\sigma}^a\sigma^{\mu\nu} + \frac{i}{4}\varepsilon^{a\mu\nu\alpha}\bar{\sigma}_\alpha\right)\lambda + \text{c.c.} + \ldots \tag{3.30d}$$

where the dots represent terms with more than two fermions. In this approach those terms are neglected because they do not contribute to the quadratic Lagrangian. This construction ensures that the new fields do not change the algebra, this is guaranteed by the transformation of the Goldstone fermion given in (3.22), note also that since this construction is Lagrangian independent and since it is invariant under all imposed symmetries, the algebra still closes off-shell. This result resambles the one obtained by M. Roček in [59].

The fields defined in (3.30) are the basic elements to carry out a Lagrangian that is invariant under Supergravity. The mechanism to build such Lagrangian is known as the Stückelberg trick.

3.2 Stückelberg Action

The previous section was devoted to the explanation of how to construct fields that non-linearly realize a broken symmetry. This prescription is particularly useful because it allows us to impose constraints on the fields that are invariant under all symmetries. In this sense, we can get rid of the auxiliary fields without imposing any equation of motion, instead we eliminate these degrees of freedom by applying some invariant constraints, meaning that our description will still be valid off-shell.

Since Supersymmetry has broken down, the degrees of freedom coming from the auxiliary fields are not longer needed and, in fact, there are not good reasons to preserve the equality between bosonic and fermionic states. All the relevant degrees of freedom are encoded in a new representation of Supergravity constituted by $e_\mu{}^a$, ψ_μ and λ. The constraints we impose to the auxiliary fields that are invariant under all symmetries are

$$M = 0 \qquad \text{and} \qquad B = 0, \tag{3.31}$$

these are solved in terms of ψ_μ and λ thus resulting in

$$m = \frac{2}{3}\lambda\sigma^{\mu\nu}\psi_{\mu\nu} + \dots \tag{3.32a}$$

$$b^a = -\bar{\psi}_{\mu\nu}\left(\bar{\sigma}^a\sigma^{\mu\nu} + \frac{i}{4}\varepsilon^{a\mu\nu\alpha}\bar{\sigma}_\alpha\right) + \text{c.c.} + \dots\lambda, \tag{3.32b}$$

with these solutions we get rid of the auxiliary fields. Specifically, after eliminating the auxiliary fields, the multiplet $\{e_\mu{}^a, \psi_\mu, \lambda\}$ transforms as

$$\delta_\epsilon e_\mu{}^a = i(\psi_\mu\sigma^a\bar{\epsilon} - \epsilon\sigma^a\bar{\psi}_\mu) + \dots \tag{3.33a}$$

$$\delta_\epsilon\psi_\mu = -2D_\mu\epsilon + \dots \tag{3.33b}$$

$$\delta_\epsilon\lambda = -\epsilon + \frac{1}{4}y\cdot(\psi - 2D\lambda) + \dots \tag{3.33c}$$

and the closure of the algebra is assured.

Now we redefine the remaining fields in order to make them to transform linearly under the complete symmetry:

$$E_\mu{}^a = e_\mu{}^a + i(\psi_\mu - D_\mu\lambda)\sigma^a\bar{\lambda} + \text{c.c.} + \dots \tag{3.34a}$$

$$\Psi_\mu = \psi_\mu - 2D_\mu\lambda + \dots, \tag{3.34b}$$

these new fields, already obtained in (3.30), are called the Stückelbergized fields and we use them to construct the Stückelberg action starting from a general action S

$$S[e,\psi,\lambda] \longrightarrow S[E,\Psi(E,\psi,\lambda),\psi,\lambda], \tag{3.35}$$

where, again, we only have to ensure that $S[e,\psi,\lambda]$ is invariant under local Lorentz transformations and diffeomorphisms. If this condition holds, then the Stückelberg action will be naturally invariant under Supersymmetry. A further simplification comes from the fact that we are neglecting terms with four fermions or more, by looking at the definitions in (3.34), we have

$$S_{\text{dS}}[E,\Psi(E,\psi,\lambda),\psi,\lambda] = S_{\text{dS}}[E,\Psi] + \dots \tag{3.36}$$

therefore, the Stückelberg trick consists on simply replacing the fields ${e_\mu}^a$ and ψ_μ with the dressed fields ${E_\mu}^a$ and Ψ_μ.

To further illustrate this method, we show bellow how to obtain the minimal model of broken Supergravity using the Stückelberg trick. Start by considering the de Sitter Lagrangian

$$S_{\mathrm{dS}} = \int \mathrm{d}^4 x \left[-\frac{1}{2} M_{\mathrm{pl}}^2 e R + \mathcal{L}_{\psi\psi} - \Lambda \right], \tag{3.37}$$

where the term

$$\mathcal{L}_{\psi\psi} = M_{\mathrm{pl}}^2 e \left[\frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} (\bar{\psi}_\mu \bar{\sigma}_\nu D_\rho \psi_\sigma) + m_{3/2} \psi_\mu \sigma^{\mu\nu} \psi_\nu + \text{c.c.} \right], \tag{3.38}$$

clearly breaks Supersymmetry by giving a mass of $m_{3/2}$ to the gravitino. Such Lagrangian can be recast as

$$\mathcal{L}_{\psi\psi} = M_{\mathrm{pl}}^2 e \left[\frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} (\bar{\psi}_\mu \bar{\sigma}_\nu \mathcal{D}_\rho \psi_\sigma) \right], \tag{3.39}$$

by introducing the generalized covariant derivative

$$\mathcal{D}_\mu \psi_\nu = D_\mu \psi_\nu - \frac{i}{2} m_{3/2}^* \sigma_\mu \bar{\psi}_\nu . \tag{3.40}$$

Now, the Stückelberg trick is performed by substituting the goldstino with

$$\psi_\mu \longrightarrow \psi_\mu - 2\mathcal{D}_\mu \lambda + \dots \tag{3.41}$$

and the Lagrangian becomes

$$\begin{aligned} \frac{1}{e M_{\mathrm{pl}}^2} (\mathcal{L}_{\psi\psi} + \mathcal{L}_{\psi\lambda} + \mathcal{L}_{\lambda\lambda}) = \\ \varepsilon^{\mu\nu\rho\sigma} \Big(\frac{1}{2} \bar{\psi}_\mu \bar{\sigma}_\nu \mathcal{D}_\rho \psi_\sigma - \bar{\psi}_\mu \bar{\sigma}_\nu [\mathcal{D}_\rho, \mathcal{D}_\sigma] \lambda + \\ + \mathcal{D}_\mu \bar{\lambda} \bar{\sigma}_\nu [\mathcal{D}_\rho, \mathcal{D}_\sigma] \lambda + \text{c.c.} \Big). \end{aligned} \tag{3.42}$$

The commutator was assembled using the antisymmetric properties of the Levi-Civita tensor. From the definition of the generalized covariant derivative, we note that this commutator can be expressed as

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] = \left(\frac{1}{2}R_{\mu\nu\alpha\beta} - |m_{3/2}|^2 g_{\mu\alpha}g_{\nu\beta}\right)\sigma^{\alpha\beta}, \tag{3.43}$$

which can be further simplified by setting our model in the context of de Sitter space, where

$$R_{\mu\nu\alpha\beta} = -\frac{\Lambda}{3M_{\text{pl}}^2}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}), \tag{3.44}$$

therefore, the commutator reduces to

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] = -\frac{1}{3}|f|^2\sigma_{\mu\nu}, \quad |f|^2 = \left(\frac{\Lambda}{M_{\text{pl}}^2} + 3|m_{3/2}|^2\right). \tag{3.45}$$

Now we have all the required elements to write the resulting Lagrangian, all the medium steps of the complete derivation are explained in Appendix G. The final form of the Lagrangian that describes the interaction between the gravitino and the Goldstone fermion in a minimal model of broken Supergravity in the de Sitter space is

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L}_{\psi\psi} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu D_\rho\psi_\sigma + m_{3/2}\psi_\mu\sigma^{\mu\nu}\psi_\nu + \text{c.c.} \tag{3.46a}$$

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L}_{\psi\lambda} = -i|f|^2\bar{\psi}_\mu\bar{\sigma}^\mu\lambda + \text{c.c.} \tag{3.46b}$$

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L}_{\lambda\lambda} = -i|f|^2\left(\bar{\lambda}\not{D}\lambda + 2im_{3/2}\lambda^2\right) + \text{c.c.} \tag{3.46c}$$

This result agrees with the Lagrangian obtained in [60, 61] using constrained superfields. We will discuss this case in Chapter IV, where the principal difference lies on how the auxiliary fields are eliminated.

For applications in cosmology, the construction of a model with broken Supersymmetry is not enough. We learned that the theory of inflation is a theory where the space-time diffeomorphisms are broken down to only time dependent spatial diffeomorphisms. According to the features of non-linearly realized symmetries, it is necessary to start from a general action that is invariantonly under Local Lorentz and space diffeomorphims, then we restore the remaining symmetries by implementing the Stückelberg trick. We will discuss how to perform such construction in the following section.

3.3 Supersymmetric EFT of Inflation

The effective field theory of inflation gives us a compelling description of the origin of the primordial fluctuations and their evolution. Such theory is constructed by imposing a symmetry that, we assume, it is present in Nature at energies above the energy scale of inflation. Then we build an inflationary action using only the fields at our disposal and declaring a priori the symmetries that are non-linearly realized. For example, C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan and L. Senatore proposed in [25] a model where inflation is thought of as period of time where time diffeomorphisms are spontaneously broken. In this case, it is assumed that the Poincaré group is a fundamental symmetry of Nature present at energies scales above H, during inflation this symmetry is spontaneously broken to time dependent spatial diffeomorphisms. The action that best describes the symmetry breaking mechanism is established using the method of nonlinear realizations, where the time translation parameter is promote to a field called π that transforms non-linearly under time diffeomorphisms. This model was discussed in the first Chapter, here we rewrite the effective action (1.66) that couples π with the graviton

$$S_{\text{eff.}} = \int \mathrm{d}^4x \left[a^3\dot{\pi}^2 - a\alpha \left(\nabla\pi\right)^2 - \frac{\beta}{aH^2}\left(\partial^2\pi\right)^2 + \dots \right], \quad (3.47)$$

where the coefficients α and β depend on time. This action represents the most general action that describes inflation seen as the theory of the breaking of the Poincaré group. However, we can extend this analysis by imposing additional symmetries of Nature at the fundamental level.

In this section we face the case where Supersymmetry is included among the symmetries of Nature. Previously we learned how to implement the Stückelberg trick to restore Supersymmetry as a nonlinear realization, here we implement this method to build the most general action when Supersymmetry and time diffeomorphisms are spontaneously broken. This procedure is performed in two steps: first we restore the Poincaré symmetry and then we reintroduce Supergravity. This procedure is valid only in that order since the Poincaré group is a subgroup of Supergravity, but Supersymmetry is not.

Here we proceed in an analogous way as in the previous section. Start by postulating a Lagrangian for the metric and the gravitino, invariant only under time dependent spatial diffeomorphisms. This Lagrangian, namely $S_{\mathcal{S}\text{eff.}}$, only represents the fermionic sector of the complete theory. Looking at the action (3.37), we note that the most general Lagrangian for the fluctuations in a de Sitter background where time diffeomorphisms and Supersymmetry are spontaneously broken must contain a standard supersymmetric kinetic term, some mass terms for the gravitino and some interaction terms between the metric and the gravitino. These terms are constructed by right combinations of ψ_μ and ψ^0. At leading order, such Lagrangian looks like

$$\begin{aligned} S_{\mathcal{S}\text{eff.}} = M_{\text{pl}}^2 \int \mathrm{d}^4 x e \Bigg[& \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \bar{\sigma}_\nu D_\rho \psi_\sigma + m_{3/2}\psi\sigma^{\mu\nu}\psi_\nu + \\ & + m_0 \psi_\mu \sigma^{\mu 0}\psi^0 + + i m_1 (\bar{\psi}^0 \bar{\sigma}^\mu \psi_\mu - \bar{\psi}_\mu \bar{\sigma}^\mu \psi^0) + \\ & + i m_2 \varepsilon^{\mu\nu\rho 0}\bar{\psi}_\mu \bar{\sigma}_\nu \psi_\rho + m_3(\psi_\mu \psi^\mu + \psi^0\psi^0) + \text{c.c.} + \dots \Bigg], \end{aligned} \tag{3.48}$$

where $m_{3/2}$, m_0, m_1, m_2 and m_3 are parameters that, in general, may depend on time. The dots represent terms of higher order in the fluctuations or the derivatives.

We can rearrange this Lagrangian and group all the quadratic terms, except for the term proportional to m_3 which for convenience we assume is zero, inside the kinetic term by introducing the new covariant derivative that acts on a spinor χ as

$$\mathcal{D}_\mu \chi = D_\mu \chi - \frac{i}{2}\left(M^* \sigma_\mu - m_0^* t_\mu \sigma^0\right)\bar{\chi} - \frac{i}{2}\left(2 m_2 t_\mu - i m_1 \sigma^0 \bar{\sigma}_\mu\right)\chi, \tag{3.49}$$

where $M = m_{3/2} + m_0 g^{00}/2$ and $t_\mu = \delta^0_\mu$. We show how to derive the former expression in Appendix H. This definition allows us to write the fermionic sector of the action as

$$\begin{aligned} S_{\mathcal{S}\text{eff.}} = M_{\text{pl}}^2 \int \mathrm{d}^4 x e \Bigg[& \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu \bar{\sigma}_\nu \mathcal{D}_\rho \psi_\sigma + \\ & + m_3(\psi_\mu \psi^\mu + \psi^0\psi^0) + \text{c.c.} + \dots \Bigg], \end{aligned} \tag{3.50}$$

which involves only the fermionic sector of the complete theory. The full action of the supersymmetric effective field theory of inflation has the form

$$S = S_{\text{eff.}} + S_{\mathcal{S}\text{eff.}}, \tag{3.51}$$

notice that the presence of the gravitino did not constrained any further the allowed terms we could write in the bosonic sector, meaning that all the terms allowed in the absence of Supersymmetry are allowed now so there are not new restrictions imposed by Supersymmetry on them.

Evidently, the action of equation (3.51) represents a theory where gauge invariance is broken, however we are interested in the energy regime where the Goldstone boson decouples and becomes strongly interacting, for this purpose it is useful to reintroduce the gauge invariance by incorporating new fields that transform non-linearly under the gauge transformation. This is the spirit of the nonlinear realizations, where the reintroduction of the gauge invariance is achieved by performing the already mentioned Stückelberg trick.

Invariance under time diffeomorphisms is restored by performing a transformation corresponding to a shift in time and promoting the transformation parameter to a field π, in a practical way this is achieved by making the replacement

$$A^0 \longrightarrow \hat{A}^0 = A^\mu \partial_\mu (t + \pi), \tag{3.52}$$

where A stands for any object with an upper 0 index. Using this formula we can work out the right replacements that will render our action invariant under time diffeomorphisms, here we bring up some examples

$$g^{00} \longrightarrow \hat{g}^{00} = \eta^{ab} e_a{}^\mu e_b{}^\nu \partial_\mu (t + \pi) \partial_\nu (t + \pi) \tag{3.53a}$$

$$\psi^0 \longrightarrow \hat{\psi}^0 = \psi^\mu \partial_\mu (t + \pi) \tag{3.53b}$$

$$\dots$$

so now and forward, the hat notation over any operator will be understood as the replacement of each object within the operator having an upper 0 index with its corresponding redefinition given by equation (3.53).

In this sense, the covariant derivative defined in (3.49) now has the form

$$\hat{\mathcal{D}}_\mu \chi = D_\mu \chi - \frac{i}{2}(\hat{M}^* \sigma_\mu - m_0^* \hat{t}_\mu \hat{\sigma}^0)\bar{\chi} - \frac{i}{2}(2m_2 \hat{t}_\mu - i m_1 \hat{\sigma}^0 \bar{\sigma}_\mu)\chi \tag{3.54}$$

where, as already mentioned, the hat over the operators indicates that all upper 0 indices have been Stückelbergized by means of the transformations (3.53). At this point, we can restore Supersymmetry by redefining the gravitino field as we did in equation (3.41), in this case the Stückelbergized gravitino is

$$\hat{\Psi}^\mu = \psi^\mu - 2\hat{\mathcal{D}}^\mu \lambda + \ldots \tag{3.55}$$

and the Stückelbergized metric is

$$\hat{E}_\mu{}^a = e_\mu{}^a + i\hat{\Psi}_\mu \sigma^a \bar{\lambda} + \text{c.c.} + \ldots \tag{3.56}$$

where we only considered terms with less that three fermions.

We want to emphasize that we are working with a Supergravity multiplet that contains the vierbein, the gravitino, the goldstino and the recently introduced Goldstone boson of time diffeomorphisms. For this reason we have to deal with the transformation law of the Goldstone boson under Supersymmetry. It is possible to assign to π a transformation law under Supergravity such that it transforms as under a time diffeomorphism. This is indeed possible because if we assume that, at the time we restore Supergravity, time diffeomorphism are part of the stability group hence we presume that the field π transforms under time diffeomorphism by a parameter that depends on the goldstino. Let us further explain this concept with more detail.

If the Goldstone field π transforms under the stability group H in a nonlinear representation D_H, then, by the argument of the CCWZ construction, we expect that this bosonic field inherits automatically a nonlinear representation of the full symmetry group S, where its transformation under an element $s \in S$ is obtained from the homomorphisms that goes from S to H, say $e^{-u'\cdot T} = h(\lambda, s)$. In this scenario we write the pure supersymmetric transformation of π as

$$\delta_\epsilon \pi = \frac{1}{2}\left(\delta_G(y) + \delta_L(\Lambda)\right)\pi + \dots \tag{3.57}$$

where we have neglect terms of higher orders. The parameters of the diffeomorphism and the Lorentz transformation are the ones obtained by the commutator $[\delta_\lambda, \delta_\epsilon]$, however the parameter λ will not affect the transformation since the field π acts like a scalar under the Lorentz group. On the other hand, the parameter y can be read from equation (D.21) and, in this case, it takes the form $y^\mu = -2i(\epsilon\sigma^\mu\bar{\lambda} - \lambda\sigma^\mu\bar{\epsilon})$. Therefore, we have the following transformation property for π

$$\delta_\epsilon \pi = \frac{1}{2}y^0 + \frac{1}{2}y^\mu \partial_\mu \pi + \dots, \tag{3.58}$$

where we also introduced the time diffeomorphism transformation $\pi \longrightarrow \pi + t$. In this sense, the field transforms under Supergravity as under time diffeomorphisms and the transformation is proportional to λ. This last point reflects the fact that it is not possible to break time diffeomorphisms without breaking Supersymmetry at the same time.

We have all the elements to write an action that is fully invariant under diffeomorphisms, local Lorentz transformations and local Supersymmetry. After performing the Stückelberg trick twice, we obtain the action

$$S_{\mathcal{S}\text{eff.}} = \frac{1}{2}M_{\text{pl}}^2 \int \mathrm{d}^4x \hat{E}\varepsilon^{\mu\nu\rho\sigma}\bar{\hat{\Psi}}_\mu \bar{\sigma}_\nu \hat{\mathcal{D}}_\rho \hat{\Psi}_\sigma + \text{c.c.} + \dots \tag{3.59}$$

which is similar to the one obtained for the case where time diffeomorphisms were kept unbroken. The difference lies on the definition of the derivative $\mathcal{D}$ and the presence of the hat notation on every operator with an upper 0 index. Similarly to the previous case, the Lagrangian takes the form

$$\frac{1}{eM_{\rm pl}^2}(\mathcal{L}_{\psi\psi}+\mathcal{L}_{\psi\lambda}+\mathcal{L}_{\lambda\lambda}) = \\ \varepsilon^{\mu\nu\rho\sigma}\left(\frac{1}{2}\bar{\psi}_\mu\bar{\sigma}_\nu\hat{\mathcal{D}}_\rho\psi_\sigma - \bar{\psi}_\mu\bar{\sigma}_\nu[\hat{\mathcal{D}}_\rho,\hat{\mathcal{D}}_\sigma]\lambda + \hat{\mathcal{D}}_\mu\bar{\lambda}\bar{\sigma}_\nu[\hat{\mathcal{D}}_\rho,\hat{\mathcal{D}}_\sigma]\lambda\right)+\ldots \tag{3.60}$$

where the ellipses represent, as usual, term with higher order in the perturbation expansion or in the number of fermions. Moreover, the commutator of derivatives $[\hat{\mathcal{D}},\hat{\mathcal{D}}]$ is expressed as

$$[\hat{\mathcal{D}}_\mu,\hat{\mathcal{D}}_\nu]\lambda = \\ \left[\frac{1}{2}R_{\mu\nu\alpha\beta} - \hat{\alpha}g_{\mu\alpha}g_{\nu\beta} + 2\hat{\beta}\hat{t}_{[\mu}g_{\nu]\alpha}\hat{t}_\beta - 2m_1\nabla_{[\mu}\hat{t}_\alpha g_{\nu]\beta}\right]\sigma^{\alpha\beta}\lambda + \\ + \left[-im_0^*\hat{t}^0_{[\mu}\left(\nabla_{\nu]}\hat{t}_\alpha\right) - im_1\hat{\gamma}\hat{t}_{[\mu}g_{\nu]\alpha} - i\partial_{[\mu}\hat{M}^*g_{\nu]\alpha}\right]\sigma^\alpha\bar{\lambda}, \tag{3.61}$$

for simplicity, in the last expression we have introduced the following coefficients:

$$\alpha = |M|^2 - m_1^2 g^{00} \tag{3.62a}$$

$$\beta = m_1^2 + \dot{m}_1 - \mathrm{Re}(Mm_0^*) \tag{3.62b}$$

$$\gamma = M^* - m_0^* g^{00}. \tag{3.62c}$$

Have in mind that these coefficients are intended to be evaluated in $t+\pi$ by means of the transformation (3.52) that acts on every upper 0 index.

We are ready to write down the final form of the constructed Lagrangian. For convenience, we separate the complete expression in three parts. The first part does not include the goldstino, in fact, this sector of the action is related to the kinetic term of the gravitino and has a similar form of the one obtained in equation (3.46a), that is

$$\frac{1}{eM_{\rm pl}^2}\mathcal{L}_{\psi\psi} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\hat{\mathcal{D}}_\rho\psi_\sigma + \text{c.c.} + \ldots \tag{3.63}$$

In this case, the mass of the gravitino, besides $m_{3/2}$, has contributions from m_0, m_1 and m_2. The second part of the Lagrangian is

$$\begin{aligned}
&\frac{1}{eM_{\rm pl}^2}\mathcal{L}_{\psi\lambda} = \\
&-i\Big\{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - 2\hat{\beta}\partial_\mu(t+\pi)\partial_\nu(t+\pi) - 2m_1\nabla_\mu\partial_\nu(t+\pi) + \\
&\quad + \Big[3|\hat{M}|^2 + g^{\alpha\beta}\partial_\alpha(t+\pi)\partial_\beta(t+\pi)\left(2\hat{\beta} - 3m_1^2\right) + \\
&\qquad + 2m_1\nabla^\alpha\partial_\alpha(t+\pi)\Big]g_{\mu\nu}\Big\}\bar{\psi}^\mu\bar{\sigma}^\nu\lambda + \\
&+\Big\{4\Big[-2i\hat{M}^* m_2 + m_1\left(\hat{M}^* - m_0^* g^{\rho\sigma}\partial_\rho(t+\pi)\partial_\sigma(t+\pi)\right) + \\
&\qquad + \frac{1}{2}m_0^*\nabla^\rho\partial_\rho(t+\pi)\Big]g_{\mu\alpha}\partial_\beta(t+\pi) + 4g_{\mu\alpha}\partial_\beta\hat{M}^* + \\
&\quad - 2m_0^*\Big[(\nabla_\mu\partial_\alpha(t+\pi))\partial_\beta(t+\pi) + \\
&\qquad + g_{\mu\alpha}\partial^\rho(t+\pi)\nabla_\beta\partial_\rho(t+\pi)\Big]\Big\}\bar{\psi}^\mu\bar{\sigma}^{\alpha\beta}\bar{\lambda} + \text{c.c.} + \ldots,
\end{aligned} \tag{3.64}$$

where the usual gravitational covariant derivative is ∇_μ and the derivative with the spinor connection is D_μ. This Lagrangian is regarded as all the mixing terms between the goldstino λ and the gravitino ψ, from equation (3.60) we have

The third part of the Lagrangian involves the goldstino kinetic and mass terms, note that it can be obtained from the previous expression by performing the replacement $\psi_\mu \to -\hat{\mathcal{D}}_\mu \lambda$, the result is

$$\begin{aligned}
&\frac{1}{eM_{\rm pl}^2}\mathcal{L}_{\lambda\lambda} = \\
&i\Big\{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - 2\hat{\beta}\partial_\mu(t+\pi)\partial_\nu(t+\pi) - 2m_1\nabla_\mu\partial_\nu(t+\pi) + \\
&\quad + \Big[3|\hat{M}|^2 + g^{\alpha\beta}\partial_\alpha(t+\pi)\partial_\beta(t+\pi)\left(2\hat{\beta} - 3m_1^2\right) + \\
&\quad + 2m_1\nabla^\alpha\partial_\alpha(t+\pi)\Big]g_{\mu\nu}\Big\}\hat{\mathcal{D}}^\mu\bar{\lambda}\bar{\sigma}^\nu\lambda + \\
&-\Big\{4\Big[-2i\hat{M}^* m_2 + m_1\left(\hat{M}^* - m_0^* g^{\rho\sigma}\partial_\rho(t+\pi)\partial_\sigma(t+\pi)\right) + \\
&\quad + \frac{1}{2}m_0^*\nabla^\rho\partial_\rho(t+\pi)\Big]g_{\mu\alpha}\partial_\beta(t+\pi) + 4g_{\mu\alpha}\partial_\beta\hat{M}^* + \\
&\quad - 2m_0^*\Big[(\nabla_\mu\partial_\alpha(t+\pi))\partial_\beta(t+\pi) + \\
&\quad + g_{\mu\alpha}\partial^\rho(t+\pi)\nabla_\beta\partial_\rho(t+\pi)\Big]\Big\}\hat{\mathcal{D}}^\mu\bar{\lambda}\bar{\sigma}^{\alpha\beta}\bar{\lambda} + \text{c.c.} + \ldots,
\end{aligned} \tag{3.65}$$

notice that both of these expressions are written in terms of the metric $\hat{g}$ instead of the Stückelbergized field $\hat{G}$, we do this because at this order in the number of fermions the contributions of the additional terms of the metric, as it can be seen from equation (3.56), are neglegible.

Symmetry Breaking II

4.1 Global Supersymmetry Breaking

This section contains general remarks on the study of constrained superfields. We focused on models that explain the spontaneous symmetry breaking of global Supersymmetry by the introduction of a chiral superfield X. This part is based on the work of N. Cribiori who wrote a nice review of the state of art of this subject in [62]. The discussion will be helpful later when this formalism is applied to the local case, which is the main topic of the next section.

So far, we focused the discussion about Supersymmetry breaking around the bottom-up effective field theory provided by the CCWZ construction. In the case where we reintroduce supersymmetry at the Lagrangian level we were able to write a general Lagrangian that describes the interaction of the gravitino and the field λ. The result is the Lagrangian of equation (3.46) that has the form:

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L}_{\psi\psi} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu D_\rho\psi_\sigma + m_{3/2}\psi_\mu\sigma^{\mu\nu}\psi_\nu + \text{c.c.} \tag{4.1a}$$

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L}_{\psi\lambda} = -i|f|^2\bar{\psi}_\mu\bar{\sigma}^\mu\lambda + \text{c.c.} \tag{4.1b}$$

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L}_{\lambda\lambda} = -i|f|^2\left(\bar{\lambda}\not{D}\lambda + 2im_{3/2}\lambda^2\right) + \text{c.c.} \tag{4.1c}$$

We will reproduce this result using an alternative, and fundamentally different, method known as the top-down effective theory. First, we study the spontaneously breaking mechanisms and nonlinearly realization of global Supersymmetry, then we will extend this prescription to local Supersymmetry.

We start by introducing the superspace formulation of the Volkov-Akulov model [55] for the minimal Supersymmetrical case. As we had discussed previously, Supersymmetry is broken by the presence of a fermionic field λ, this field can be thought of as the component of a chiral superfield X which has to be an irreducible representation of Supersymmetry. Chiral superfields are characterized by the condition

$$\bar{D}_{\dot{\alpha}}X = 0, \tag{4.2}$$

where the covariant derivative D was defined, for the rigid case, in equations (2.10). Solving the condition (4.2) leads us to the expansion in θ and $\bar{\theta}$ of the chiral superfield X in terms of its components A, G_α and F:

$$X = A + \sqrt{2}\theta^\alpha G_\alpha + \theta^2 F, \tag{4.3}$$

where A represents a scalar field, G_α is a Weyl fermion that eventually will be related with λ and F is a complex auxiliary field. The sense of introducing this superfield is to spontaneously break Supersymmetry for cosmological purposes.

We can construct with these ingredients a Lagrangian that is invariant under Supersymmetry that breaks, spontaneously, the ground state of the spectrum. Such Lagrangian has the form

$$\mathcal{L} = \int \mathrm{d}^4\theta K(X, \bar{X}) + \left(f \int \mathrm{d}^2\theta W(X, \bar{X}) + \text{c.c.} \right) \qquad (4.4)$$

where $K(X, \bar{X})$ is a general Kähler potential and $W(X, \bar{X})$ is a superpotential.

For illustrative purposes, consider the simplest case by taking K to be the canonical Kähler potential and W to be a linear potential. When these assumptions are incorporated in the theory, we obtain the Lagrangian

$$\mathcal{L} = \int \mathrm{d}^4\theta X\bar{X} + f \int \mathrm{d}^2\theta X + f^* \int \mathrm{d}^2\bar{\theta}\bar{X}, \qquad (4.5)$$

after integrating over superspace, this Lagrangian is expressed in terms of the component fields as

$$\mathcal{L} = \partial_\mu A \partial^\mu \bar{A} - i\bar{G}\bar{\sigma}^\mu \partial_\mu G + F\bar{F} + fF + f^*\bar{F}, \qquad (4.6)$$

and note that the presence of the auxiliary field F is responsible of the Supersymmetry breaking, as a matter of fact one can integrate out F using the equations of motion

$$F = -f \qquad \text{and} \qquad \bar{F} = -f^*, \qquad (4.7)$$

thus producing a constant and positive definite scalar potential of the form

$$\mathcal{L} = \partial_\mu A \partial^\mu \bar{A} - i\bar{G}\bar{\sigma}^\mu \partial_\mu G - f^2, \qquad (4.8)$$

which leads to a spontaneous symmetry breaking of the ground sate at the energy scale of $\sqrt{f}$.

So far, we developed a toy model of spontaneous symmetry breaking by introducing a chiral superfield X with a superptoential W. In this scenario, the supersymmetric transformation of G_α with parameter ϵ is defined as

$$\delta_\epsilon G_\alpha = \epsilon^\alpha D_\alpha X\Big|_\theta \tag{4.9a}$$

$$\delta_\epsilon G_\alpha = -\sqrt{2} f \epsilon_\alpha + i\sqrt{2}(\sigma^\mu \bar{\epsilon})_\alpha \partial_\mu A, \tag{4.9b}$$

which leaves the Lagrangian $\mathcal{L}$ invariant. In addition, we see that G_α transforms inhomogeneously as soon as the auxiliary field F acquires a vacuum expectation value $\langle F \rangle = -f$, hence we recognize that G_α is the Goldstone fermion of the symmetry breaking triggered by the auxiliary field when taking a non-zero expectation value.

Following with this example, we intuit that a low energy effective theory for this model should be possible and it must be obtained by integrating out the massive degrees of freedom[1]. In this approach, however, both the scalar field A and the fermionic field G_α are massless.In addition, we know that the vanishing mass of the Goldstone fermion is protected by the Goldstone theorem of symmetry breaking, leaving the scalar A as the only field that can acquire mass.

At this stage, if we want an effective theory that describes the Goldstone fermion, then we should consider a different Lagrangian coming from a Kähler potential with a curvature term. Take as an example the curved Kähler form

$$dK(X, \bar{X}) = \left(X\bar{X} - \frac{1}{\Lambda^2} X^2 \bar{X}^2 \right) \mathrm{d}^4\theta, \tag{4.10}$$

where the parameter Λ has dimensions of mass, and place it in the Lagrangian (4.4), after some algebraic developement one can

[1]We are looking for reliable models that can be applied at low energies, where the Goldstone fermion is effectively decoupled from additional fields of the multiplet.

show that the Lagrangian turns out to be

$$\begin{aligned}\mathcal{L} = & -\kappa^2\partial_\mu A\partial^\mu \bar{A} - i\kappa^2(G\sigma^\mu\partial_\mu G) + \frac{4i}{\Lambda^2}A\partial_\mu\bar{A}(G\sigma^\mu\bar{G})+ \\ & -\frac{G\bar{G}}{\Lambda^2} - \frac{2f}{\Lambda^2\kappa^2}\left(G^2\bar{A}+\bar{G}^2A\right) - \frac{4}{\Lambda^4\kappa^2}\left(G^2\bar{G}^2A\bar{A}\right) + \\ & -\frac{f^2}{\kappa^2} + \left|F\kappa + \frac{f}{\kappa} + \frac{2}{\Lambda^2\kappa^2}G^2\bar{A}\right|^2,\end{aligned} \tag{4.11}$$

where we defined the quantity κ as

$$\kappa^2 = 1 - 4A\bar{A}/\Lambda^2. \tag{4.12}$$

The integration of the auxiliary fields follows immediately. Note that the mass m_A of the scalar field A can be obtained by taking the low energy limit $E \ll f/\Lambda$, which is equivalent to the substitution

$$\kappa \sim \frac{1}{\Lambda} \longrightarrow \infty, \tag{4.13}$$

together with the replacement of the auxiliary field with its own expectation value $\langle F\rangle = f$. This can be seen by the expansion of the term with the squared modulus in (4.11), which leads to

$$\mathcal{L} = -\kappa^2\partial_\mu A\partial^\mu\bar{A} - \frac{f^2}{\kappa^2} + f^2\kappa^2 + \frac{f^2}{\kappa^2} + 2f^2 + (\text{terms with } G), \tag{4.14}$$

henceforth, the mass of the scalar field A is

$$m_A = 4f^2/\Lambda^2. \tag{4.15}$$

If we restrict our analysis to an energy regime bellow f/Λ we find another important feature of the theory. To effectively eliminate the degrees of freedom coming from the field A, we use its equation of motion, namely

$$A = \frac{G^2}{2F}, \tag{4.16}$$

so the effective Lagrangian takes the following form

$$\mathcal{L}_{\text{eff.}} = -i\left(G\sigma^{\mu}\partial_{\mu}\bar{G}\right) + F\bar{F} + f(F+\bar{F}) + \frac{\bar{G}^2\partial^2 G^2}{4F\bar{F}}. \tag{4.17}$$

Equation (4.17) describes a model where the field G is embedded in a potential defined by F and Supersymmetry is spontaneously broken: notice that the spectrum is not supersymmetric as there is only one fermionic field. Moreover, this Lagrangian does not depend on the parameter Λ, this is not a surprise since the information of the high energy regime has been lost.

Now we place our attention on the field F. We know that this field does not propagate so it represents auxiliary degrees of freedom that are not physical and, therefore, we use its equation of motion, which is

$$F = -f - \frac{1}{4f^3}\bar{G}^2\partial^2 G^2 + \frac{3}{16f^7}\bar{G}^2 G^2\partial^2\bar{G}^2\partial^2 G^2, \tag{4.18}$$

to write an effective Lagrangian that only contains the field G:

$$\mathcal{L}_{\text{eff.}} = -f^2 + i\partial_{\mu}\bar{G}\bar{\sigma}^{\mu}G + \frac{1}{4f^2}\bar{G}^2\partial^2 G^2 - \frac{1}{16f^6}G^2\bar{G}^2\partial^2 G^2\partial^2\bar{G}^2. \tag{4.19}$$

This describes the dynamics of the Goldstone fermion of spontaneous Supersymmetry breaking bellow the decoupling limit. The Lagrangian is invariant under the non-linear transformation

$$\delta_{\epsilon}G_{\alpha} = \sqrt{2}F\epsilon_{\alpha} + i\sqrt{2}(\sigma^{\mu}\bar{\epsilon})_{\alpha}\partial_{\mu}\left(\frac{G^2}{2F}\right) \tag{4.20}$$

which is, moreover, non homogeneous; this signals the fact that G is the Goldstone fermion of the symmetry breaking mechanism.

With this model, we described the interactions of the Goldstone fermion G_{α} at low energies. for its derivation, start from a simple Lagrangian written in the language of superfields, and then integrate out the scalar component of the superfield. The result resembles the one obtained by D. V. Volkov and V. P. Akulov in [55] using only geometric arguments, thus strengthening the fact that the Volkov-Akulov model is universal.

So far, our analysis has been restricted to a theory that is model dependent. However, we can generalize our result by noticing that equation (4.16) is actually an inherent property of any Lagrangian obtained from the Volkov-Akulov model. In this sense, we can construct a general model of spontaneous symmetry breaking by ensuring that the scalar component of the superfield is determined by (4.16). This idea is fundamental when constructing a top down supersymmetric effective theory of inflation.

It is possible to construct a superfield which contains the fermion G_α the auxiliary field F and whose lowest component is the specific combination $G^2/2F$ as in equation (4.16). It turns out that such field is, in fact, a chiral field that satisfies an additional constrain. The corresponding constraint was first found in [59] and it was linked with the nonlinear realization of Supersymmetry in [63]. It was found that, in order to build a general model of broken Supersymmetry, it was necessary to introduce a superfield X with the following constraints

$$\bar{D}_{\dot{\alpha}} X = 0 \qquad \text{and} \qquad X^2 = 0, \tag{4.21a}$$

where the first equation indicates that X is chiral and the second one[2] is imposed to eliminate the scalar field A. In Appendix I we show how to solve such constraint, the field X turns out to be

$$X^2 = 0 \longrightarrow X = \frac{G^2}{2F} + \sqrt{2}\theta^\alpha G_\alpha + \theta^2 F, \tag{4.22}$$

[2]Notice that this constraint is indeed supersymmetric and its solution is still a chiral superfield.

as we intended. The logic behind this analysis is that we can remove components of a superfield by imposing specific contraints, these components will be expressed in terms of the remaining fields.

With the use of constrained superfields, effective theories can be constructed without carrying out the equations of motion of the heavy fields. As a matter of fact, the Lagrangian of equation (4.17) can be produced from the initial Lagrangian (4.4) by replacing the lower component of X with $G^2/2F$, this is exactly what the implementation of the constraint means. The result is a Lagrangian that is invariant under non-linearly realized Supersymmetry and where Supersymmetry is spontaneously broken with G_α being the Goldstone fermion.

Constrained superfields are also useful when we want to eliminate components of additional superfields. In [64] it was established a method to correctly select appropriate constraints that remove any selected component from a generic superfield. These tools allow us to carry out models where Supersymmetry is non-linearly realized via the method of constrained superfields, i.e. using the language of linear Supersymmetry. This is the prime advantage of the method of constrained superfields.

The model previously presented is only an illustrative example made to motivate the method of constrained superfields. Nevertheless, in the following stage we will study the case of minimal constrained Supergravity, which is the area of interest in the field of cosmology.

4.2 Local Supersymmetry Breaking

Shortly after the Volkov-Akulov model was introduced in [55], S. Samuel and J. Wess proved in [65, 66] that this action was not unique in curved space. They derived the local Supersymmetry transformation law for the Volkov-Akulov field[3] for $\mathcal{N} = 1$ Supergravity.

It is interesting to study the nonlinear realization formalism in the context of Supergravity theories. It allows us to derive effective models that have applications in cosmology. For instance, some inflationary models have been developed by extending the tools of nonlinear realization of Supersymmetry to the local case (see for example [67, 68, 69, 70, 71, 72]).

These models relay on the equations of motion of the components of the Goldstino superfield, thereby they study an effective action in a determined energy regime. However, a more suitable procedure is the one of constrained superfields. This prescription consists on imposing equations that are invariant under Supergravity and that define a constraint directly on the superfields involved in the theory. Consequently, the degrees of freedom coming from heavy particles are removed as if they were integrated out. Recently, the interest on this topic has increased because it allows us to construct cosmological models with nonlinear Supergravity. Special attention has been placed on minimal constrained Supergravity [73, 74], which is the main subject of this work.

We devote this section to discuss the proceedings of creating a general model of broken Supergravity. If Supergravity is realized in Nature, then it must be spontaneously broken above some energy scale, hence the mechanisms of nonlinear realized Supergravity becomes of phenomenological relevance. We construct here a general model where Supergravity is spontaneously broken by the presence of a fermionic field $\mathcal{G}_\alpha$ that transforms non-linearly under supersymmety, in this regard we can identify it with the associated Goldsone fermion.

The authors of [75] constructed a Lagrangian that is invari-

[3] Which is related, but is not the same, with the Goldstone fermion λ introduced in Chapter III.

ant under Supergravity transformations which also reduces to the pure supersymmetric counterpart, equation (4.4), in the flat space limit. Here, we use their result and specialized in the case of interest where only a chiral superfield is present.

To define such superfield it is compulsory to introduce the covariant constraint condition of chirality in curved space, we say that X is a chiral superfield if

$$\bar{\mathcal{D}}_{\dot{\alpha}} X = 0, \tag{4.23}$$

where the covariant derivative $\mathcal{D}$ depends on the metric field and it was defined in (2.30). For simplicity, we are slightly changing the notation from $\hat{D}$ to $\mathcal{D}$[4]. In addition, we impose the familiar nilpotent constraint

$$X^2 = 0, \tag{4.24}$$

as we did for the rigid case. Following the main argument of the previous section, we know that the former constraint (4.24) implies that the chiral field X has the form

$$X = \frac{\mathcal{G}^2}{2F} + \sqrt{2}\Theta^\alpha \mathcal{G}_\alpha + \Theta^2 F, \tag{4.25}$$

as we show in Appendix I. We say that such constraint eliminates the degrees of freedom coming from the lowest component of the superfield without using any equation of motion.

The new Θ variables introduced in (4.25) are defined such that the coefficients of the expansion of chiral superfields are precisely

[4] Here we use the curvy $\mathcal{D}$ to denote the covariant derivative that contains the vierbein (and the gravitino) instead of using the hat notation $\hat{D}$ of Chapter II. We did not use that notation then to avoid confusion with the generalized covariant derivative of Chapter III.

the following covariant derivatives:

$$\frac{\mathcal{G}^2}{2F} = X\big|_{\theta=0} \tag{4.26a}$$

$$\mathcal{G} = \frac{1}{\sqrt{2}}\mathcal{D}_\alpha X\big|_{\theta=0} \tag{4.26b}$$

$$F = -\frac{1}{4}\mathcal{D}^\alpha\mathcal{D}_\alpha X\big|_{\theta=0}. \tag{4.26c}$$

These new variables allow us to construct invariant actions. Before that, it is useful to introduce the chiral density, namely $\mathcal{E}$, which is a function of superspace with a transformation law chosen so the product of a chiral field with the chiral density is again a chiral density. For instance,

$$(\mathcal{E}\cdot X) \longrightarrow \mathcal{E}'(X). \tag{4.27}$$

Moreover, we say that $\mathcal{E}$ transforms to a total derivative, thus allowing us to construct an action with the structure

$$\int \mathrm{d}^4x\mathcal{L} = \int \mathrm{d}^4x\mathrm{d}^2\Theta\mathcal{E}\cdot f(X), \tag{4.28}$$

where $f(X)$ is an arbitrary chiral function of X. We know that such action, by construction, satisfies

$$\delta\int \mathrm{d}^4x\mathcal{L} = 0, \tag{4.29}$$

which is a desirable property if we want to build models that are invariant under Supergravity. In this sense, if one wants to write general actions invariant under the action of a symmetry group, attention must be placed on finding the appropriate chiral density.

In the context of Supergravity, the expression of the chiral density was found long ago in [76, 77], here we rewrite it as

$$2\mathcal{E} = e\left[1 + i\Theta\sigma^\mu\bar{\psi}_\mu - \Theta^2(3\bar{m} + \bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu)\right], \tag{4.30}$$

where the fields e, ψ and m are components of the Supergravity multiplet $\{e^a{}_\mu, \psi_\mu, m, b_\mu\}$. A Lagrangian that describes the couplings between these fields and that is invariant under Supergravity transformations can be written in curved space as

$$\frac{1}{M_{\rm pl}^2}\mathcal{L} = \int \mathrm{d}^2\Theta 2\mathcal{E}\left[\frac{1}{2}\mathcal{P}\Omega(X,\bar{X}) + W(X,\bar{X})\right] + \text{c.c.}, \tag{4.31}$$

where the chiral projector $\mathcal{P}$ generalized to curved space is

$$\mathcal{P} = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 8\mathcal{R}), \tag{4.32}$$

and the curvature $\mathcal{R}$ has a component expansion of the form

$$\begin{aligned} 6\mathcal{R} = -3m - \Theta&\left[2\sigma^{\mu\nu}\psi_{\mu\nu} - 3i\sigma^\mu\bar{\psi}_\mu m + i\psi_\mu b^\mu\right] + \\ &+ \Theta^2\left[\frac{1}{2}R - i\bar{\psi}^\mu\bar{\sigma}^\nu\psi_{\mu\nu} - 6|m|^2 - \frac{1}{3}b^2 + i\mathcal{D}_\mu b^\mu - \frac{3}{2}\bar{\psi}^2 m + \right.\\ &\left.\quad + \frac{1}{2}\psi_\mu\sigma^\mu\bar{\psi}_\nu b^\nu - \frac{1}{8}\varepsilon^{\mu\nu\rho\sigma}\left(\bar{\psi}_\mu\bar{\sigma}_\nu\psi_{\rho\sigma} + \psi_\mu\sigma_\nu\bar{\psi}_{\rho\sigma}\right)\right]. \end{aligned} \tag{4.33}$$

The formula (4.31) also contains a real function $\Omega(X,\bar{X})$ that is related with the Kähler potential by

$$\Omega(X,\bar{X}) = -3e^{-K(X,\bar{X})/3}. \tag{4.34}$$

We will see later that the expression of the Lagrangian will be significantly simplified when the degrees of freedom coming from the auxiliary fields m and b_μ are eliminated.

Regarding the previous section, the auxiliary field was integrated out using its equation of motion, here we decided to follow a different path. In order to eliminate the fields m and b_μ we set constraints directly on the superfield $\mathcal{R}$ of curvature. Have in mind that, as discussed in Appendix E, there are two superfields that encode the information of Supergravity, these are the curvature $\mathcal{R}$ and the torsion K. Instead of using the equations of motion of the auxiliary fields, we can restrict the Supergravity multiplet by imposing geometrical constraints.

To correctly eliminate both auxiliary fields m and b_μ we only need to impose constraints over the curvature $\mathcal{R}$. It was shown in [73] that the degrees of freedom coming from the field m are removed by requiring

$$X\mathcal{R} = 0, \tag{4.35}$$

whereas the degrees of freedom of b_μ are eliminated by the constraint

$$X\bar{X}G_{\alpha\dot{\alpha}} = 0. \tag{4.36}$$

Here, the hermitian vector superfield $G_{\alpha\dot{\alpha}}$ indicates the curvature superfield in the spinor representation and is defined in (E.8). These constraints have been known for a while already. Equation (4.35) was first introduced in [78] for global Supersymmetry and then extended in [79] for the local case. In [80] the authors proposed and solved the constraint of equation (4.36).

One advantage of using the formalism of constrained superfields is that it significantly simplifies the calculations that regard the Lagrangian (4.31). For instance, since we imposed the nilpotency of the superfield X, then we know that, without any loss of generality, the Kähler potential and the superpotential have the form

$$K(X,\bar{X}) = X\bar{X} \qquad \text{and} \qquad W(X,\bar{X}) = m_{3/2} + fX, \tag{4.37}$$

where $m_{3/2}$ is a mass scale and $-f$ is the expectation value of F. Therefore, the Lagrangian reduces to the sum of three terms. The usual Einstein-Hilbert action

$$\frac{1}{M_{\text{pl}}^2}\mathcal{L}_1 = -6\int \mathrm{d}^2\Theta\mathcal{E}\mathcal{R} + \text{c.c.}, \tag{4.38}$$

the Kähler term

$$\frac{1}{M_{\mathrm{pl}}^2}\mathcal{L}_2 = \int \mathrm{d}^2\Theta \mathcal{E} X \mathcal{P} \bar{X} + \mathrm{c.c.}, \tag{4.39}$$

and the superpotential

$$\frac{1}{M_{\mathrm{pl}}^2}\mathcal{L}_3 = \int \mathrm{d}^2\Theta 2\mathcal{E} W + \mathrm{c.c.} \tag{4.40}$$

We are able to write the Lagrangian in terms of the component fields $\mathcal{G}$ and F using only the expression of X given in equation (4.25). The Lagrangian becomes

$$\begin{aligned}\frac{1}{eM_{\mathrm{pl}}^2}\mathcal{L} = &-\frac{1}{2}R + \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\psi_{\rho\sigma} - \frac{i}{2}\mathcal{G}\sigma^\mu\mathcal{D}_\mu\bar{\mathcal{G}}+\\ &-\frac{i}{\sqrt{2}}f\mathcal{G}\sigma^\mu\bar{\psi}_\mu + \mathrm{c.c.}\\ &+\frac{1}{4}\left[(\psi_\mu\sigma_\nu\bar{\psi}^\mu)(\mathcal{G}\sigma^\nu\bar{\mathcal{G}}) + i\varepsilon^{\mu\nu\rho\sigma}(\psi_\mu\sigma_\nu\bar{\psi}_\rho)(\mathcal{G}\sigma_\sigma\bar{\mathcal{G}})\right]+\\ &-3m\bar{m} - \frac{1}{2}\left(\bar{m}\mathcal{G}^2\frac{\bar{F}}{F} + \mathrm{c.c.}\right)+\\ &+\frac{1}{3}b_\mu b^\mu + |F+\bar{f}|^2 - |f|^2+\\ &-\left(m_{3/2} + f\frac{G^2}{2F}\right)\left(3\bar{m} + \bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu\right) + \mathrm{c.c.} + \ldots,\end{aligned} \tag{4.41}$$

where the ellipses denote terms of order $\mathcal{O}(\mathcal{G}^3)$. Of course, the complete expansion for the Lagrangian is long and intricate, however for the purpose of this work we only wrote the terms up to order two in the Goldstone fermion expansion, that is to say: only terms with at most two $\mathcal{G}$'s.

As it was discussed previously, this Lagrangian spontaneously breaks Supersymmetry when the auxiliary field F takes the expectation value $\langle F \rangle = -f$. It is not possible to remove this component field by imposing a constraint over the curvature since F does not belong to the Supergravity multiplet. F is the highest component of the chiral field X introduced to break Supergravity and therefore it is not related with the curvature $\mathcal{R}$. We eliminate F using its equation of motion which we write here as

$$F = -\bar{f} - \frac{1}{4}\frac{1}{f}\bar{\mathcal{G}}^2\psi_\mu\sigma^{\mu\nu}\psi_\nu + m_{3/2}\frac{1}{f}\bar{\mathcal{G}}^2 + \frac{1}{2}m^*_{3/2}\frac{1}{\bar{f}}\mathcal{G}^2 + \dots \tag{4.42}$$

and now we are ready to implement the known constraints over $\mathcal{R}$ and $G_{\alpha\dot{\alpha}}$ that remove the auxiliary fields m and b_μ. We will see later that this is not the only path to eliminate F.

Solving the constraints (4.35) and (4.36) is not a simple task. In [73] the authors solved both constraints and found one equation that resolve m and one equation that resolve b_μ in terms of $\mathcal{G}$ and F. For example, the solution for m is

$$m = \frac{\sqrt{2}}{3}\mathcal{G}\sigma^{\mu\nu}\psi_{\mu\nu} + \frac{i}{3\sqrt{2}F}b^\mu\mathcal{G}\psi_\mu + \dots \tag{4.43}$$

Since we do not want F to be in the final solution, we replace the result of equation (4.42) in the expressions of both m and b_μ. Expanding bellow order $\mathcal{O}(\mathcal{G}^3)$ and doing an iterative process, the auxiliary fields of the Supergravity multiplet become

$$m = -m_{3/2} + \frac{f}{\bar{f}}\mathcal{G}^2 + \dots \tag{4.44a}$$

$$b_\mu = \frac{1}{4}\mathcal{G}\sigma_\mu\bar{\mathcal{G}} + \dots \tag{4.44b}$$

and the Lagrangian takes the simplified form

$$\begin{aligned}\frac{1}{eM_{\mathrm{pl}}^2}\mathcal{L} = -\frac{1}{2}R + \frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\psi_{\rho\sigma}+ \\ -\frac{i}{2}\mathcal{G}\sigma^\mu\mathcal{D}_\mu\mathcal{G} - \frac{i}{\sqrt{2}}f\mathcal{G}\sigma^\mu\bar{\psi}_\mu+ \\ \left(\frac{1}{2}\frac{f}{\bar{f}}\mathcal{G}^2 - m_{3/2}\right)\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu - m_{3/2}^*\frac{f}{\bar{f}}\mathcal{G}^2 + \text{c.c.}+ \\ -|f|^2 + 3|m_{3/2}|^2 + \ldots\end{aligned} \tag{4.45}$$

where, furthermore, we are neglecting terms with more than two fermions as they will not contribute to the quadratic Lagrangian. One can go further and consider terms with higher order, however our purpose here is to match this procedure with the renown CCWZ construction illustrated in Chapter III, where only terms with two or less fermions were considered.

The Lagrangian (4.45) describes a model of spontaneously broken and non-linearly realized Supersymmetry, where the field $\mathcal{G}$ transforms as

$$\delta\mathcal{G}_\alpha = -\sqrt{2}\bar{f}\epsilon_\alpha + \frac{i}{\sqrt{2}}\frac{1}{\bar{f}}(\mathcal{G}\sigma^\mu\bar{\epsilon})\mathcal{D}_\mu G_\alpha + \ldots \tag{4.46}$$

Notice that such transformation is nonlinear and non homogeneous, so $\mathcal{G}$ has the status of a Goldstone particle. However, $\mathcal{G}$ is not exactly the same field as the Volkov-Akulov fermion γ of the rigid case. We know that the transformation of γ (extended to the local case) is

$$\delta\gamma = f\epsilon - \frac{i}{f^2}(\gamma\sigma^\mu\bar{\epsilon})\mathcal{D}_\mu\gamma + \ldots, \tag{4.47}$$

and thereby we are lean to make the following identification:

$$\mathcal{G}_\alpha = \sqrt{2}\frac{F}{f}\gamma_\alpha = -\sqrt{2}\frac{\bar{f}}{f}\gamma_\alpha + \ldots \tag{4.48}$$

which is supported by the fact that the quantity $\sqrt{2}F\gamma_\alpha/f$ transforms identically as the field $\mathcal{G}_\alpha$[5].

These considerations serve us to spell out the action in terms of the γ field, after rather simple algebraic steps and integrating by parts some of the terms, we get the formula

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L} = -\frac{1}{2}R - \frac{\Lambda}{M_{\text{pl}}^2} + \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\mathcal{D}_\rho\psi_\sigma - i\gamma\sigma^\mu\mathcal{D}_\mu\bar{\gamma} + \\ + m_{3/2}\psi_\mu\sigma^{\mu\nu}\psi_\nu + if\bar{\gamma}\bar{\sigma}^\mu\psi_\mu + 2m_{3/2}\frac{\bar{f}}{f}\gamma^2 + \text{c.c.} + \ldots, \tag{4.49}$$

where we have introduced the cosmological constant Λ by the identification

$$\frac{\Lambda}{M_{\text{pl}}^2} = |f|^2 - 3|m_{3/2}|^2, \tag{4.50}$$

the sharp reader will notice that this corresponds exactly with the definition of f of equation (3.45) provided in Section 3.2! As a matter of fact, the Lagrangian (3.46) matches exactly with the effective Lagrangian (4.49) up to one linear field redefinition, namely

$$\gamma = f\lambda, \tag{4.51}$$

which corresponds to a change in the units. There is, however, a subtlety on this replacement. We know that the transformation of γ does not mix γ with $\bar{\gamma}$, contrary to λ whose transformation does mix λ with $\bar{\lambda}$. Nevertheless, in Appendix J we show to all orders the complete correspondence between both fields, we found that equation (4.51) is valid at first order.

It is straightforward to see that the former action is constituted by the usual Einstein-Hilbert term, the cosmological con-

[5]Here we show that this identity only works at first order. However, one can show by an iterative process that this is, in fact, accurate to all orders.

stant and the following three elements:

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L}_{\psi\psi} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\mathcal{D}_\rho\psi_\sigma + m_{3/2}\psi_\mu\sigma^{\mu\nu}\psi_\nu + \text{c.c.} \tag{4.52a}$$

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L}_{\psi\lambda} = -i|f|^2\bar{\psi}_\mu\bar{\sigma}^\mu\lambda + \text{c.c.} \tag{4.52b}$$

$$\frac{1}{eM_{\text{pl}}^2}\mathcal{L}_{\lambda\lambda} = -i|f|^2\left(\bar{\lambda}\bar{\not{\mathcal{D}}}\lambda + 2im_{3/2}\lambda^2\right) + \text{c.c.} \tag{4.52c}$$

These are precisely the terms obtained in Section 3.2 except that here we used a completely different derivation.

The resulting action describes all the interactions of the Goldstone fermion λ in a Supergravity setup. Though the analysis of this Lagrangian was due in the previous Chapter, it is important to notice one additional subtlety. If we fix the gauge to the unitary gauge, which corresponds to

$$\lambda = 0, \tag{4.53}$$

then we see that the gravitino acquires a mass with value $m_{3/2}$. However, the interpretation of $m_{3/2}$ as a gravitino mass in the complete action (4.52) is not straightforward; the presence of a mixing term between λ and ψ may produce a different value for the gravitino's mass. Notice that, since both Lagrangians are equivalent, all the phenomenological implications must be the same, the only distinction is the theoretical technique involved on each derivation.

We constructed a general model of broken Supergravity where the symmetry is non-linearly realized by a fermionic field. To construct such effective action, we started from a general Lagrangian that is invariant under Supergravity and where the only ingredient was a chiral superfield X which is the responsible of the symmetry breaking mechanism.

Imposing a constraint that respects Supergravity over the chiral field X we were able to write the action in terms of its component fields: $\mathcal{G}$ and F. The lowest component of the chiral superfield was removed by the constraint $X^2 = 0$. Since we were dealing with local Supersymmetry, gravity became an important factor.

To treat gravity we used the already known Supergravity multiplet $\{e^a{}_\mu, \psi_\mu, m, b_\mu\}$, here only the graviton and the gravitino are physical degrees of freedom, the fields m and b_μ were removed by imposing additional constraints directly on the curvature superfield. This operation is analogous to the elimination of auxiliary fields performed in the CCWZ construction. There, we imposed constraints over the Stückelberg fields M and B that were invariant under all symmetries. This analogy between the methods to remove the auxiliary fields of Supergravity is shown schematically as follows:

$$M = 0 \qquad \longleftrightarrow \qquad X\mathcal{R} = 0 \tag{4.54a}$$

$$B = 0 \qquad \longleftrightarrow \qquad X\bar{X}G_{\alpha\dot{\alpha}} = 0 \tag{4.54b}$$

Meaning that, fixing correctly the auxiliary fields is equivalent to impose geometrical constraints directly over the curvature superfield.

Finally, we also found a relationship between the field $\mathcal{G}$, introduced as a component of the chiral superfield X, and the field λ introduced by the CCWZ construction. We interpret both fields as the Goldstone fermions originated by the symmetry breaking mechanism. This gives evidence to our thesis that both approaches are equivalent at least to first order. Extending this result to highest orders would be an interesting research area for further projects.

The purpose of this section was to explain the mechanism of supersymmetry breaking by the introduction of a chiral superfield

X that is constrained, we referred to this construction as a top-down effective theory. At this stage, it is important to bind the current discussion with an application in cosmology and, more specifically, a description of inflation based on the breaking of supersymmetry.

One disadvantage of using the top-down description is that, at the current stage of the research, we do not have a procedure that allows us to construct a general action where time diffeomorphisms are also a broken symmetry that drives inflation or, in other words, we do not know how to derive an action similar to (3.64) and (3.65) that represents the most general action that describes inflation in a supersymmetric scenario. However, there are several models of inflation that include supersymmetry, in the following section we will highlight one important example.

4.3 Nilpotent Inflation

Several models of inflationary cosmology that implement a minimal formulation of supergravity have been proposed recently, for a general review see for example [67]. Here we present a way to implement the previous discussion about the breaking of supergravity due to the presence of a chiral superfield subject to a nilpotent constraint in the context of inflation. More specifically, we want to represent the Starobinsky model [39] in terms of a theory with spontaneously symmetry breaking, as done by R. Kallosh and A. Linde in [81].

Let us first recall some important aspects about the Starobinsky model. There are two different, though equivalent, interpretations about the Starobinsky model; it was first introduced as a theory based on an extension of general relativity where the Hilbert-Einstein action is replaced by a more general action using the scalar $R+R^2$ instead of the habitual Ricci scalar R. Such extension generates new couplings between the metric and the inflaton field that leads to inflation. Alternatively, the Starobinsky model is conformally equivalent to a theory of general relativity where the metric is coupled to a scalar field, the inflation ϕ, which has a potential with the specific form

$$V(\phi) = V_0 \left(1 - e^{-\sqrt{2/3}\phi}\right)^2 , \tag{4.55}$$

where V_0 is the scale of the potential and has a value of 10^{-9} in Planck units [82]. These two models give the same phenomenological predictions, thus establishing a duality between models of higher curvature and models with an inflation embedded in the potential of equation (4.55).

One way to understand such duality is thinking that conformally invariant models with a higher curvature ($R + R^2$ models) behave as if there where an additional scalar degree of freedom subject to a potential with a scale related with the curvature. Such scalar degree of freedom is connected with the fact that we are assuming that the theory is invariant under conformal transformations[6].

To show how these type of models are related with each other we start by considering the simplest model with higher curvature which is the Starobinsky model with a Lagrangian that reads

$$\mathcal{L} = M_{\rm pl}^2 \sqrt{-g}\frac{1}{2}\left(R + \frac{R^2}{8V_0}\right), \tag{4.56}$$

if the theory is scale invariant, one can perform a conformal transformation of the type

$$g'_{\mu\nu} = \left(1 + \frac{\varphi}{4V_0}\right) g_{\mu\nu}, \tag{4.57}$$

[6]This feature resembles the already discussed equivalence between the two methods that we used to eliminate the auxiliary fields of Supergravity, namely (4.54)

and a field redefinition given by

$$\phi = \sqrt{\frac{3}{2}} \ln \left(1 + \frac{\varphi}{4V_0}\right), \tag{4.58}$$

the overall result will be a Lagrangian for the field ϕ in the presence of a potential V, according to equations (4.57) and (4.58), the Lagrangian can be recast as

$$\mathcal{L} = M_{\rm pl}^2 \sqrt{-g'} \left[\frac{1}{2}R' - \frac{1}{2}(\partial\phi)^2 - V_0 \left(1 - e^{-\sqrt{2/3}\phi}\right)^2\right]. \tag{4.59}$$

This model predicts accurately the parameters of inflation measured experimentally. We see that, as anticipated, there is a correspondence between models that use an extended version of general relativity and the Starobinsky model.

Having discussed briefly about such important class of inflationary theories, we would like to extend the analysis by shifting from gravity to Supergravity in order to connect what we already study during this Chapter with inflation.

The insertion of $R + R^2$ theories in Supergravity is sensitive to the choice of auxiliary fields. Two distinct classes of minimal models arise depending on the choice of auxiliary fields, one set is based on the old-minimal formulation of off-shell Supergravity [83, 84], and the other is based on the new-minimal formulation of off-shell Supergravity [85, 86, 87].

The problem with these models is that it is difficult to obtain a supersymmetry breaking scale at the end of inflation much lower than the Hubble scale during inflation. One way to avoid such complication is to introduce a nilpotent constraint on the chiral superfield X so that the Goldstino lacks a scalar superpartner, this chiral field is the analogue of the Volkov-Akulov field that non-linearly realizes Supergravity [88, 89]. This is essentially the mechanism we developed throughout the previous section, but now we want to include a field that produces inflation.

Contrary to the CCWZ construction, studied on Chapter III, where inflation was a result of the symmetry breaking of time diffeomorphisms, here we need to introduce by hand a field responsible of inflation. This field must be a component of a chiral superfield Φ, if we want to incorporate separate scales of supersymmetry breaking during and at the end of inflation, then it is convenient to choose the flat Kähler potential

$$K(\Phi, X) = \frac{1}{2}\left(\Phi + \bar{\Phi}\right)^2 + X\bar{X}, \tag{4.60}$$

and, consequently, the superpotential has the form

$$W(\Phi, X) = m_{3/2} + f(\Phi)X. \tag{4.61}$$

Notice that these are only extensions of the already analyzed potentials of equation (4.37), but in this case we include the superfield Φ.

Two classes of models arise from this approach depending on the specific form of the function $f(\Phi)$. In the first class, developed mainly in [90], the function $f(\Phi)$ is

$$f(\Phi) = M^2(1 + g^2(\phi)) \tag{4.62}$$

where the function $g(\Phi)$ vanishes when $\Phi = 0$ and M is some mass scale. This class of models provide a simple description of inflation and match the currently measured acceleration of the universe in the context of Supergravity.

The second class of models was developed in [91] in the search for a consistent and realistic model of large-field inflation where Supergravity is manifest. In this case, the function $f(\Phi)$ of equation (4.61) satisfies the conditions

$$\bar{f}(\Phi) = f(-\bar{\Phi}), \qquad f(0) \neq 0, \qquad f'(0) = 0, \tag{4.63}$$

and the superfield Φ has the form

$$\Phi = \frac{1}{\sqrt{2}}\,(a + i\phi) \qquad (4.64)$$

where ϕ is the inflaton and a is an additional field that it is driven to zero during inflation, therefore it is not a relevant degree of freedom for our purpose. One relevant choice for the function $f(\Phi)$ subscribed to this class of models is

$$f(\Phi) = \alpha - i\beta\phi + \gamma e^{i\sqrt{2/3}\Phi}, \qquad (4.65)$$

which reproduces the Starobinsky potential, this is a nice feature for realistic models because of its predictive power.

Conclusions

We are not sure if inflation actually happened. However, we feel that the Universe might originate from a process where the energy density was higher than all energy regimes tested in any experiment. Even worse, a fundamental theory that describes Nature at all energy scales does not exists, therefore the physics involved in the process of inflation remain obscure. At most, we can require that whatever the theory of Nature is, it must reduce to the known theories that have been tested at low energies. In other words, we follow the hypothesis that inflation must be described by an effective field theory.

Throughout this work, we focused on the meaning of effective theories and recognized that there are two possible approaches: the first one was presented in Chapter III, there we assumed a symmetry and constructed the most general action which is invariant under such symmetry (bottom-up). The second one, developed in Chapter IV, consists on writing a general action starting from a fundamental principle, from where we integrate out all the degrees of freedom that are not relevant at the energy scale of interest (top-down). We found that, up to some redefinition, both effective theories were equivalent within the energy regime of validity. Here we stress about this idea.

First of all, let us recapitulate what we know about inflation. Inflation is regarded as a phase where the Universe expanded as an exponential function of time. The fact that such phase actually ended indicates that there is a preferred clock that measures cosmological time, in this sense, during inflation the Universe was not fully symmetrical. To see the underlying idea, it suffices to consider a model where time diffeomorphisms are treated as a spontaneously broken local symmetry. We know from quantum field theory that any gauge symmetry can be parametrized by a field that, in spite of the gauge liberty, does not represent a physical degree of freedom whatsoever. A field of this kind is known as a Goldstone boson (fermion) even though, strictly speaking, it does not have the physical significance of a particle. However, we know that the correlation functions of the energy density of the inflaton[1] can be obtained directly from the correlation functions of the Goldstone boson field, meaning that all the information about the primordial fluctuations can be extracted from the dynamics of this field instead of using the dynamics of the scalar field that produces inflation.

Regarding the bottom-up approach, at the end of Chapter I we showed how to construct an effective theory of inflation in the absence of Supersymmetry. We started by defining the local parameter of time diffeomorphisms. If we promote such parameter to a local field π then new interactions will emerge between π and the field content of the theory. We motivate that this field can be gauged away so it does not represent, by itself, a physical degree of freedom. However, in spite of the Goldstone equivalence theorem [43], it gives new polarization states to the remaining fields of the theory. We say that, at low energies, the theory of spontaneous local symmetry breaking is completely described by the metric fields, such fields have acquired an additional polarization state coming from the degrees of freedom of π, less precisely

[1] The field that responsible of inflation.

we say that π has been *eaten* by those fields. When the energy exceeds such limit, the metric decouples from π and then it behaves like a massless particle, the additional degree of freedom is, therefore, in the field π.

To describe the dynamics of π we constructed a general action invariant under all space-time symmetries by performing a nonlinear time translation. The result was equation (1.66). Curiously enough, the correlation function of π is related with the correlation of the density fluctuations field, such feature is useful when one does phenomenological computations. This effective theory is valid bellow energy scales of order H.

Despite having an effective model that describes the primordial density fluctuations, it is not clear what is the appearance of the UV-complete theory it comes from. Though this is a standard problem for all bottom-up effective theories, we feel that knowing the underlying fundamental principles will endorse our understanding of the phenomenon. It is natural then to consider a complete theory of gravity and find its low energy limit to see if it matches with the already constructed bottom-up effective model. This argument was our principal motivation to push forward the present thesis, later we will comment about our results.

A complete theory of gravity does not exist yet. Nevertheless, for several different reasons, we include Supersymmetry among the symmetries of Nature. One of the main reasons is that it enables us to formulate a consistent UV completion of gravity, so a supersymmetric effective theory of inflation becomes relevant in the context of unification. We were interested in Supersymmetry primarily because it is the only possible extension of the space-time symmetries given by the Poincaré group [47]. However, if Supersymmetry is realized in Nature it must be spontaneously broken at low energies. The study of this subject from the bottom-up and top-down perspectives was the main subject of this work.

To construct an action that describes a model where Supersymmetry is spontaneously broken but time diffeomorphisms are present, we implemented the renown Callan-Coleman-Wess-Zumino construction (CCWZ) based on the nonlinear realization of Supergravity. This prescription reduces to construct the most general Lagrangian that non-linearly realizes Supergravity. One feature of this prescription is that it allow us to formulate a theory of spontaneously Supersymmetry breaking that is model independent. An important simplification yield by the fact that Supersymmetry is spontaneously broken is that the auxiliary fields are removed away and, therefore, the obtained action listed in (3.46) effectively describes the dynamics of the metric without any field taking a vacuum expectation value. We deduced in Section 3.2 that the mixing term of the action (3.46) becomes more and more irrelevant above energies of order

$$E^2_{\text{mix}} \sim \frac{\Lambda}{M^2_{\text{pl}}} + 3|m_{3/2}|^2, \tag{5.1}$$

this non-trivial result was already discussed in [92]. However, this model does not describe the breaking of time diffeomorphisms and, hence, it does not truly describes inflation; it was used as an illustrative example.

In order to describe the dynamics of the primordial fluctuations during inflation, we extended the previous example using the following logic. Since inflation breaks time diffeomorphisms, then, because of the algebra, the inflationary background breaks also Supersymmetry. We considered an action invariant only under the residual symmetry group, which are time dependent spatial diffeomorphisms and local Lorentz invariance. Then, we reintroduced time diffeomorphisms and Supersymmetry by performing two Stückelberg transformations: one for time diffeomorphisms and one for Supersymmetry in that order[2]. Each trans-

[2] The order is important. Since Supersymmetry is not a subgroup of

formation introduces a Goldstone particle in the Lagrangian, we referred to these particles as the Goldstone boson π and the Goldstone fermion λ. Because of the Stückelberg trick, new operators, or interactions, are obtained. The resulting action is the one of equations (3.64) and (3.65).

One important simplification coming from this formalism is that the auxiliary fields of Supergravity can be removed away without using their equations of motion. Setting an invariant constraint is enough to obtain an expression of these fields in terms of the Goldstone fermion and the gravitino. There is no need for the auxiliary fields to take any vacuum expectation value as they usually do in any theory of Supersymmetry breaking, see for example [93], [94] and more recently [95].

The Stückelberg trick is also useful because it allows us to extract the spin-1/2 degree of freedom from the gravitino. As stressed before (during the discussion of the Goldstone equivalence theorem), we know that such polarization state decouples at high energies and it is carried by the Goldstone fermion λ. The energy limit from where the decouple occurs is reached when the commutator of two covariant derivatives (evaluated in the unitary gauge $\pi = 0$) vanishes. Looking at the expression of such commutator provided in equation (3.61) one can realize that solving it may represent a difficult task. As a matter of fact, we do not have a solution for such equation in the general case, so any exact bound for the decoupling limit is computed in a model-dependent theory where the background is assumed *a priori*.

An approximate estimation of the decoupling limit could be guessed by assuming that all the parameters in the general action, constructed by following the CCWZ prescription, satisfy

$$m_i \lesssim H, \tag{5.2}$$

super-Poincaré, the CCWZ construction is only valid if we reintroduce first the time diffeomorphisms and then Supersymmetry.

where m_i represents any parameter. In this case, the mixing term in the Lagrangian (3.64) takes the form

$$\mathcal{L}_{\psi\lambda} \sim H\psi^{\mu}\sigma_{\mu}\lambda \tag{5.3}$$

after a canonical transformation and after fixing the unitary gauge, which is equivalent to set $\pi = 0$. This implies that the decoupling between the gravitino and the Goldstone fermion will occur at energies above

$$E_{\text{mix}} \sim H, \tag{5.4}$$

for any background solution. This is true only if two conditions are satisfied. First, we assumed that all the parameters are of order H, this choice is justified because if we move away from the condition (5.2), the speed of sound for the Goldstone fermion is superluminal. Second, we assume that inflation takes place at an energy scale of order H, however this is not necessarily the case. Two exception are considered in [96, 97] where the time diffeomorphisms are not softly broken but considered as a discrete symmetry.

Now we shift the discussion to the top-down approach developed on this work. Starting from a Lagrangian of a chiral field X which is invariant under Supergravity, we imposed a constraint on such chiral superfield that removes its lowest component. This procedure also significantly simplifies the action and it allows us to write it in terms of the components of the Supergravity multiplet. At this stage, the theory still depends on the auxiliary fields, which is an uncomfortable property if we want a common frame between this theory and the previously mentioned EFT of spontaneously supersymmetry breaking.

The auxiliary fields come naturally from the spectrum of Supergravity. Their presence is inevitable to equate the number of degrees of freedom between an on-shell and an off-shell description of Supergravity. However, we are interested in the case where Supergravity is spontaneously broken and therefore we do not require such equality.

To define a common frame for this approach and the previously studied bottom-up construction, we need to integrate out the auxiliary fields and then compare the results obtained in both derivations. In this scenario, we achieved this by imposing constraints over the curvature. We interpreted these constraints as geometrical restrictions set *a priori* who act directly on the curvature superfield. In this sense, compared with the bottom-up construction, we are shifting the problem of removing the auxiliary fields by finding the Stückelberg auxiliary fields and setting them to zero, to the one of finding the corresponding geometrical restrictions that effectively eliminates such fields. The analogy is made explicit in the correspondence (4.54):

$$M = 0 \qquad \longleftrightarrow \qquad X\mathcal{R} = 0 \tag{5.5a}$$

$$B = 0 \qquad \longleftrightarrow \qquad X\bar{X}G_{\alpha\dot{\alpha}} = 0. \tag{5.5b}$$

Applying the current constraints into the Lagrangian produces an effective action for the metric given in equation (4.49) which is invariant under Supergravity and that describes the metric in an inflationary background. An equivalence between the resulting action and the one calculated using the CCWZ construction is manifest. We showed how the Goldstone fermions of each theory are related. The overall correspondence is the main result of the present work. We found that the most general Lagrangian that describes the process of Supersymmetry breaking can be obtained from a theory of minimal constrained Supergravity where some constraints of the curvature are assumed by hand. This result provides a faithful frame to develop models of Supersymmetry breaking that describe inflation. An interesting point, meant to be studied in further research projects, is how different constraints may interfere with the current result, in particular, is there any constraint that results in a more general action?

One point to highlight for both theories is that, as effective models, they most break at some energy scale. We know that the cut-off energy of the symmetry breaking is of order H, however it may happen that the unitary condition on tree-level scattering amplitudes breaks bellow H. We do not present here a formal computation of such unitary bound. One way to give an approximate value is to use the Goldstone equivalence theorem, where it is stated that at high energies the physical amplitudes of the gravitino are asymptotically equal to those of the Goldstone fermion.

In [98] the authors claimed that the S-matrix for interactions involving gravitons with helicities $\pm 1/2$ and any other fields, are all equivalent up to corrections of order $\mathcal{O}(m_{3/2}/M_{\rm pl})$. In this order of ideas, the worst energy behavior comes from the terms with four Goldstone fermions in the Lagrangian. From (4.45), we see that the terms with four fermions are of the form

$$\frac{1}{f^2}\bar{\mathcal{G}}^2\partial^2\mathcal{G}^2, \tag{5.6}$$

we guess, therefore, that these terms produce an amplitude of the type

$$\text{Amplitude} \sim \frac{E^4}{f^2}, \tag{5.7}$$

in turn, this amplitude implies the following upper bound for the energy:

$$E \lesssim \sqrt{f} \sim \sqrt{m_{3/2}M_{\rm pl}}, \tag{5.8}$$

where the last step was obtained by evaluating the limit given by the equivalence theorem. This result matches with the one obtained in [99] using a very different description. One last observation about this estimate bound is that it must be above the mixing energy, implying that

$$m_{3/2} > \frac{H^2}{M_{\rm pl}}, \tag{5.9}$$

this is the only effective restriction for the mass parameter obtained by the present analysis.

Further topics of research emerge from these physical discussions that are not treated in this work. Our emphasis here was to create a common frame between two different descriptions that met one another in the Language of effective theories. The most notorious enhancement that can be done to this work is an extension to higher orders in the fermionic expansion, for simplicity we stopped at terms with two fermions, but new physics may appear at higher orders.

THE END

Appendix

A Friedmann Equations

We devote this section to demonstrate the Friedmann equations. Starting from the following explicitly covariant expression for the stress-energy tensor

$$T^{\mu}{}_{\nu} = (\rho + P)U^{\mu}U_{\nu} - P\delta^{\mu}_{\nu}, \tag{A.1}$$

where ρ is the total energy density, P is the pressure and U^{μ} is the relative four-velocity which, for a comoving observer, takes the value $U^{\mu} = (1, 0, 0, 0)$. On this reference frame, the stress-energy tensor is

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}. \tag{A.2}$$

On the other hand, by considering the de Sitter metric of equation (1.1), the non-vanishing component of the Ricci tensor are

$$R_{00} = -3\frac{\ddot{a}}{a} \tag{A.3a}$$

$$R_{ij} = -\left[\frac{\ddot{a}}{a} + 2H^2 + 2\frac{k}{a^2}\right] g_{ij}, \tag{A.3b}$$

where k is the curvature. The Ricci scalar turns out to be

$$R = -6\left[\left(\frac{\ddot{a}}{a}\right)^2 + H^2 + \frac{k}{a^2}\right], \tag{A.4}$$

and therefore, the non-zero components of the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - (R/2)g_{\mu\nu}$ are

$$G^0{}_0 = 3\left[H^2 + \frac{k}{a^2}\right] \tag{A.5a}$$

$$G^i{}_j = \left[2\frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2}\right]\delta^i_j. \tag{A.5b}$$

Now we can replace our former results into the Einstein field equations

$$G_{\mu\nu} = \frac{1}{M_{\rm pl}^2} T_{\mu\nu}, \tag{A.6}$$

and we obtained the two independent Friedmann equations

$$H^2 = \frac{1}{3M_{\rm pl}^2}\rho - \frac{k}{a^2} \tag{A.7a}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{\rm pl}^2}(\rho + 3P). \tag{A.7b}$$

Here, the energy density ρ and the pressure P should be taken as the sum of all contributions of every different type of energy, i.e. cold dark matter, radiation and dark energy.

B Spinor Algebra

Here we write some results on spinor algebra that are essential for the development of supersymmetric theories.

Following Wess and Bagger notation, we define the metric using the sign convention $\eta_{\mu\nu} \sim (-1,1,1,1)$, we also use the conventional Pauli matrices

$$\sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

This set of matrices form a complete basis of $SL(2,\mathbb{C})$. To manipulate the Pauli matrices we set the Lorentz vectors σ^μ and $\bar{\sigma}^\mu$ as:

$$\sigma^\mu_{\alpha\dot{\alpha}} = (\sigma^0, \boldsymbol{\sigma}^i)_{\alpha\dot{\alpha}} \tag{B.1a}$$

$$\bar{\sigma}^{\mu\dot{\alpha}\alpha} = \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}\sigma^\mu_{\beta\dot{\beta}}, \tag{B.1b}$$

where the Greek indices denote spinors, dotted indices refer to the $(0, \frac{1}{2})$ representation of the Lorentz group.

Consider a representation of $SL(2,\mathbb{C})$ consisting of a set of matrices M under which the two-component spinors ψ—with Grassmann components—transform according to

$$\psi'_\alpha = M_\alpha{}^\beta \psi_\beta \qquad \bar{\psi}'_{\dot{\alpha}} = M^*{}_{\dot{\alpha}}{}^{\dot{\beta}} \bar{\psi}_{\dot{\beta}}$$

$$\psi'^\alpha = M^{-1}{}_\beta{}^\alpha \psi^\beta \qquad \bar{\psi}'^{\dot{\alpha}} = (M^*)^{-1}{}_{\dot{\beta}}{}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}}.$$

In this case, the matrices M and M^* belong respectively to the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations.

Spinors with upper and lower indices are related by the complete antisymmetric ε-tensor that satisfies

$$\begin{aligned}\varepsilon^{12} &= \varepsilon_{21} = 1\\ \varepsilon^{21} &= \varepsilon_{12} = -1\\ \varepsilon_{11} &= \varepsilon_{22} = 0,\end{aligned} \tag{B.2}$$

and consequently, the product of two ε-tensors can be expressed in terms of the Kronecker δ as

$$\varepsilon_{\alpha\beta}\varepsilon^{\gamma\delta} = \delta_\alpha{}^\delta\delta_\beta{}^\gamma - \delta_\alpha{}^\gamma\delta_\beta{}^\delta \tag{B.3a}$$

$$\varepsilon_{\dot\alpha\dot\beta}\varepsilon^{\dot\gamma\dot\delta} = \delta_{\dot\alpha}{}^{\dot\delta}\delta_{\dot\beta}{}^{\dot\gamma} - \delta_{\dot\alpha}{}^{\dot\gamma}\delta_{\dot\beta}{}^{\dot\delta}. \tag{B.3b}$$

With these relations, it can be shown that[3]:

$$\varepsilon_{\alpha\beta}\varepsilon^{\beta\gamma} = \delta_\alpha{}^\gamma \tag{B.4a}$$

$$\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta} + \varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta} + \varepsilon_{\alpha\delta}\varepsilon_{\gamma\beta} = 0. \tag{B.4b}$$

The general rule to rise and lower indices is summarized in here:

$$\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta \qquad\qquad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta \tag{B.5a}$$

$$\bar\psi^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta}\bar\psi_{\dot\beta} \qquad\qquad \bar\psi_{\dot\alpha} = \varepsilon_{\dot\alpha\dot\beta}\psi^{\dot\beta}. \tag{B.5b}$$

With these conventions, the products $\psi^\alpha\psi_\alpha$ and $\bar\psi_{\dot\alpha}\bar\psi^{\dot\alpha}$ are Lorentz scalars; *descending* contracted *undotted* indices ${}^\alpha{}_\alpha$ and *ascending* contracted *dotted* indices ${}_{\dot\alpha}{}^{\dot\alpha}$ can be suppressed. Some convenient identities that follow are

$$\psi^\alpha\psi^\beta = -\frac{1}{2}\varepsilon^{\alpha\beta}\psi^2 \tag{B.6a}$$

$$\bar\psi_{\dot\alpha}\bar\psi_{\dot\beta} = -\frac{1}{2}\varepsilon_{\dot\alpha\dot\beta}\bar\psi^2 \tag{B.6b}$$

$$(\psi\sigma^a\bar\psi)(\psi\sigma^b\bar\psi) = -\frac{1}{2}\psi^2\bar\psi^2\eta^{ab} \tag{B.6c}$$

[3] In order to perform the proof of these identities better recall that $\delta_\alpha{}^\alpha = 2$.

The behavior of the products of two spinors χ and ζ under complex conjugation can be summarized in the formulae

$$(\chi\Sigma\zeta)^\dagger = \bar{\zeta}\Sigma_r\bar{\chi} \tag{B.7a}$$
$$(\chi\Sigma\bar{\zeta})^\dagger = \zeta\Sigma_r\bar{\chi}, \tag{B.7b}$$

where Σ stands for any alternating sequence of σ and $\bar{\sigma}$ matrices and Σ_r represents the same sequence reversed.

From the definition of the σ-matrices we find that

$$(\sigma^a\bar{\sigma}^b + \sigma^b\bar{\sigma}^a)_\alpha^{\ \beta} = -2\eta^{ab}\delta_\alpha^{\ \beta} \tag{B.8a}$$
$$(\bar{\sigma}^a\sigma^b + \bar{\sigma}^b\sigma^a)^{\dot{\alpha}}_{\ \dot{\beta}} = -2\eta^{ab}\delta^{\dot{\alpha}}_{\ \dot{\beta}}, \tag{B.8b}$$

and also the following completeness relations:

$$\mathrm{Tr}\sigma^a\bar{\sigma}^b = -2\eta^{ab} \tag{B.9}$$
$$\sigma^a_{\alpha\dot{\alpha}}\bar{\sigma}_a^{\dot{\beta}\beta} = -2\delta_\alpha^{\ \beta}\delta_{\dot{\alpha}}^{\ \dot{\beta}}, \tag{B.10}$$

similarly

$$\sigma^a_{\alpha\dot{\alpha}}\sigma_{a\beta\dot{\beta}} = -2\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}} \tag{B.11a}$$
$$\bar{\sigma}^{a\dot{\alpha}\alpha}\bar{\sigma}_a^{\dot{\beta}\beta} = -2\varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}. \tag{B.11b}$$

It is also convenient to work out some identities that are meant to be used in the development of a supersymmetric effective theory. We start by defining the Lorentz group generators in the spinor representation as

$$\sigma^{ab} = \frac{1}{4}(\sigma^a\bar{\sigma}^b - \sigma^b\bar{\sigma}^a) \tag{B.12a}$$
$$\bar{\sigma}^{ab} = \frac{1}{4}(\bar{\sigma}^a\sigma^b - \bar{\sigma}^b\sigma^a), \tag{B.12b}$$

from where it follows that

$$\varepsilon^{abcd}\sigma_{cd} = -2i\sigma^{ab} \tag{B.13a}$$

$$\varepsilon^{abcd}\bar{\sigma}_{cd} = 2i\bar{\sigma}^{ab}, \tag{B.13b}$$

and hence

$$\sigma^{ab}{}_{\alpha}{}^{\alpha} = 0 \tag{B.14a}$$

$$\sigma^{ab}{}_{\alpha}{}^{\beta}\varepsilon_{\beta\gamma} = \sigma^{ab}{}_{\gamma}{}^{\beta}\varepsilon_{\beta\alpha}. \tag{B.14b}$$

The contraction of two group generators can be simplified using the result:

$$(\sigma^{ab})_{\alpha}{}^{\beta}(\sigma_{ab})_{\gamma}{}^{\tau} = -2\delta_{\alpha}{}^{\tau}\delta_{\gamma}{}^{\beta} + \delta_{\alpha}{}^{\beta}\delta_{\gamma}{}^{\tau} \tag{B.15a}$$

$$(\bar{\sigma}^{ab})^{\dot{\alpha}}{}_{\dot{\beta}}(\bar{\sigma}_{ab})^{\dot{\gamma}}{}_{\dot{\tau}} = -2\delta^{\dot{\alpha}}{}_{\dot{\tau}}\delta^{\dot{\gamma}}{}_{\dot{\beta}} + \delta^{\dot{\alpha}}{}_{\dot{\beta}}\delta^{\dot{\gamma}}{}_{\dot{\tau}} \tag{B.15b}$$

$$(\bar{\sigma}^{ab})^{\dot{\alpha}}{}_{\dot{\beta}}(\sigma_{ab})_{\gamma}{}^{\tau} = 0. \tag{B.15c}$$

We also also bring here other important relations that involve the product of the σ matrices:

$$\bar{\sigma}^{a}\sigma^{b}\bar{\sigma}^{c} - \bar{\sigma}^{c}\sigma^{b}\bar{\sigma}^{a} = -2i\varepsilon^{abcd}\bar{\sigma}_{d} \tag{B.16a}$$

$$\sigma^{a}\bar{\sigma}^{b}\sigma^{c} - \sigma^{c}\bar{\sigma}^{b}\sigma^{a} = 2i\varepsilon^{abcd}\sigma_{d}, \tag{B.16b}$$

and

$$\bar{\sigma}^{a}\sigma^{b}\bar{\sigma}^{c} + \bar{\sigma}^{c}\sigma^{b}\bar{\sigma}^{a} = 2\left[-\eta^{ab}\bar{\sigma}^{c} - \eta^{bc}\bar{\sigma}^{a} + \eta^{ac}\bar{\sigma}^{b}\right] \tag{B.17a}$$

$$\sigma^{a}\bar{\sigma}^{b}\sigma^{c} + \sigma^{c}\bar{\sigma}^{b}\sigma^{a} = 2\left[-\eta^{ab}\sigma^{c} - \eta^{bc}\sigma^{a} + \eta^{ac}\sigma^{b}\right], \tag{B.17b}$$

from where it is possible to compute the contractions

$$(\bar{\sigma}_{b})^{\dot{\alpha}\beta}(\sigma^{ab})_{\beta}{}^{\alpha} = \frac{3}{2}(\bar{\sigma}^{a})^{\dot{\alpha}\alpha}, \tag{B.18a}$$

$$(\sigma_{b})_{\alpha\dot{\beta}}(\bar{\sigma}^{ab})^{\dot{\beta}}{}_{\dot{\alpha}} = \frac{3}{2}(\bar{\sigma}^{a})_{\alpha\dot{\alpha}}. \tag{B.18b}$$

The symmetric and antisymmetric combinations of two σ-matrices can be also computed

$$\sigma^a_{\alpha\dot{\alpha}}\sigma^b_{\beta\dot{\beta}} - \sigma^b_{\alpha\dot{\alpha}}\sigma^a_{\beta\dot{\beta}} = 2\left[(\sigma^{ab}\varepsilon)_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}} + (\varepsilon\bar{\sigma}^{ab})_{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta}\right] \tag{B.19a}$$

$$\sigma^a_{\alpha\dot{\alpha}}\sigma^b_{\beta\dot{\beta}} + \sigma^b_{\alpha\dot{\alpha}}\sigma^a_{\beta\dot{\beta}} = -\eta^{ab}\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}} + 4(\sigma^{ac}\varepsilon)_{\alpha\beta}(\varepsilon\bar{\sigma}^{bc})_{\dot{\alpha}\dot{\beta}}. \tag{B.19b}$$

If χ, ζ and η are Grassmann spinors, it is straightforward to show that

$$\chi\zeta = \zeta\chi \qquad \bar{\chi}\bar{\zeta} = \bar{\zeta}\bar{\chi} \tag{B.20a}$$

$$\chi\sigma^{ab}\zeta = -\zeta\sigma^{ab}\chi \qquad \chi\sigma^a\bar{\zeta} = -\bar{\zeta}\bar{\sigma}^a\chi \tag{B.20b}$$

and this allows us to compute the following identities:

$$(\chi\sigma^a\bar{\zeta})(\sigma_a\bar{\eta})_\alpha = -2\chi_\alpha(\bar{\zeta}\bar{\eta}) \tag{B.21a}$$

$$(\bar{\chi}\bar{\sigma}^a\zeta)(\sigma_a\bar{\eta})_\alpha = 2\zeta_\alpha(\bar{\chi}\bar{\eta}) \tag{B.21b}$$

$$(\bar{\chi}\bar{\sigma}^{ab}\bar{\zeta})(\sigma_{ab}\eta)_\alpha = 0 \tag{B.21c}$$

$$(\chi\sigma^{ab}\zeta)(\sigma_{ab}\eta)_\alpha = \zeta_\alpha(\chi\eta) - (\zeta\eta)\chi_\alpha \tag{B.21d}$$

$$(\chi\sigma^{[a}\not{p}\sigma^{b]}\bar{\zeta})(\sigma_{ab}\eta)_\alpha = (\bar{\zeta}\not{p}\chi)\eta_\alpha \tag{B.21e}$$

It is also possible to relate two-component spinors with four-component spinors. To perform such a transformation, we introduce another representation of $SL(2,\mathbb{C})$, which is mapped from the Pauli representation by means of

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \tag{B.22}$$

this is the Dirac representation in which the Dirac spinors contain two Weyl spinors:

$$\Psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}. \tag{B.23}$$

In the case of massless particles the Majorana basis is used. Here the four-component spinors are written in terms of one Weyl spinor:

$$\Psi_M = \begin{pmatrix} \chi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}. \tag{B.24}$$

Another device used for anticommuting spinor is the algebra of the Grassmann variables θ and $\bar{\theta}$. In some sense, integrating over one Grassmann variable η can be thought as a derivative:

$$\int \mathrm{d}\eta = 0 \qquad \text{and} \qquad \int \eta \mathrm{d}\eta = 1. \tag{B.25}$$

For the case with two Grassmann variables we define the volume differentials

$$\mathrm{d}^2\theta = -\frac{1}{4}\mathrm{d}\theta^\alpha \mathrm{d}\theta^\beta \varepsilon_{\alpha\beta} \tag{B.26a}$$

$$\mathrm{d}^2\bar{\theta} = -\frac{1}{4}\mathrm{d}\bar{\theta}_{\dot{\alpha}} \mathrm{d}\theta_{\dot{\beta}} \varepsilon^{\dot{\alpha}\dot{\beta}} \tag{B.26b}$$

$$\mathrm{d}^4\theta = \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}, \tag{B.26c}$$

and then it is straightforward to show that

$$\int \mathrm{d}^2\theta\theta^2 = 1 \tag{B.27a}$$

$$\int \mathrm{d}^2\theta(\chi\theta)(\psi\theta) = -\frac{1}{2}(\chi\psi). \tag{B.27b}$$

Taking derivatives can be more delicate, to ensure success have in mind the following identities

$$\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\beta} = -\frac{\partial}{\partial\theta_\alpha}, \qquad \varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta}\theta^2 = 4 \tag{B.28a}$$

$$\varepsilon_{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}_{\dot{\beta}}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}, \qquad \varepsilon_{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\beta}}}\bar{\theta}^2 = 4. \tag{B.28b}$$

C Representations of Supersymmetry

Following the discussion held in Section 2.1, we will present a schematic prescription to build the Supersymmetry irrep. for 1) massive particles and 2) massless particles. For each particular case we have to find the appropriate vacuum Ω of the algebra over which the action of the creation operator $\bar{Q}$ will span the particle spectra.

Massive particles

In the rest frame of a massive particle, where the momentum is $P_m = (-M, 0, 0, 0)$, the algebra (2.2) reduces to the Clifford algebra

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} \propto \delta_{\alpha\dot{\beta}}, \tag{C.1}$$

up to some rescaling of Q. The vacuum is Ω_j with spin j allows us to construct the spectrum by acting $\bar{Q}$ on this vacuum. Take as an example the the case with Ω_0, here the generator $\bar{Q}$ will produce the states

$$\Omega_0 \tag{C.2a}$$

$$\bar{Q}\Omega_0 \tag{C.2b}$$

$$\frac{1}{\sqrt{2}}\bar{Q}_{\dot{\alpha}}\bar{Q}_{\dot{\beta}}\Omega_0, \tag{C.2c}$$

that correspond to one fermion with spin 1/2 and one complex scalar (two states with spin 0). Different vacua will produce different states of particles, we gather some relevant cases in the following table

s	Ω_0	$\Omega_{1/2}$	Ω_1	$\Omega_{3/2}$
0	2	1		
1/2	1	2	1	
1		1	2	1
3/2			1	2
2				1

Massles particles

The particles that intermediate gravity and Supergravity are massless so we will show particular interest on this case. To work out the spectrum go to the fixed light-like frame $P_m = (-E, 0, 0, E)$ and here the algebra will look like this

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = \begin{pmatrix} 4E & 0 \\ 0 & 0 \end{pmatrix}. \tag{C.3}$$

The opperators Q_1 and $\bar{Q}_{\dot{1}}$ have the only non vanishing anticommutator relation, that resemblance the Clifford algebra

$$\{Q_1, \bar{Q}_{\dot{1}}\} = 4E, \tag{C.4}$$

which vacuum can be bosonic or fermionic. The states are represented by the helicity λ and are generated by the forementioned operators. As before, we span the multiplet starting from different vacua, the result is

λ	Ω_0	$\Omega_{1/2}$	Ω_1	$\Omega_{3/2}$
0	1			
1/2	1	1		
1		1	1	
3/2			1	1
2				1

One important observation is that the multiplets are, in general, not CPT invariant. This means that in order to have a theory which is invariant under CPT one should double the multiplet constructed by adding its CPT conjugate. If the multiplet is already invariant, this procedure is not needed. To complete the previous table we add the negative helicity representations summarized here:

λ	$\Omega_{-1/2}$	Ω_{-1}	$\Omega_{-3/2}$	Ω_{-2}
0	1			
-1/2	1	1		
-1		1	1	
-3/2			1	1
-2				1

For instance, one relevant case is the one generated by the vacuum $\Omega_{3/2}$, it will produce one particle with the degrees of freedom of the graviton, which has an helicity of 2, and a particle with helicity 3/2 known as the gravitino which is indeed the supersymmetric partner of the graviton. The multiplet looks like this

$$\{+2, +3/2\} \oplus \{-2, -3/2\}. \tag{C.5}$$

The exposed examples are meant to clarify the meaning of a multiplet in Supersymmetry.

D Algebra of Supergravity

We devote this section to show that the algebra of space-time diffeomorphism, Lorentz transformations and Supersymmetry is closed. Here we compute the commutator of two supersymmetric transformations applied to the vierbien ${e_\mu}^a$, we expect to obtain the closure relation

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta_G(y) + \delta_L(\Lambda) + \delta(\hat{\epsilon}). \tag{D.1}$$

The parameters y^μ, Λ^{ab} and $\hat{\epsilon}$ have to be determined and they can depend on the supermultiplet fields. Let us start by applying the infinitesimal transformations δ_ϵ to the supermultiplet consisting of the *vierbein* field e^a_μ, the gravitino ψ_μ and the auxiliary fields m and b_μ. These fields transform as

$$\delta_\epsilon {e_\mu}^a = i\psi_\mu \sigma^a \bar{\epsilon} + \text{c.c.} \tag{D.2}$$

$$\delta_\epsilon \psi_\mu = -2\hat{D}_\mu(\epsilon) + i\left[m\sigma_\mu \bar{\epsilon} + b_\mu \epsilon + \frac{1}{3} b^\nu (\sigma_\mu \bar{\sigma}_\nu \epsilon)\right] \tag{D.3}$$

$$\delta_\epsilon m = -\frac{1}{3}\epsilon(\sigma^\mu \bar{\sigma}^\nu \psi_{\mu\nu} + ib^\mu \psi_\mu - 3i\sigma^\mu \bar{\psi}_\mu m) \tag{D.4}$$

$$\begin{aligned} \delta_\epsilon b^a = & \frac{3}{8}(\bar{\psi}_{\mu\nu}\sigma^a \bar{\sigma}^\mu \sigma^\nu \epsilon) - \frac{1}{8}(\bar{\psi}_{\mu\nu}\bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^a \epsilon) + \\ & - \frac{3i}{2} m^*(\epsilon\psi^a) - \frac{i}{8} b_c (\epsilon\sigma^c \bar{\sigma}^a \sigma^\mu \bar{\psi}_\mu) + \\ & + \frac{i}{4} b^a (\epsilon \sigma^\mu \bar{\psi}_\mu) + \frac{i}{8} b^c (\bar{\psi}_\mu \bar{\sigma}^a \sigma_c \bar{\sigma}^\mu \epsilon) + \text{c.c.}, \end{aligned} \tag{D.5}$$

with $\hat{D}_\mu$ being the covariant derivative defined by the complete connection ${\hat{\omega}_\mu}^{ab} = {\omega_\mu}^{ab} + {K_\mu}^{ab}$. The first term is what we call the spin connection ${\omega_\mu}^{ab}$, and the second term is the torsion ${K_\mu}^{ab}$. The advantage of this notation is that the spin connection is torsion-free and therefore it is defined as

$${\omega_\mu}^{ab} = {e^b}_\nu \nabla_\mu e^{a\nu}, \tag{D.6}$$

which in the $\left(\frac{1}{2},0\right)$ and $\left(0,\frac{1}{2}\right)$ representation takes the form

$$D_\mu^{\left(\frac{1}{2},0\right)} = \partial_\mu + \frac{1}{2}\omega_\mu{}^{ab}\sigma_{ab} \tag{D.7}$$

$$D_\mu^{\left(0,\frac{1}{2}\right)} = \partial_\mu + \frac{1}{2}\omega_\mu{}^{ab}\bar{\sigma}_{ab}. \tag{D.8}$$

In the adjoint representation the covariant derivative is

$$D_\mu e_\nu{}^a = \partial_\mu e_\nu{}^a + \omega_\mu{}^{ab}\eta_{bc}e_\nu{}^c. \tag{D.9}$$

Now we proceed to perform two Supergravity transformations. For simplicity we apply these transformation to the vierbein $e_\mu{}^a$, thus giving

$$\begin{aligned}[\delta(\epsilon_1),\delta(\epsilon_2)]e_\mu{}^a =& -2i\hat{D}_\mu(\epsilon_1)(\sigma^a\bar{\epsilon}_2) + \text{c.c.}+\\ &-\Big[m(\sigma_\mu\bar{\epsilon}_1)(\sigma^a\bar{\epsilon}_2) + b_\mu\epsilon_1(\sigma^a\bar{\epsilon}_2)+\\ &+\frac{1}{3}b^\nu(\sigma_\mu\bar{\sigma}_\nu\epsilon_1)(\sigma^a\bar{\epsilon}_2) + \text{c.c.}\Big] - (1\leftrightarrow 2),\end{aligned} \tag{D.10}$$

and now we compute all the terms in squared brackets separately. For $m(\sigma_\mu\bar{\epsilon}_1)(\sigma^a\bar{\epsilon}_2)$, we make explicit the $(1\leftrightarrow 2)$ term:

$$\begin{aligned}-m(\sigma_\mu\bar{\epsilon}_1)(\sigma^a\bar{\epsilon}_2) - (1\leftrightarrow 2) =&\\ =& -m\left[\varepsilon^{\alpha\beta}\sigma_{\beta\dot{\beta}\mu}\bar{\epsilon}_1^{\dot{\beta}}\sigma^a_{\alpha\dot{\alpha}}\bar{\epsilon}_2^{\dot{\alpha}} - \varepsilon^{\alpha\beta}\sigma^a_{\beta\dot{\beta}}\bar{\epsilon}_1^{\dot{\beta}}\sigma_{\alpha\dot{\alpha}\mu}\bar{\epsilon}_2^{\dot{\alpha}}\right]\\ =& -m\bar{\epsilon}_2^{\dot{\alpha}}\varepsilon^{\alpha\beta}\left[-\sigma_{\beta\dot{\beta}\mu}\sigma^a_{\alpha\dot{\alpha}} + \sigma^a_{\beta\dot{\beta}}\sigma_{\alpha\dot{\alpha}\mu}\right]\bar{\epsilon}_1^{\dot{\beta}}\\ =& -m\bar{\epsilon}_2^{\dot{\alpha}}\varepsilon^{\alpha\beta}\left[-\sigma^b_{\beta\dot{\beta}}\sigma^a_{\alpha\dot{\alpha}} + \sigma^a_{\beta\dot{\beta}}\sigma^b_{\alpha\dot{\alpha}}\right]\bar{\epsilon}_1^{\dot{\beta}}\eta_{bc}e_\mu{}^c,\end{aligned}$$

using (B.19) and (B.14), we have

$$-m(\sigma_\mu\bar{\epsilon}_1)(\sigma^a\bar{\epsilon}_2) - (1\leftrightarrow 2) = -(4m\bar{\epsilon}_2\bar{\sigma}^{ab}\bar{\epsilon}_1)\eta_{bc}e_\mu{}^c. \tag{D.11}$$

We proceed now to compute $b_\mu \epsilon_1(\sigma^a \bar{\epsilon}_2)$:

$$\begin{aligned}
-b_\mu \epsilon_1(\sigma^a \bar{\epsilon}_2)+\text{c.c.} - (1 \leftrightarrow 2) = & \\
&= -b_\mu \left[\epsilon_1 \sigma^a \bar{\epsilon}_2 + \epsilon_2 \sigma^a \bar{\epsilon}_1 - \epsilon_2 \sigma^a \bar{\epsilon}_1 - \epsilon_1 \sigma^a \bar{\epsilon}_2\right] \\
&= 0. \qquad \text{(D.12)}
\end{aligned}$$

For the third term, we write $\frac{1}{3} b^\nu (\sigma_\mu \bar{\sigma}_\nu \epsilon_1)(\sigma^a \bar{\epsilon}_2)$ explicitly as

$$\begin{aligned}
-\frac{1}{3} b^\nu (\sigma_\mu \bar{\sigma}_\nu \epsilon_1)(\sigma^a \bar{\epsilon}_2) - (1 \leftrightarrow 2) + \text{c.c} = & \\
&= -\frac{1}{3}\left[\sigma^b \not{b} \epsilon_1 \sigma^a \bar{\epsilon}_2 - \sigma^b \not{b} \epsilon_2 \sigma^a \bar{\epsilon}_1\right] \eta_{bc} e_\mu{}^c + \text{c.c.} \\
&= -\frac{1}{3}\left[\epsilon_2(\sigma^a \not{b} \sigma^b - \sigma^b \not{b} \sigma^a)\bar{\epsilon}_1\right] \eta_{bc} e_\mu{}^c + \text{c.c} \\
&= -\frac{2}{3}\left(\epsilon_2 \sigma^{[a} \not{b} \sigma^{b]} \bar{\epsilon}_1\right) \eta_{bc} e_\mu{}^c + \text{c.c} \qquad \text{(D.13)}
\end{aligned}$$

Replacing (D.11), (D.12) and (D.13) in (D.10), we obtain the expression

$$\begin{aligned}
[\delta(\epsilon_1), \delta(\epsilon_2)] e_\mu{}^a = & -2i\hat{D}_\mu(\epsilon_1)(\sigma^a \bar{\epsilon}_2) + \text{c.c.} - (1 \leftrightarrow 2) + \\
& - \left[4m \bar{\epsilon}_2 \bar{\sigma}^{ab} \bar{\epsilon}_1 + \frac{2}{3} \epsilon_2 \sigma^{[a} \not{b} \sigma^{b]} \bar{\epsilon}_1 + \text{c.c.}\right] \eta_{bc} e_\mu{}^c. \qquad \text{(D.14)}
\end{aligned}$$

At last, we proceed to compute the term with derivatives in the commutator (D.10), this gives

$$\begin{aligned}
-2i\hat{D}_\mu(\epsilon_1)(\sigma^a\bar{\epsilon}_2) + \text{c.c.} - (1 \leftrightarrow 2) = \\
&= -2i\Big[\hat{D}_\mu(\epsilon_1)(\sigma^a\bar{\epsilon}_2) - \epsilon_2\sigma^a\hat{D}_\mu(\bar{\epsilon}_1) + \\
&\qquad - \hat{D}_\mu(\epsilon_2)\sigma^a\bar{\epsilon}_1 + \epsilon_1\sigma^a\hat{D}_\mu(\bar{\epsilon}_2)\Big] \\
&= -2i\left[\hat{D}_\mu(\epsilon_1\sigma^a\bar{\epsilon}_2) - \hat{D}_\mu(\epsilon_2\sigma^a\bar{\epsilon}_1))\right] \\
&= \hat{D}_\mu\left[-2i(\epsilon_1\sigma^\nu\bar{\epsilon}_2 - \epsilon_2\sigma^\nu\bar{\epsilon}_1)e_\nu{}^a\right] \\
&= \hat{D}_\mu(y^\nu e_\nu{}^a),
\end{aligned}$$

where it has been defined the parameter of the transformation as

$$y_\mu \equiv -2i(\epsilon_1\sigma_\mu\bar{\epsilon}_2 - \epsilon_2\sigma_\mu\bar{\epsilon}_1). \tag{D.15}$$

It is possible to reorganize the former result by adding and subtracting $y^\nu\hat{D}_\nu e^a_\mu$, thus giving

$$\hat{D}_\mu(y^\nu e^a_\nu) = (\hat{D}_\mu y^\nu)e^a_\nu + y^\nu\hat{D}_\nu e^a_\mu + y^\nu(\hat{D}_\mu e^a_\nu - \hat{D}_\nu e^a_\mu).$$

Expanding the covariant derivative—by means of the definition (D.9)—one verifies the expression

$$\begin{aligned}
\hat{D}_\mu(y^\nu e^a_\nu) =&(\partial_\mu y^\nu)e^a_\nu + y^\nu(\partial_\nu e^a_\mu) + y^\nu(\omega_\nu{}^{ab} + K_\nu{}^{ab})\eta_{bc}e^c_\mu + \\
&+ y^\nu(\hat{D}_\mu e^a_\nu - \hat{D}_\nu e^a_\mu),
\end{aligned} \tag{D.16}$$

where the first two terms constitute a diffeomorphism transformation with parameter y^ν, i.e. $\delta^{\text{diff}}_y e^a_\mu$. The last term is evaluated using the relation

$$\hat{D}_\mu e^a_\nu - \hat{D}_\nu e^a_\mu = -\frac{i}{2}\psi_\mu\sigma^a\bar{\psi}_\nu, \tag{D.17}$$

obtained by the definition of ${K_\mu}^{ab}$; as a result, the expression

$$\begin{aligned}\hat{D}_\mu e^a_\nu - \hat{D}_\nu e^a_\mu &= ({K_\mu}^{ab} e^c_\nu - {K_\nu}^{ab} e^c_\mu)\eta_{bc} \\ &= -\frac{i}{4}\psi^a \sigma_\nu \bar{\psi}_\mu - \frac{i}{8}\psi^a \sigma_\mu \bar{\psi}_\nu + \frac{i}{4}\psi^a \sigma_\mu \bar{\psi}_\nu + \\ &\quad + \frac{i}{8}\psi^a \sigma_\nu \bar{\psi}_\mu + \frac{i}{4}\psi_\nu \sigma^a \bar{\psi}_\mu + \frac{i}{8}\psi_\nu \sigma_\mu \bar{\psi}^a + \\ &\quad - \frac{i}{4}\psi_\mu \sigma^a \bar{\psi}_\nu - \frac{i}{8}\psi_\mu \sigma_\nu \bar{\psi}^a \\ &= -\frac{i}{2}\psi_\mu \sigma^a \bar{\psi}_\nu,\end{aligned}$$

is replaced in (D.16) and thus

$$\hat{D}_\mu (y^\nu e^a_\nu) = \delta_G(y) e^a_\mu + y^\nu {\hat{\omega}_\nu}^{ab} \eta_{bc} e^c_\mu + i\psi_\mu \sigma^a (\psi_\nu y^\nu /2)^\dagger, \quad \text{(D.18)}$$

where the last term is simply a Supergravity transformation with parameter $\hat{\epsilon} = \psi_\nu y^\nu /2$. Finally we plug equation (D.18) in (D.10) and the commutator takes the form

$$\begin{aligned}[\delta(\epsilon_1), \delta(\epsilon_2)] e^a_\mu &= \delta_G(y) e^a_\mu + \delta(\hat{\epsilon}) e^a_\mu + y^\nu {\hat{\omega}_\nu}^{ab} \eta_{bc} e^c_\mu + \\ &\quad - \left[4m \bar{\epsilon}_2 \bar{\sigma}^{ab} \bar{\epsilon}_1 + \frac{2}{3}\epsilon_2 \sigma^{[a} \not{b} \sigma^{b]} \bar{\epsilon}_1 + \text{c.c.}\right] \eta_{bc} e^c_\mu . \\ &= \delta_G(y) e^a_\mu + \delta(\hat{\epsilon}) e^a_\mu + \Lambda^{ab} \eta_{bc} e^c_\mu ,\end{aligned}$$

note that, for simplicity, it has been defined

$$\Lambda^{ab} \equiv y^\nu {\hat{\omega}_\nu}^{ab} - \left[4m \bar{\epsilon}_2 \bar{\sigma}^{ab} \bar{\epsilon}_1 + \frac{2}{3}\epsilon_2 \sigma^{[a} \not{b} \sigma^{b]} \bar{\epsilon}_1 + \text{c.c.}\right]. \quad \text{(D.19)}$$

Identifying the term $\Lambda^{ab}\eta_{bc} e^c_\mu$ with the lorentz transformation $\delta_L(\Lambda) e^a_\mu$, we determine that the expression of the commutator is

$$[\delta(\epsilon_1), \delta(\epsilon_2)] = \delta_G(y) + \delta(\hat{\epsilon}) + \delta_L(\Lambda), \quad \text{(D.20)}$$

with

$$y^\mu = -2i(\epsilon_1\sigma^\mu\bar{\epsilon}_2 - \epsilon_2\sigma^\mu\bar{\epsilon}_1) \tag{D.21}$$

$$\Lambda^{ab} = y^\mu\hat{\omega}_\mu{}^{ab} - \left[4m\bar{\epsilon}_2\bar{\sigma}^{ab}\bar{\epsilon}_1 + \frac{2}{3}\epsilon_2\sigma^{[a}\not{b}\sigma^{b]}\bar{\epsilon}_1 + \text{c.c.}\right] \tag{D.22}$$

$$\hat{\epsilon} = \psi_\nu y^\nu/2. \tag{D.23}$$

Now that we have shown that the algebra closes, it is useful to compute the second order commutator

$$[\delta(\epsilon_1), [\delta(\epsilon_1), \delta(\epsilon_2)]] = [\delta(\epsilon_1), \delta(\hat{\epsilon})] + [\delta(\epsilon_1), \delta_G(y)] + [\delta(\epsilon_1), \delta_L(\Lambda)]. \tag{D.24}$$

For simplicity, we expand the commutators in the right hand side of the previous equation stopping at second order in ϵ_1, the result is

$$\begin{aligned} &\frac{1}{2}\delta\left(y^\mu\delta_{\epsilon_1}\psi_\mu\right) + \delta\left(y^\mu\partial_\mu\epsilon_1\right) + \frac{1}{2}\delta\left(\Lambda_{ab}\sigma^{ab}\epsilon_1\right) + \ldots = \\ &\quad = \delta\left[\frac{1}{2}y^\mu(2\partial_\mu\epsilon_1 + \delta_{\epsilon_1}\psi_\mu) + \frac{1}{2}\Lambda_{ab}\sigma^{ab}\epsilon_1\right] + \ldots, \end{aligned} \tag{D.25}$$

the ellipses denote terms of higher order. We conclude that the second order commutator is

$$[\delta(\epsilon_1), [\delta(\epsilon_1), \delta(\epsilon_2)]] = \delta(\varsigma) + \ldots, \tag{D.26}$$

where the transformation parameter is

$$\varsigma = \frac{1}{2}y^\mu(2\partial_\mu\epsilon_1 + \delta_{\epsilon_1}\psi_\mu) + \frac{1}{2}\Lambda_{ab}\sigma^{ab}\epsilon_1. \tag{D.27}$$

E Curvature and Torsion

Here we expand the discussion of gauging Supersymmetry, in particular we review the geometrical implications, meaning the curvature and torsion fields of superspace.

The vierbein superfield consists of one local Lorentz index A, we indicate this superfield as E^A, moreover we can obtain the world components of this tensor using the base z^M:

$$E^A = dz^M E_M{}^A. \tag{E.1}$$

This field is locally Lorentz covariant:

$$\delta_L E_M{}^A = E_M{}^B \Lambda_B{}^A(z), \tag{E.2}$$

the z-dependence of Λ indicates that the group is gauged, thus inducing a connection ϕ and a covariant derivative $\hat{D}$ in the manifold, the effect is that the geometry is not flat anymore.

In this sense, the connection ϕ generates a curvature R which by definition is its own external derivative

$$R = d\phi + \phi\phi, \tag{E.3}$$

and as usual, the curvature is a Lie algebra valued two-form that encapsulates the geometrical structure of the manifold, here we write it explicitly in terms of the connection's components:

$$R_A{}^B = dz^M dz^N \partial_N \phi_{MA}{}^B + dz^M \phi_{MA}{}^C dz^N \phi_{NC}{}^B. \tag{E.4}$$

The external derivative defined by ϕ allows us to understand the geometrical structure of superspace. For instance, the external derivative of E is, in general, not zero; this means that space is endowed with torsion. The torsion can be computed as the derivative

$$K^A = dE^A + E^B \phi_B{}^A, \tag{E.5}$$

to obtain its components we recall that, in the z^M base, the tensor K^A can be expressed as

$$K^A = \frac{1}{2} E^C E^B K_{BC}{}^A, \tag{E.6}$$

explicitly, this becomes

$$\begin{aligned} K_{NM}{}^A =& \partial_N E_M{}^A - (-)^{nm} \partial_M E_N{}^A \\ &+ (-)^{n(b+m)} E_M{}^B \phi_{NB}{}^A - (-)^{mb} E_N{}^B \phi_{MB}{}^A \end{aligned} \tag{E.7}$$

Both the torsion and curvature are direct consequences of the connection, the last can be obtained from the structure group transformations of superspace[4].

One characteristic of the curvature R is that it contains the auxiliary fields m and b^a on its lowest component. In the spinor representation, the curvature can be spelled in terms of an hermitian vector superfield G_a

$$R_{\beta\dot{\delta}\gamma\alpha} = \varepsilon_{\alpha\beta} G_{\gamma\dot{\delta}} + \varepsilon_{\gamma\beta} G_{\alpha\dot{\delta}}, \tag{E.8}$$

where $G_{\alpha\dot{\alpha}}$ represents the field G in the spinor base:

$$G_{\alpha\dot{\alpha}} = \sigma^a_{\alpha\dot{\alpha}} G_a. \tag{E.9}$$

The non-physical degrees of freedom that complete the Supergravity multiplet are defined as the following projection

$$R(z)\Big|_{\theta=0} = -\frac{1}{18} m(x) \tag{E.10}$$

$$G_a(z)\Big|_{\theta=0} = -\frac{1}{3} b_a(x). \tag{E.11}$$

These conventions are held because with them the Supergravity transformation of both fields m and b_a looks cleaner.

[4]Naturally, we are talking about the space-time diffeomorphisms and local Lorentz invariance $G \times L$, these are the elements of the supergauge transformation.

F Goldstino Transformation

Consider that a group S is broken down to a subgroup H. The elements of the coset group S/H with broken generators Q can be identically parametrized by

$$se^{-\lambda\cdot Q} = e^{-\lambda'(x')\cdot Q}h(\lambda), \tag{F.1}$$

where s is an element of S and h is an element of the subgroup H. Take s to be a pure supersymmetric transformation with parameter ϵ, in this case, the transformation of the Goldstone field will be determined by

$$\delta_\epsilon\lambda(x) = \lambda'(x) - \lambda(x), \tag{F.2}$$

and the expression of $\lambda'(x')$ is given in terms of the fields in the Supergravity multiplet. In [57] A. A. Kapustnikov noted that the transformation of the goldstino has the structure

$$\lambda'(x) - \lambda(x) = \eta + \alpha^\mu\partial_\mu\lambda + \frac{1}{2}\beta_{ab}\sigma^{ab}\lambda, \tag{F.3}$$

where the parameters η, α^μ and β_{ab} dependend on the field content of the theory. To obtain these parameters, Kapustnikov also recognized that they are, respectively, the resulting parameters of a Supergravity transformation generated by[5]

$$\frac{\exp(\delta_\lambda) - 1}{\delta_\lambda} \wedge \delta(\eta) = -\delta(\epsilon) + \delta_G(\alpha^\mu) + \delta_L(\beta_{ab}). \tag{F.4}$$

[5]The notation $f(X) \wedge Y$ is meant to be understood as an expansion of nested commutators, for example the quantity $e^X \wedge Y$ is equivalent to

$$e^X \wedge Y = Y + [X, Y] + \frac{1}{2!}[X, [X, Y]] + \frac{1}{3!}[X, [X, [X, Y]]] + \dots$$

The left hand side of the forementioned equation can be computed using the Baker Campbell Hausdorff formula, thereby resulting in the expression

$$\delta_\eta + \frac{1}{2}[\delta_\lambda, \delta_\eta] + \frac{1}{3!}[\delta_\lambda, [\delta_\lambda, \delta_\eta]] + \dots, \tag{F.5}$$

and the problem is now shifted to find the commutators that can be computed directly from the structure of the group. In fact, following the analysis of Appendix D, we compute the commutators of (F.5) using the relations (D.20) and (D.26) as reference:

$$[\delta_\lambda, \delta_\eta] = \delta(\hat{\eta}) + \delta_L(y) + \delta_G(\Lambda) \tag{F.6a}$$

$$[\delta_\lambda, [\delta_\lambda, \delta_\eta]] = \delta(\varsigma) + \dots, \tag{F.6b}$$

where the transformation parameters are

$$y^\mu = -2i(\lambda\sigma^\mu\bar{\eta} - \eta\sigma^\mu\bar{\lambda}) \tag{F.7a}$$

$$\Lambda^{ab} = y^\mu \hat{\omega}_\mu{}^{ab} - \left[4m\bar{\eta}\bar{\sigma}^{ab}\bar{\lambda} + \frac{2}{3}\eta\sigma^{[a}\not{b}\sigma^{b]}\bar{\lambda} + \text{c.c.}\right] \tag{F.7b}$$

$$\hat{\eta} = \frac{1}{2}y^\mu\psi_\mu \tag{F.7c}$$

$$\varsigma = \frac{1}{2}y^\mu(2\partial_\mu\lambda + \delta_\lambda\psi) + \frac{1}{2}\Lambda_{ab}\sigma^{ab}\lambda. \tag{F.7d}$$

Since our purpose is to determine the values of η, α^μ and β_{ab} in terms of ϵ and λ, it is necessary to identify the transformation parameters of the left hand side of the equation (F.4) with each parameter in the right hand side:

$$\eta = -\epsilon - \frac{1}{2}\hat{\eta} - \frac{1}{3!}\varsigma + \dots \tag{F.8a}$$

$$\alpha^\mu = \frac{1}{2}y^\mu + \dots \tag{F.8b}$$

$$\beta_{ab} = \frac{1}{2}\Lambda_{ab} + \dots, \tag{F.8c}$$

in this case, the ellipses denote terms of second order in λ, these terms will not contribute to the second order transformation of the goldstino. Note that the forementioned expressions have to be evaluated in ϵ instead of η, doing an iterative process it is possible to determine that

$$\eta = -\epsilon + \frac{1}{2}\hat{\epsilon} + \frac{1}{3!}\varsigma + \dots \tag{F.9a}$$

$$\alpha^\mu = -\frac{1}{2}y^\mu + \dots \tag{F.9b}$$

$$\beta_{ab} = -\frac{1}{2}\Lambda_{ab} + \dots, \tag{F.9c}$$

where the parameters ς, y and Λ are the same as in (F.7) except that instead of evaluating in η, it is used ϵ.

We are ready to compute the transformation of the goldstino λ by replacing our previous result in equation (F.3). After some algebraic steps we reduce the expression to

$$\lambda' = -\epsilon + \frac{1}{4}y \cdot \psi + \frac{1}{3!}\left[-2y \cdot \partial\lambda + \frac{1}{2}y \cdot \delta_\lambda \psi - \Lambda \cdot \sigma\lambda\right], \tag{F.10}$$

which can be further simplified by means of the identities in (B.21). First we compute the explicit expression

$$\Lambda^{ab}\sigma_{ab}\lambda = y \cdot \hat{\omega}^{ab}\sigma_{ab}\lambda - \gamma, \tag{F.11}$$

where

$$\gamma_\alpha = 12m^*(\lambda\epsilon)\lambda_\alpha + \frac{2}{3}\left(\bar{\lambda}\not{b}\epsilon - \bar{\epsilon}\not{b}\lambda\right)\lambda_\alpha, \tag{F.12}$$

and replace it in the transformation above, the terms proportional to y^μ can be combined, now the transformation looks like this:

$$\lambda' = -\epsilon + \frac{1}{4}y \cdot \psi + \frac{1}{3!}\left[y \cdot \left(-2\hat{D}\lambda + \frac{1}{2}\delta_\lambda\psi\right) + \gamma\right]. \tag{F.13}$$

To eliminate the derivative of λ, we use the transformation law for the gravitino, which we rewrite here as

$$\delta_\lambda \psi_\mu = -2\hat{D}_\mu \lambda + i a_\mu, \tag{F.14}$$

with

$$a_\mu = m\sigma_\mu \bar{\lambda} + b_\mu \lambda + \frac{1}{3}(\sigma_\mu \not{b} \lambda). \tag{F.15}$$

Putting all together and isolating the terms with auxiliary fields, we find the expression

$$\lambda' = -\epsilon + \frac{1}{4} y \cdot (\psi + \delta_\lambda \psi) + \frac{1}{3!}\left[-iy \cdot a + \gamma\right] + \ldots, \tag{F.16}$$

which, after treating carefully the terms with auxiliary fields and after some algebraic steps, takes the final form of

$$\lambda' = -\epsilon + \frac{1}{4} y \cdot \left[(\psi + \delta_\lambda \psi) - \frac{2i}{3} m\sigma\bar{\lambda}\right] + 2m^*(\lambda\epsilon)\lambda - \frac{4}{3}(\lambda\epsilon)(\bar{\lambda}\not{b}) + \ldots \tag{F.17}$$

This transformation of the Goldstone fermion non-linearly realizes the symmetry of Supergravity.

G Generalized Covariant Derivative I

In this appendix we derive the Lagrangian (3.46) where the degrees of freedom coming from the goldstino are include, this is done by applying the Stückelberg trick. First we define the generalized covariant derivative that acts on a spinor χ as

$$\mathcal{D}_\mu \chi = D_\mu \chi - \frac{i}{2} m^* \sigma_\mu \bar{\chi} \tag{G.1}$$

$$\mathcal{D}_\mu \bar{\chi} = D_\mu \bar{\chi} - \frac{i}{2} m \bar{\sigma}_\mu \chi. \tag{G.2}$$

In this case, for simplicity, we are indicating the mass of the gravitino as m.

To obtain the field strength we compute the commutator of the generalized covriant derivative, $[\mathcal{D}_\mu, \mathcal{D}_\nu]$. It is easy to see that the quantity

$$\mathcal{D}_\mu \mathcal{D}_\nu \chi = D_\mu D_\nu \chi - \frac{i}{2} m^* (D_\mu \sigma_\nu + \sigma_\mu D_\nu) \bar{\chi} - \frac{1}{4} |m|^2 \sigma_\mu \bar{\sigma}_\nu \chi, \tag{G.3}$$

after making the μ and ν indices antisymmetric, leads to the commutator

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] \chi = \left(\frac{1}{2} R_{\mu\nu\alpha\beta} - |m|^2 g_{\mu\alpha} g_{\nu\beta} \right) \sigma^{\alpha\beta} \chi, \tag{G.4}$$

where the Riemann tensor was introduced by the commutator

$$[D_\mu, D_\nu] \chi = (1/2) R_{\mu\nu\alpha\beta} \sigma^{\alpha\beta} \chi. \tag{G.5}$$

For a generic de Sitter background—with cosmological constant Λ—the Riemann tensor is

$$R_{\mu\nu\alpha\beta} = -\frac{\Lambda}{3 M_{\rm pl}^2} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}), \tag{G.6}$$

which can be replaced in expression (G.4) to derive the final form of the commutator:

$$[\mathcal{D}_\mu, \mathcal{D}_\nu]\chi = -\left(\frac{\Lambda}{3M_{\rm pl}^2} + |m|^2\right)\sigma_{\mu\nu}\chi. \tag{G.7}$$

The new generalized covariant derivative can be used to write a kinetic term in the action that is manifestly invariant under the linearly realized symmetries. The gravitino sector of the Lagrangian has the form of equation (3.38) which we rewrite here as

$$\frac{1}{eM_{\rm pl}^2}\mathcal{L}_\psi = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\left(\bar{\psi}_\mu\bar{\sigma}_\nu\mathcal{D}_\rho\psi_\sigma + \text{c.c.}\right). \tag{G.8}$$

To restore all symmetries we must write the action à la Stückelberg, this is achieved by performing the replacement

$$\psi \longrightarrow \Psi_\mu = \psi - 2\mathcal{D}_\mu\lambda + \ldots, \tag{G.9}$$

that leads to the Lagrangian

$$\begin{aligned}\frac{1}{eM_{\rm pl}^2}(\mathcal{L}_{\psi\psi} + \mathcal{L}_{\psi\lambda} + \mathcal{L}_{\lambda\lambda}) =& \\ \varepsilon^{\mu\nu\rho\sigma}\bigg(\frac{1}{2}\bar{\psi}_\mu\bar{\sigma}_\nu\mathcal{D}_\rho\psi_\sigma - \bar{\psi}_\mu\bar{\sigma}_\nu\mathcal{D}_\rho\mathcal{D}_\sigma\lambda +& \\ - \mathcal{D}_\rho\bar{\psi}_\sigma\bar{\sigma}_\nu\mathcal{D}_\mu\lambda + 2\mathcal{D}_\mu\bar{\lambda}\bar{\sigma}_\nu\mathcal{D}_\rho\mathcal{D}_\sigma\lambda + \text{c.c.}\bigg).&\end{aligned} \tag{G.10}$$

This result can be embellish using the antisymmetric property of $\varepsilon^{\mu\nu\rho\sigma}$ and combining the second and the conjugate of the third term:

$$
\begin{aligned}
\frac{1}{eM_{\rm pl}^2}&(\mathcal{L}_{\psi\psi}+\mathcal{L}_{\psi\lambda}+\mathcal{L}_{\lambda\lambda}) = \\
&\varepsilon^{\mu\nu\rho\sigma}\Bigg(\frac{1}{2}\bar{\psi}_\mu\bar{\sigma}_\nu\mathcal{D}_\rho\psi_\sigma - \bar{\psi}_\mu\bar{\sigma}_\nu[\mathcal{D}_\rho,\mathcal{D}_\sigma]\lambda + \\
&+\mathcal{D}_\mu\bar{\lambda}\bar{\sigma}_\nu[\mathcal{D}_\rho,\mathcal{D}_\sigma]\lambda + \text{c.c.}\Bigg).
\end{aligned}
\tag{G.11}
$$

Looking at this Lagrangian, one can note that in the pure AdS Supergravity—where the commutator (G.7) is exactly zero—there is no kinetic term for the goldstino, this property is related with the fact that the action before the Stückelberg trick is already invariant under local Supersymmetry transformations.

In general, the commutator (G.7) is different from zero and, in spite of the identity (B.18), we can determine the quadratic Lagrangian to be

$$
\frac{1}{eM_{\rm pl}^2}\mathcal{L}_{\psi\psi} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu D_\rho\psi_\sigma + m\psi_\mu\sigma^{\mu\nu}\psi_\nu \tag{G.12}
$$

$$
\frac{1}{eM_{\rm pl}^2}\mathcal{L}_{\psi\lambda} = -i\left(\frac{\Lambda}{M_{\rm pl}^2}+3|m|^2\right)\bar{\psi}_\mu\bar{\sigma}^\mu\lambda \tag{G.13}
$$

$$
\frac{1}{eM_{\rm pl}^2}\mathcal{L}_{\lambda\lambda} = -i\left(\frac{\Lambda}{M_{\rm pl}^2}+3|m|^2\right)\left(\bar{\lambda}\not{D}\lambda + 2im\lambda^2\right), \tag{G.14}
$$

in addition with its complex conjugate. The slashed derivative is defined as $\not{D} = \bar{\sigma}^\mu D_\mu$.

H Generalized Covaraint Derivative II

We continue the discussion about the construction of a general covariant derivative, in this case we study the scenario where the time diffeomorphism are broken. Ignoring the standard kinetic term $\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu D_\rho\psi_\sigma/2$ and the term proportional to m_3, the quadratic Lagrangian that is invariant only under spatial diffeomorphisms is

$$\begin{aligned}\mathcal{L}/M_{\rm pl}^2 =& m^*(\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu) + m_0^*(\bar{\psi}_\mu\bar{\sigma}^{\mu 0}\bar{\psi}^0) + \text{c.c.}+\\ &+ im_1(\bar{\psi}^0\bar{\sigma}^\mu\psi_\mu - \bar{\psi}_\mu\bar{\sigma}^\mu\psi^0)+\\ &+ im_2\varepsilon^{\mu\nu\rho 0}(\bar{\psi}_\mu\bar{\sigma}_\nu\psi_\rho),\end{aligned} \tag{H.1}$$

where, again, the mass of the gravitino is $m \equiv m_{3/2}$.

We want to manipulate this expression so every term has a prefactor of the form $\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu$. After using the identities

$$\varepsilon^{abcd}\bar{\sigma}_{cd} = 2i\bar{\sigma}^{ab} \qquad \text{and} \qquad \varepsilon^{abcd}\varepsilon_{abef} = 2(\delta^c_e\delta^d_f - \delta^c_f\delta^d_e),$$

we can recast the original Lagrangian as [6]

$$\begin{aligned}\mathcal{L}/M_{\rm pl}^2 =& m^*\bar{\psi}_\mu\left(-\frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\sigma}_{\nu\rho}\right)\bar{\psi}_\sigma+\\ &+ m_0^*\bar{\psi}_\mu\left(-\frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\sigma}_{\nu\rho}t_\sigma t_\lambda\right)\bar{\psi}^\lambda+\\ &+ \frac{1}{2}m_1\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\left(-\frac{i}{2}\varepsilon_{\nu\rho\alpha\lambda}\bar{\sigma}^\lambda\right)t^\alpha\psi_\sigma+\\ &+ \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu(-im_2t_\rho)\psi_\sigma + \text{c.c.},\end{aligned} \tag{H.2}$$

where t_μ is understood as $\partial_\mu t = \delta^0_\mu$. We use the antisymmetric properties of the Levi-Civita tensor to rewrite the first term, in

[6] Since the two last terms are real we can decompose them as $A = (1/2)A +$ c.c.

the second term we express the symmetric tensor $t_\sigma t_\lambda$ in terms of $g_{\sigma\lambda}$. For the third term we apply the identity (B.16b) to obtain the expression

$$\begin{aligned}\mathcal{L}/M_{\text{pl}}^2 = &\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\left(-\frac{i}{2}m^*\sigma_\rho\right)\bar{\psi}_\sigma+\\ &+\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\left[-\frac{i}{2}m_0^*\left(\frac{1}{2}g^{00}\sigma_\rho - t_\rho\sigma^0\right)g_{\sigma\lambda}\right]\bar{\psi}^\lambda+\\ &+\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\left(-\frac{1}{2}m_1\sigma^0\bar{\sigma}_\rho\right)\psi_\sigma+\\ &+\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu(-im_2t_\rho)\psi_\sigma + \text{c.c.},\end{aligned} \tag{H.3}$$

note that we have factorize the gravitino field ψ_σ in all terms and now the Lagrangian can be accommodated as

$$\mathcal{L}/M_{\text{pl}}^2 = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\mathcal{D}_\rho\psi_\sigma + \text{c.c.}. \tag{H.4}$$

The kinetic term has been reintroduced and we defined the new generalized covariant derivative as

$$\begin{aligned}\mathcal{D}_\rho\psi_\sigma = D_\rho\psi_\sigma - \frac{i}{2}\Bigg[&\left(m^*\sigma_\rho + \frac{1}{2}m_0^*g^{00}\sigma_\rho - m_0^*t_\rho\sigma^0\right)\bar{\psi}_\sigma+\\ &+\left(-im_1\sigma^0\bar{\sigma}_\rho + 2m_2t_\rho\right)\psi_\sigma\Bigg]\end{aligned} \tag{H.5}$$

The generalized covariant derivative is used to perform a field redefinition that depends on the goldstino λ. We are looking for a field that transforms as

$$\delta_\epsilon\psi'_\mu = -2\mathcal{D}_\mu\epsilon + (2\text{ ferm.})\epsilon \tag{H.6}$$

under infinitesimal Supersymmetry. The shifted field turns out to be

$$\begin{aligned} \psi'_\mu = \psi_\mu - i\left(m^*\sigma_\mu + \frac{1}{2}m_0^* g^{00}\sigma_\mu - m_0^* t_\mu \sigma^0\right)\bar{\lambda} + \\ -i\Big(- i m_1 \sigma^0 \bar{\sigma}_\mu + 2 m_2 t_\mu\Big)\lambda, \end{aligned} \tag{H.7}$$

since the infinitesimal transformations of ψ and λ are

$$\delta_\epsilon \psi_\mu = -2 D_\mu \epsilon + \dots \tag{H.8a}$$

$$\delta_\epsilon \lambda = -\epsilon + \dots \tag{H.8b}$$

hence the transformation of the field ψ' is exactly the transformation given in equation (H.6) as we wanted.

I Local Constrained Superfield

Supersymmetry breaking requires the existance of a fermionic field which can be described by means of a chiral superfield

$$X = \mathcal{A} + \sqrt{2}\Theta\mathcal{G} + \Theta^2\mathcal{F}, \tag{I.1}$$

where we impose the constraint

$$X^2 = 0. \tag{I.2}$$

In order to eliminate undesired degrees of freedom. Since Θ is an anticommuting variable, all functions of Θ are of order $\mathcal{O}(\Theta^2)$. We also know that the combination X^2 is also a chiral superfield expressed as

$$\begin{aligned} X^2 &= \mathcal{A}^2 + \sqrt{2}\Theta(\mathcal{A}\mathcal{G} + \mathcal{G}\mathcal{A}) + 2(\Theta\mathcal{G})^2 + 2\Theta^2\mathcal{A}\mathcal{F} \\ &= \mathcal{A}^2 + \sqrt{2}\Theta(\mathcal{A}\mathcal{G} + \mathcal{G}\mathcal{A}) - \Theta^2\mathcal{G}^2 + 2\Theta^2\mathcal{A}\mathcal{F} \\ &= \mathcal{A}^2 + \sqrt{2}\Theta(\mathcal{A}\mathcal{G} + \mathcal{G}\mathcal{A}) + \Theta^2(-\mathcal{G}^2 + 2\mathcal{A}\mathcal{F}), \end{aligned} \tag{I.3}$$

therefore $X^2 = 0$ is satisfied if $2\mathcal{A}\mathcal{F} = \mathcal{G}^2$. Consequently, we prove that the constraint (I.2) implies the relation

$$A = \frac{\mathcal{G}^2}{2\mathcal{F}}, \tag{I.4}$$

so the superfield in equation (I.1) has the following form:

$$X = \frac{\mathcal{G}^2}{2\mathcal{F}} + \sqrt{2}\Theta\mathcal{G} + \Theta^2\mathcal{F}, \tag{I.5}$$

where the lowest component has been replaced by an expression written in terms of the remaining componets.

J Goldstino Redefinition

This section is rather technical. We show how to derive an expression for the field γ in terms of the Goldstone fermion λ. Remember that λ is the fermionic field needed to correctly implement the CCWZ construction developed in Chapter III. On the other hand, using the language of constrained superfields, we defined the field γ in relation with the component $\mathcal{G}$ of the chiral superfield X. The relation between $\mathcal{G}$ and γ is

$$\mathcal{G}_\alpha = \sqrt{2}\frac{F}{f}\gamma_\alpha, \tag{J.1}$$

so we need to find an expression of γ in terms of λ to completely relate both effective theories.

Notice that the main difference between both fields is that the transformation of λ mixes λ with $\bar{\lambda}$, whereas the transformation of γ is only a function of itself[7].

We can go from one representation to the other by making the coordinate transformation

$$y^\mu = x^\mu - ik^2 v^\mu(y) \tag{J.2}$$

where we introduced the dimensionless constant k to keep track of the order of λ's and we also defined the vector v^μ as

$$v_\mu = \lambda\sigma^\mu\bar{\lambda}. \tag{J.3}$$

The new coordinate y^μ helps us to transform λ into γ as

$$\gamma(x) = \lambda(y), \tag{J.4}$$

[7]This can be seen directly from equation (3.22) for λ and from equation (4.20) fro γ.

so our problem is reduced to write $\lambda(y)$ evaluated in x^μ. Expanding equation (J.2) we obtain

$$\begin{aligned} y^\mu &= x^\mu - ik^2 v^\mu - k^4 \left(v^\alpha - ik^2 v^\beta \partial_\beta v^\alpha\right) \partial_\alpha v^\mu + \\ &\quad + \frac{1}{2} ik^6 v^\alpha v^\beta \partial_\alpha \partial_\beta + \ldots \\ &= x^\mu - ik^2 v^\mu - k^4 v^\alpha \partial_\alpha v^\mu + ik^6 v^\beta \partial_\beta v^\alpha \partial_\alpha v^\mu + \\ &\quad + \frac{1}{2} ik^6 v^\alpha v^\beta \partial_\alpha \partial_\beta v^\mu + \ldots \end{aligned} \tag{J.5}$$

where the dots represent terms with higher order in k^2 that will not contribute to the final result since they will produce terms with λ^3 which is exactly zero as λ is a Grassmann spinor.

Equation (J.4) turns out to be

$$\gamma(x) = \lambda + (y-x)^\mu \partial_\mu \lambda + \frac{1}{2}(y-x)^\mu (y-x)^\alpha \partial_\mu \partial_\alpha \lambda + \ldots \tag{J.6}$$

After replacing the expansion of $(y-x)^\mu$ provided by (J.5) and ignoring terms with λ^3 we obtain the final result:

$$\begin{aligned} \gamma = \lambda &- ik^2 v^\mu \partial_\mu \lambda - k^4 v^\alpha (\partial_\alpha v^\mu) \partial_\mu \lambda - \frac{1}{2} k^4 v^\mu v^\alpha \partial_\mu \partial_\alpha \lambda + \\ &+ \frac{1}{2} ik^6 v^\alpha v^\beta (\partial_\alpha \partial_\beta v^\mu) \partial_\mu \lambda + ik^6 v^\beta (\partial_\beta v^\alpha)(\partial_\alpha v^\mu) \partial_\mu \lambda \end{aligned} \tag{J.7}$$

where all the fields are evaluated in x^μ.

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