Freight-Transit Tour Synthesis

Diana Patricia Moreno Palacio

Universidad Nacional de Colombia

Facultad de Minas, Departamento Ingeniería Civil

Medellín, Colombia

2023

Freight-Transit Tour Synthesis

Diana Patricia Moreno Palacio

Tesis presentada como requisito parcial para optar al título de:

Doctor en Ingeniería – Ingeniería Civil

Advisor:

Carlos A. González-Calderón, Ph.D.

Co-Advisors:

John Jairo Posada Henao, Ph.D.

Héctor Andrés López-Ospina, Ph.D

Línea de Investigación: Transporte de carga

Universidad Nacional de Colombia

Facultad de Minas, Departamento Ingeniería Civil

Medellín, Colombia

2023

Para mi Abue, el amor de mi vida, mi luz.

Para mi tio, el padre que me recompensó la vida, mi ejemplo e inspiración.

Para mi madre, que me enseñó la perseverancia.

Para mi Caty, mi ancla y mi adoración.

Y para mi Agus por rescatarme tantas veces con su amor.

To my Granny, the love of my life, my light.

To my uncle, the father who gifted me life, my example and inspiration.

To my mother, who taught me perseverance.

To my Caty, my anchor and my adoration.

And to my Agus, for rescuing me so many times with his love.

Acknowledgements

Undoubtedly, the doctoral journey has been one of the most challenging endeavors I have undertaken in my academic and personal life. The possibility of penning these words was a moment that often felt unreachable. This work has demanded significant effort on professional, emotional, and even health fronts. Nonetheless, the cherished moment finally arrived, and its realization wouldn't have been possible without the support, affection, and guidance of all those who stood by my side, offering their hand and the encouragement I needed to press forward.

Allow me to begin by expressing my gratitude to my advisors and mentors, Professors Carlos A. González-Calderón, John Jairo Posada Henao, and Héctor López Ospina, for their invaluable contributions, references, and timely advice in the development of this research. In particular, I want to emphasize Professor González, who consistently believed in me and went beyond his role as a teacher, becoming a fellow researcher and a companion on this path. He not only shared his knowledge but also his understanding and trust.

Likewise, I wish to acknowledge and extend my gratitude to my fellow doctoral candidates for their friendship and companionship, with special mention of Claudia Muñoz, a friend who illuminated this process with her crucial suggestions and affection. I must also express my appreciation to my colleagues from the Freight Transport and Logistics research group at Universidad Nacional de Colombia, who contributed to significant accomplishments in my doctoral journey, particularly Jhan Kevin Gil, a valued collaborator. Likewise, in a broader sense, I want to thank the Universidad Nacional de Colombia, at Medellin, for having become my home throughout this process, and welcoming me while allowing me to walk towards my goal.

It is of utmost importance to convey my gratitude to my alma mater, Universidad de Antioquia, the Faculty of Engineering, and the Environmental School for affording me the opportunity to undertake this monumental project and achieve the goal of completing my doctoral studies under dignified conditions, knowing that my university awaits my return to share all the knowledge I have acquired. I want to offer special mention to Rector John Jairo Arboleda for his support during pivotal moments and for his belief in me.

I cannot overlook expressing my gratitude to the Volvo Research and Educational Foundation (VREF) for the support they provided, enabling me to expand my networks and academic knowledge. I am especially thankful to Professor José Holguín-Veras, whose wisdom always imparts something new and whose warmth is ever available.

Additionally, I extend my thanks to Caliper Co and Andrés Rabinowicz for allowing us to use their academic simulation tools.

Now, I wish to direct my gratitude to the true driving forces of this journey: my family and dearest friends. The love of my grandmother and mother, the trust and support of my uncle Rigo, the unwavering companionship of my sister, as well as the assistance and backing of Eduardito, and of course, my Agus with her exceptional joy and sweetness, were my strength in the toughest moments and the inspiration to persevere.

My lovely friends, always ready with open arms and attentive ears, ensured that joy and hope never faded completely. Alex, Diego, Memo, Marcela, Klaus, little Janeth, my dear "Sombrilleros": Georgi, Luisa, Manu, Isa, María, Pao, Sari; besides Jessed, my girls from UdeA, and my treasured Professor Carlos Palacio. To each of you, my heartfelt gratitude for your genuine affection and unwavering support.

The acknowledgments may be long, just as the journey has been. A journey that I didn't traverse alone. It wouldn't have been possible.

Agradecimientos

Indudablemente, el doctorado ha representado uno de los desafíos más exigentes que he enfrentado en mi trayectoria académica y personal. La posibilidad de escribir estas palabras era un momento que muchas veces parecía inalcanzable. Este trabajo ha requerido un gran esfuerzo en términos profesionales, emocionales e incluso de salud. Sin embargo, finalmente llegó el anhelado momento y su realización no habría sido posible sin el apoyo, afecto y guía de todos los que estuvieron a mi lado, brindándome su mano y el ánimo necesario para seguir adelante.

Quiero comenzar expresando mi agradecimiento a mis directores y asesores, los profesores Carlos A. González-Calderón, John Jairo Posada Henao y Héctor López Ospina, por sus valiosas contribuciones, referencias y consejos oportunos en el desarrollo de esta investigación. En particular, deseo destacar al profesor González-Calderón, quien siempre creyó en mí y más allá de su papel como profesor, fue un compañero de investigación y de camino. No solo compartió su conocimiento, sino también su comprensión y confianza.

Asimismo, deseo reconocer y agradecer a mis compañeros de doctorado por su amistad y compañía, con especial mención a Claudia Muñoz, quien ha sido una amiga que iluminó este proceso con sus sugerencias y afecto fundamentales. También debo expresar mi gratitud a los compañeros del semillero de investigación en Transporte de Carga y Logística de la Universidad Nacional de Colombia, quienes contribuyeron a importantes logros en mi formación doctoral, especialmente a Jhan Kevin Gil, un valioso compañero de trabajo. Así mismo, quiero agradecer, de manera general, a la Universidad Nacional de Colombia Sede Medellín, por haberse convertido en mi casa durante todo este proceso, y acogerme mientras me permitía caminar hacia mi objetivo.

Es de suma importancia expresar mi agradecimiento a mi alma mater, la Universidad de Antioquia, a la Facultad de Ingeniería y la Escuela Ambiental por brindarme la oportunidad de llevar a cabo este monumental proyecto y cumplir el objetivo de concluir mis estudios doctorales en condiciones dignas, sabiendo que mi universidad me espera nuevamente para poner al servicio todo el conocimiento adquirido. Quiero hacer una mención especial al señor Rector John Jairo Arboleda por su apoyo en momentos cruciales y por creer en mí.

No puedo dejar de agradecer a la Volvo Research and Educational Foundation (VREF) por el respaldo que me brindaron, permitiéndome ampliar mis redes y conocimientos académicos.

Agradezco especialmente al Profesor José Holguín-Veras, cuya sabiduría siempre brinda algo nuevo y cuya calidez está siempre disponible.

Asimismo, agradezco a Caliper Co y a Andrés Rabinowicz por permitirnos el uso de sus herramientas de simulación académica.

Deseo ahora dirigir mi gratitud a quienes han sido los verdaderos motores de este proceso: mi familia y mis más queridos amigos. El amor de mi abuela y mi madre, la confianza y apoyo de mi tío Rigo, la complicidad y compañía incondicional de mi hermana, así como la ayuda y respaldo de Eduardito y, por supuesto, mi Agus con su alegría y dulzura excepcionales, fueron mi fuerza en los momentos más difíciles y la inspiración para seguir adelante.

Mis entrañables amigos, siempre dispuestos con brazos abiertos y oídos atentos, aseguraron que la alegría y la esperanza nunca se desvanecieran por completo. Alex, Diego, Memo, Marcela, Klaus, Janeth, mis amados Sombrilleros: Georgi, Luisa, Manu, Isa, María, Pao, Sari; además Jessed, mis niñas de la UdeA y mi querido profesor Carlos Palacio. A cada uno de ustedes, mi sincero agradecimiento por su afecto genuino y apoyo incondicional.

Largos los agradecimientos, así como largo ha sido este camino, que no atravesé sola, no habría sido posible.

Abstract

This research introduces a multi-class demand synthesis model for transit and freight, utilizing entropy maximization and fuzzy logic. The model incorporates traffic data and fuzzy parameters to accommodate uncertainty. The use of fuzzy logic enhances classical modeling by providing flexibility and addressing data uncertainty, a critical aspect in resource-constrained decision-making scenarios.

Finite resources such as road capacity necessitate optimal decision-making. Flexible models are essential, as not all constraints can be fully met. Fuzzy logic excels in handling variability and uncertainty, improving results' reliability. It aids in estimating congestion patterns, emissions levels, and accidents, thereby providing valuable insights to decision-makers.

Fuzzy logic's flexibility is crucial for real-world adaptability. It enhances transportation planning, benefiting urban mobility. Results' accuracy directly impacts decisions, and fuzzy logic incorporates real-world variability into models.

The research focuses on triangular membership functions, a commonly used approach. Fuzzy logic's adaptability is compared with deterministic models, demonstrating superior performance. It helps in finding satisfactory solutions when full constraint satisfaction is unfeasible.

Pareto frontiers indicate multi-objective optimization. Decision-makers can use this frontier to choose the right model based on accomplishment versus entropy trade-offs. Fuzzy logic accommodates partial solutions when strict constraints cannot be met.

Trials with a developed model show that capacity and cost significantly influence outcomes. Sensitivity analyses reveal the model's robustness. The model's application is promising for shared lanes and infrastructure optimization, handling data variability and uncertainty. It aids in decision-making for urban transportation planning and infrastructure development.

Government agencies must strategize mobility elements. Accurate data are crucial for decisions related to routes, traffic management, and infrastructure. Fuzzy logic can guide decisions about shared lanes and resource allocation, enhancing urban transportation planning and development.

Keywords: *Entropy*, *Freight Transportation*, *Freight Tour Synthesis, Transit Tour Synthesis*, *Freight and Transit Tour Synthesis*, *Fuzzy Logic, Sioux Falls Network.*

Resumen

Esta investigación presenta un modelo de síntesis de demanda multiclase para tránsito y carga, utilizando maximización de entropía y lógica difusa. El modelo incorpora datos de tráfico y parámetros difusos para adaptarse a la incertidumbre. El uso de la lógica difusa mejora el modelado clásico al proporcionar flexibilidad y abordar la incertidumbre de los datos, un aspecto crítico en escenarios de toma de decisiones con recursos limitados.

Los recursos finitos, como la capacidad de las vías, requieren una toma de decisiones óptima. Los modelos flexibles son esenciales, ya que no todas las restricciones pueden cumplirse por completo. La lógica difusa se destaca en el manejo de la variabilidad y la incertidumbre, mejorando la confiabilidad de los resultados. Ayuda a estimar los patrones de congestión, los niveles de emisiones y los accidentes, proporcionando así información valiosa a los responsables de la toma de decisiones.

La flexibilidad de la lógica difusa es crucial para la adaptabilidad al mundo real. Mejora la planificación del transporte, beneficiando la movilidad urbana. La precisión de los resultados impacta directamente en las decisiones, y la lógica difusa incorpora la variabilidad del mundo real en los modelos.

La investigación se centra en las funciones de pertenencia triangulares, un enfoque de uso común. La adaptabilidad de la lógica difusa se compara con modelos deterministas, lo que demuestra un rendimiento superior. Ayuda a encontrar soluciones satisfactorias cuando la satisfacción total de la restricción es inviable.

Las fronteras de Pareto indican optimización multiobjetivo. Los tomadores de decisiones pueden usar esta frontera para elegir el modelo correcto en función de las compensaciones entre logros y entropía. La lógica difusa acomoda soluciones parciales cuando no se pueden cumplir restricciones estrictas.

Los ensayos con el modelo desarrollado muestran que la capacidad y el costo influyen significativamente en los resultados. Los análisis de sensibilidad revelan la solidez del modelo. La aplicación del modelo es una alternativa prometedora en el uso de infraestructura compartida (carriles y bahías) y la optimización de la misma, al incluir la variabilidad e incertidumbre de los datos, pudiendo ser de ayuda en la toma de decisiones para la planificación del transporte urbano y el desarrollo de infraestructura.

Las agencias gubernamentales deben diseñar estrategias para los elementos de movilidad. Los datos precisos son cruciales para las decisiones relacionadas con las rutas, la gestión del tráfico y la infraestructura. La lógica difusa puede guiar las decisiones sobre carriles compartidos y asignación de recursos, mejorando la planificación y el desarrollo del transporte urbano.

Palabras claves: *Entropía, Transporte de carga, Síntesis de toures de carga, Síntesis de toures de buses, Síntesis de toures de carga y buses, Lógica difusa, Red de Sioux Falls.*

TITULO EN ESPAÑOL: Síntesis de toures de carga y de buses de transporte público

Contents

1. Introduction ... 1
1.1. The aim and objectives of the research ... 7
1.2. Structure of the thesis ... 7
1.3. References ... 9

2. Freight Tour Synthesis based on Entropy Maximization with Fuzzy Logic constraints .. 13
2.1. Introduction ... 14
2.2. Background ... 16
2.2.1. Freight demand synthesis ... 16
2.2.2. Freight OD synthesis ... 18
2.2.3. Freight tour synthesis - FTS ... 19
2.2.4. Fuzzy Logic (FL) and transportation modeling ... 19
2.3. Modeling formulation ... 21
2.3.1. Entropy-based FTS deterministic formulation ... 21
2.3.2. Entropy-based FTS formulation with fuzzy parameters ... 23
2.4. Numerical Experiment ... 30
2.4.1. Case study ... 30
2.4.2. Modeling approach ... 31
2.4.3. Results and analysis ... 32
2.5. Concluding Remarks and Planning Implications ... 44
2.6. References ... 47

3. Entropy-based Transit Tour Synthesis using Fuzzy Logic ... 53
3.1. Introduction ... 54
3.2. Background ... 56
3.2.1. Transit demand synthesis ... 56
3.2.2. EM in transportation modeling ... 57
3.2.3. Fuzzy logic and membership functions in transportation modeling ... 59
3.3. Estimation of transit tours based on entropy maximization and fuzzy logic ... 61
3.3.1. Transit tour synthesis formulation (TTS) using EM ... 62
3.3.2. TTS formulation using the fuzzy logic formulation ... 67
3.4. Numerical experiments: TTS using EM-FL ... 71
3.5. Conclusions and discussion ... 84
3.6. References ... 86

4. Freight-Transit Tour Synthesis Entropy-Based Formulation: Sharing Infrastructure for Buses and Trucks ... 91
4.1. Introduction ... 92
4.2. Literature review ... 93
4.2.1. Freight Tours Transportation ... 93
4.2.2. Transit tours transportation ... 94
4.2.3. Exclusive or dedicated lanes (Bus and trucks) ... 94
4.2.4. Integration between freight and transit ... 96
4.2.5. Entropy maximization (EM) in tour transportation modeling ... 97
4.2.5.1. FTS - Deterministic and Flexible Modeling ... 97
4.2.5.2. TTS - Deterministic and Flexible Modeling ... 98
4.3. Freight transit tour synthesis (FTTS) modelling: Deterministic and Flexible models ... 98
4.3.1. Link performance function estimation ... 99
4.3.2. FTTS Entropy Maximization Formulation ... 101

4.3.3. FTTS Entropy Maximization Formulation using fuzzy parameters 106
4.4. Numerical experiments of FTTS (GAMS) using FL 109
4.4.1. Network.. 110
4.4.2. Integrated link performance function.. 111
4.4.3. Analysis and Results 112
4.4.3.1. Scenario 1: Analysis without capacity and cost constraints................................ 112
4.4.3.2. Scenario 2: Analysis with capacity constraint 117
4.4.3.3. Scenario 3: Analysis with cost constraint 118
4.5. Concluding Remarks.. 119
4.6. References.. 120

5. Conclusions and recommendations 125
5.1. Conclusions.. 125
5.2. Recommendations.. 129

List of figures

Figure 2-1 Membership function example (triangular) 25
Figure 2-2 Sioux Falls network (Gonzalez-Calderon & Holguín-Veras, 2019) 31
Figure 2-3. Pareto frontier. Entropy vs. accomplishment level 34
Figure 2-4. Traffic change for each solution 35
Figure 2-5. Truck tour volumes for different ε values 36
Figure 2-6. Accomplishment of the node-production 37
Figure 2-7. Percentage of the accomplishment node-production constraint 38
Figure 2-8. Percentage of the accomplishment link-volume constraint 39
Figure 2-9. Histogram of % of accomplishment in comparison FL and No FL cases 43
Figure 3-1. Sioux Falls Network. Source: (Wei et al., n.d.) 72
Figure 3-2. Transit Pareto frontier (Entropy vs. Accomplishment level). 73
Figure 3-3. Bus traffic changes for each different solution. 74
Figure 3-4. Bus tour volumes for different ε values. 75
Figure 3-5. TTS - Level (%) of accomplishment in nodes production. 76
Figure 3-6. TTS - Percentage accomplishment in node-production constraint. 77
Figure 3-7. TTS - Percentage accomplishment in link-volume constraint. 77
Figure 4-1. Central transitway diagram (BRT) 95
Figure 4-2. Curb side bus lane diagram 96
Figure 4-3. Sioux Falls Network 110
Figure 4-4. Pareto frontier Entropy vs. Epsilon. 113
Figure 4-5. Change percentage of the flow by tour – Buses and trucks without capacity constraint 113
Figure 4-6. Volume by link - buses and trucks for different solutions 114
Figure 4-7. Volume by link - buses and trucks for different solutions 115
Figure 4-8. Volume by tour - buses and trucks for different solutions 116
Figure 4-9. Variation of the Cost for every epsilon value 117

List of tables

Table 2-1. Results of testing statistically differences between all feasible solutions. 33
Table 2-2. Descriptive analysis of the accomplishment node-production constraint....................... 38
Table 2-3. Descriptive analysis of the accomplishment link-volume constraint 39
Table 2-4. Percentage of accomplishment FL model and No FL models – Nodes (production)..... 41
Table 2-5. Percentage of accomplishment FL model and No FL models – Links (Volumes)......... 42
Table 2-6. Percentage of accomplishment - Descriptives 44
Table 2-7. Quantity of solutions inside and outside the range 44
Table 3-1. TTS–Descriptive analysis of accomplishment. 78
Table 3-2. Percentage Distance from the target value (Δ_{Va}) of the flow per link 82
Table 3-3. Percentage Distance (Δ_{Va}) Descriptive - Links (volume) 83
Table 3-4. Percentage Distance from the target value (Δ_{Oi}) of the production per node 83
Table 3-5. Percentage Distance (ΔOi) Descriptive - Nodes (production)....................................... 84
Table 4-1. Impact of the capacity changes on the solutions 118
Table 4-2. Impact of the cost changes on the solutions 119

1. Introduction

The freight system is both extensive and intricate, necessitating consideration of all available freight modes along with their associated infrastructure and operations. Freight flows stand as tangible expressions of the manufacturing and consumer economies that underpin modern life (Holguin-Veras et al., 2015). Nonetheless, freight activity generates external impacts, some of which are negative, owing to the fact that freight-vehicle traffic engenders congestion, pollution, noise, and infrastructure degradation. Public policy endeavors to optimize the net social advantages stemming from freight activity, aiming to maximize the benefits of reliable freight flows while minimizing the adverse externalities associated with freight-vehicle traffic.

The movement of goods constitutes a daily societal requirement, thereby giving rise to freight trips, whose core purpose is to efficiently transport goods using vehicles, primarily trucks. These movements culminate in what is termed freight trip generation (FTG), encompassing both inbound freight trips arriving at establishments for delivering goods (referred to as freight trip attraction—FTA) and the outbound trips departing from establishments (referred to as freight trip production—FTP) (Bastida & Holguin-Veras, 2009; Board et al., 2016; Campbell et al., 2018). These freight trips are specialized according to the type and size of the goods they carry, often utilizing trucks for their efficient conveyance. These commercial vehicles are integral to cargo transport, requiring specific facilities for transit and parking, while adhering to special policies and regulations. Their impact encompasses both positive and negative externalities, necessitating careful consideration. Furthermore, trucks often necessitate multiple stops, as they undertake various deliveries in a single journey termed a "tour." Thus, an in-depth study of truck movements as tours, commencing from a home base and encompassing sequential stopping points along a designated route, becomes crucial, particularly in urban locales.

A prevailing technique employed for estimating FTG distribution within urban networks involves the origin–destination (OD) synthesis through the entropy maximization (EM) approach. This procedure yields the most plausible OD matrix based on available system information, which may include data regarding trips' origins or destinations. This method stands as an efficient and cost-effective means of deriving freight flows, leveraging traffic counts for estimations.

In the domain of freight transportation, the focus on trip chains or tours takes precedence over merely considering origins and destinations. A freight tour delineates the sequence that freight vehicles must adhere to in fulfilling assigned deliveries or pick-ups. Such a sequence involves nodes linked by arcs, representing delivery or pick-up locations, a distinctive attribute of urban freight transport. This process of obtaining tour flows within freight tours is recognized as freight tour synthesis (FTS) (Gonzalez-Calderon & Holguín-Veras, 2019; Holguin-Veras et al., 2020; Sanchez-Diaz et al., 2015). FTS emerges as a valuable tool for representing the movement of goods within society, considering diverse factors such as human behavior, geographical location, and specific requirements that shape the modeling process's methodologies, inputs, and outputs.

Freight Origin-Destination (OD) Synthesis offers an economical approach for obtaining OD matrices, recognizing the substantial expenses and labor associated with data collection, such as freight surveys (Holguín-Veras & Jaller, 2014). Despite this, if the objective of freight planning entails understanding freight movements and their patterns within urban settings (Holguin-Veras & Patil, 2005), freight tour synthesis (FTS) provides a wealth of information. FTS can be realized through methodologies like the Entropy Maximization (EM) technique, which involves specifying freight tours undertaken by trucks based on secondary data.

A pivotal methodology employed in formulating Freight Tour Synthesis (FTS) models revolves around entropy maximization. This technique targets the derivation of the most plausible distribution of tour flows for freight (trucks) within a network, reliant on traffic counts. However, in actuality, elements like costs, traffic counts, and truck demands exhibit diverse variations, uncertainties, and potential ambiguities arising from human behavior, complexities that deterministic models inadequately capture. Introducing Fuzzy Logic (FL) enables the incorporation of such dynamic variabilities into the modeling process. The inherent adaptability of FL equips the model to yield solutions where some or all constraints may not be fully satisfied yet remain closely aligned with their expected values. This process involves approximating an optimal value within an acceptable range through the application of membership functions.

Incorporating variability brings FL solutions into closer alignment with reality, considering that street truck counts fluctuate daily due to a variety of reasons, as mentioned earlier. These reasons encompass congestion (which can influence logistical decisions), daily freight demand based on zones (where truck flow varies with the type of goods transported), and external factors like weather and the day of the week (with truck counts differing on weekends and workdays). Consequently, solutions achieved using FL are dynamic and offer improved approximations compared to conventional static solutions. While decision-makers recognize the dynamic nature of transportation, data collection in this context remains challenging. As such, alternatives to static solutions are embraced, where information is gathered during peak hours to encompass various scenarios. Nonetheless, even within this framework, variations between consecutive days are still evident.

This research analyzes the impact of parameter variabilities on freight transport system planning through the application of FL within the EM procedure. This is achieved by implementing membership functions to model constraints. The novelty of this methodology lies in developing a more flexible FTS model formulation using FL, allowing for the identification of solutions that achieve a certain level of constraint fulfillment, even when complete satisfaction is unattainable. This approach enhances the entropy-based FTS modeling introduced by Gonzalez-Calderon and Holguin-Veras (2019), which also integrated fuzzy logic parameters to account for uncertainty in the FTS model based on EM. To the best of the authors' knowledge, the application of FL theory to estimate tour flows in the FTS process represents a novel contribution.

On the other hand, various other modes of transportation serve the purpose of moving people through the city's streets. Among these, collective public transportation, mainly buses, plays a significant role.

The urban transit network within an urban area is typically determined by planning decision-makers, and its allocation is contingent on factors like passenger demand and street capacity. Transit networks possess distinctive features, such as nodes, each with a different nature. Origin nodes signify trip starting points, stop nodes represent stations, and destination nodes mark trip endpoints (Kurauchi et al., 2003). These nodes are interconnected by links defined by performance functions, which express associated tour costs (e.g., travel time contingent on traffic flow). Urban transit tours encompass the utilization of urban infrastructure, including lanes, curb space, and bays.

The significance of public transportation cannot be understated, as it profoundly impacts people's daily lives. When the public transportation system operates efficiently, the entire city functions smoothly, contributing to an improved quality of life. For instance, it can enhance job access and expedite travel (Nagy et al., 2019). The transit system is indispensable for moving people within urban areas and encompasses a broad spectrum of modes, ranging from various bus services to trams, light rail transit (LRT), commuter rail, and metropolitan rail (metro) systems (Wirasinghe et al., 2013). According to these authors, the planning process for urban public buses, as well as transit in general, aims to provide a satisfactory level of service while maintaining a balanced, cost-effective approach for both operators and passengers.

In the pursuit of alleviating congestion, numerous strategies have been proposed and implemented, such as expanding road capacity through construction or optimizing traffic control. These strategies aim to optimize, either entirely or partially, the overall vehicle flow within the system. However, implementing such measures may lead to unintended consequences as flow increases, posing challenges despite the intended system improvement. When traffic reaches capacity, the issue resurfaces, signifying the need for alternative strategies. Public transportation emerges as a potential solution, or at least a part of it, capable of reshaping demand behavior and contributing to congestion management. Therefore, the prioritization of the assignment problem in the metropolitan public transportation system becomes essential. Addressing this requires the development of effective solutions for an urban bus transit system that aligns with physical, social, and economic structures (Fan & Machemehl, 2006; Soehodo & Koshi, 1999).

Transit networks are characterized by specific points or nodes with distinct roles: origin nodes signify trip start points, stop nodes represent stations, and destination nodes indicate trip endpoints. Acknowledging the importance of maximizing street utilization, it becomes evident that the two interconnected systems—transit and road networks—significantly influence the overall transportation system. In this regard, the entropy maximization technique proves valuable and applicable in transportation modeling. The entropy maximization (EM) approach has found wide application as an optimization technique in both passenger and freight analysis, encompassing OD synthesis and tour synthesis distribution modeling. However, despite extensive literature on estimating OD matrices from traffic counts in road networks, limited attention has been directed toward the transit passenger OD estimation problem (Lam et al., 2003). This observation underscores the scarcity of research in the realm of transit OD estimation, a gap that requires attention and investigation.

The pursuit of effective models also raises concerns regarding the models' accuracy in representing the real world. While perfection may remain elusive, narrowing the gap holds promise. Examining the impact of parameter variability on modeling and transport planning can be accomplished through various methodologies. Fuzzy Logic (FL), for example, has been explored within the realm of transportation, as seen in the estimation of OD synthesis distribution matrices for passenger cars (López-Ospina et al., 2021), employing EM and fuzzy parameters within constraints. This incorporation of uncertainty stemming from data variability enables flexibility in the constraints, acknowledging that not all constraints are universally satisfied, let alone achieving

100% fulfillment. Deterministic problems do not have the capacity to incorporate variability of data such are costs, traffic counts and volumes, in those cases the solution could be represented in the use of stochastic problems. However, those latest problems could imply more complexity in the resolution. Then, FL is a tool which can work out the problems being more realistic than deterministic but being less complex than stochastic ones.

To the best of the authors' knowledge, an analysis of bus tour flows (transit tour flows) through the lens of EM remains an unexplored territory. Despite EM's utilization in obtaining OD matrices for transit (buses) systems, there is an opportunity to employ it for identifying the most probable bus tour flows within an urban network using traffic counts. Furthermore, this dissertation introduces, for the first time, both the deterministic entropy-based Transit Tour Synthesis (TTS) formulation and the flexible entropy-based TTS incorporating Fuzzy Logic (FL) as tools to estimate tour flows within this system.

Freight Tour Synthesis (FTS) and Transit Tour Synthesis (TTS) represent crucial methodologies in the realm of transportation planning. These techniques offer cost-effective means to estimate tour flows in urban areas. Conventional data collection methods like surveys often prove expensive and time-intensive. As a response, techniques such as OD synthesis have emerged, employing approaches like Entropy Maximization (EM), a widely adopted modeling methodology. The entropy could be understood as the representation of the number of ways that freight vehicle tours or transit (buses) tours could be arranged. Then, EM strives to deduce the most probable OD matrix or tours distribution based on observed link flows. This is due to Its application extends beyond passenger car demand, encompassing freight demand estimation, providing reliable outcomes even amidst the intricacies of congestion.

Likewise, Transit Tour Synthesis (TTS) applies entropy-based modeling to predict the distribution of bus tour flows. The inclusion of fuzzy logic into the TTS model facilitates the integration of uncertainties arising from data variations and human behavior. This integration enhances the model's precision and adaptability. This approach addresses challenges in public transportation planning by embracing frequency-based and schedule-based strategies. TTS delves into the impacts of diverse parameters, such as capacity and maximum cost constraints, on solutions derived through fuzzy parameters. By considering various scenarios involving subjective values of time (SVT), TTS underscores its resilience in optimizing traffic management and infrastructure utilization.

In certain instances, the same infrastructure employed by freight trucks to fulfill their objective of urban distribution in the final mile is shared. Given this context, it becomes pertinent to scrutinize the conduct of both freight tours and transit tours when utilizing common infrastructure. Furthermore, investigating whether such infrastructure sharing could yield benefits for the systems under different conditions holds significance.

The proposition of the Freight Transit Tour Synthesis (FTTS) model, showcased in this work, introduces a novel approach to the estimation of tour flows, utilizing entropy maximization (EM) and fuzzy logic (FL). It draws inspiration from a model for transit tour flow estimation based on EM and FL (Moreno-Palacio et al., 2022), as well as the truck tour flow estimation presented in Gonzalez-Calderon (2014), incorporating fuzzy parameters in recent research (Moreno-Palacio et al., 2023). The Freight Transit Tour Synthesis (FTTS) model, incorporating FL, strives to analyze both systems as multiclass categories, exploring the hypothetical situation in which they share designated infrastructure like dedicated lanes. As far as the author's knowledge extends, no previous development of this model appears to exist, making this presentation the first of its kind.

It is important to note that this paper's formulations pertain specifically to the freight (truck) and transit (bus) systems, excluding other modes of transport. This selection stems from the compatibility of both systems with tour-based modeling, rendering them suitable for making comparisons regarding flows, routes, stop points, and more.

In this dissertation, the proposed Freight Transit Tour Synthesis (FTTS) model amalgamates elements from both freight and transit systems. The integration of fuzzy logic and entropy maximization enhances the accuracy and adaptability of these models, leading to better traffic management and infrastructure optimization.

The research contributes to the optimization of tour flows for each system individually and within the context of shared infrastructure, aiding urban transportation planning and sustainable resource allocation. Through robust experimentation and analysis, the study provides a valuable insight into the dynamics of freight and transit operations, offering potential solutions to enhance the efficiency and sustainability of urban transportation networks.

The FTTS model meticulously explores scenarios wherein buses and trucks share infrastructure, presenting a comprehensive approach to optimizing transportation systems. Ultimately, the novel approach of FTTS further extends these benefits by considering shared infrastructure and offering a comprehensive solution for urban transportation planning.

1.1. The aim and objectives of the research

This research aim is to estimate a multi-class demand synthesis model for both transit and freight by applying analytical techniques considering trip chain behavior using secondary data. The research aim will be achieved through the following objectives:

Specific Objectives	Papers	Contributions
To propose shared lanes —and its respective bays— for both transit and freight considering sensitivity analysis based on traffic flows.	Paper 1, 2: Published Paper 3: Under review	• Dissertation Hypothesis. • FTS model including fuzzy parameters. • TTS model deterministic. • TTS using fuzzy parameters. • FTTS model using fuzzy parameters.
To analyze capacity and level of service of the infrastructure in the network when multi-class demand model for freight and transit is applied.	Paper 3: Under review	The FTTS model using FL
To examine the model's feasibility considering congestion throughout a case study.	Paper 1, 2: Published Paper 3: Under review	• FTS model including fuzzy parameters. • TTS model deterministic. • TTS using fuzzy parameters. • FTTS model using fuzzy parameters.

The above objectives were developed using simulated tours as data input in testing modelling.

1.2. Structure of the thesis

This thesis is composed of five chapters. From this dissertation three papers were produced. The document structure is as follows: Chapter 1 corresponds to a general introduction of the dissertation. Chapter 2 and 3 correspond to two papers already published in freight tour synthesis and transit tour synthesis, respectively. Chapter 4 contains a submitted paper for evaluation (integration of freight-transit tour synthesis). The document finalizes with Chapter 5 corresponding to general conclusions of the research. Thus, the main body of this research is contained in three chapters (2, 3 and 4). Introduction, literature review, conclusion, and references are self-contained in each chapter.

Chapters 2 is focused in presenting the FTS using FL formulation. Chapter 3 presents the TTS deterministic model and TTS using FL (flexible model). Subsequently, based on chapters 2 and 3, the FTTS model is presented in chapter 4. The performance of the proposed formulations was assessed in the Sioux Falls network using Caliper software TransModeler. To solve the problem, the models were run in General Algebraic Modeling System (GAMS), applying the ε approach. This network was used in the numerical experiments of the three chapters. Different research contributions were found in the three chapters as follows:

Chapter 2 delves into the realm of Freight Tour Synthesis (FTS), a methodology aimed at capturing the complexities of urban freight movements. Freight systems are a critical component of urban life, facilitating the movement of goods across various sectors. The demand for accurate freight tour flow estimation necessitates innovative approaches that can contend with the uncertainty and variability inherent in freight systems. To address these challenges, this thesis explores the integration of FTS with fuzzy logic (FL). Fuzzy logic's inherent adaptability makes it a suitable candidate for modeling the uncertainty and variations in factors such as costs, traffic counts, and truck demands. By incorporating FL into the FTS model, this chapter seeks to provide a robust framework that can enhance the accuracy and reliability of tour flow estimations in freight systems.

The main contribution of this chapter is to include the impact of parameter variabilities on freight transport system planning using FL by implementing the FL theory in the EM procedure. To do so, membership functions are applied to model constraints. The novelty in the methodology is represented in developing a more flexible FTS model formulation using FL, which allows finding some solutions that achieve a certain level of accomplishment for the constraints when it is not possible to achieve complete satisfaction.

Chapter 3 shifts its focus to the realm of Transit Tour Synthesis (TTS). Public transportation, particularly buses, plays a vital role in alleviating congestion and promoting sustainable urban mobility. However, the optimization of transit systems presents unique challenges, ranging from frequency-based to schedule-based approaches. To navigate these complexities, this chapter introduces the concept of using fuzzy logic in TTS. Fuzzy logic's capability to accommodate uncertainties and variabilities inherent in transit systems positions it as a powerful tool for modeling accurate tour flow estimations. By integrating fuzzy logic with TTS, this chapter aims to enhance the adaptability and precision of transit tour flow predictions, contributing to more efficient and effective public transportation planning.

The models allow to obtain the most probable transit (bus) tour flow distribution in the network based on traffic counts. Since the FL permits to include in modeling data variability, obtaining solutions where some or all the constraints do not entirely satisfy their expected value but are close to it due to the flexibility this method provides to the model. This optimization problem was transformed into a bi-objective problem when the optimization variables were the membership and entropy. The numerical experiment proves that the inclusion of the FL and EM approaches to estimate bus tour flow, applying the synthesis method (traffic counts), improves the quality of the tour estimation.

Chapter 4 of this thesis marks the convergence of freight and transit systems within the framework of Freight-Transit Tour Synthesis (FTTS). As urban areas witness the dynamic interplay between freight and transit movements, optimizing shared infrastructure becomes a pressing concern. This chapter introduces the concept of FTTS, a groundbreaking methodology that explores scenarios where freight and transit systems coexist and share resources. By harnessing the power of fuzzy logic, FTTS offers a comprehensive solution for tour flow estimation that transcends the boundaries of individual systems. This chapter delves into the nuances of integrating fuzzy logic with FTTS, shedding light on how this holistic approach can redefine urban transportation planning. Through FTTS, decision-makers gain insights into resource allocation, traffic management, and infrastructure efficiency, paving the way for more sustainable and interconnected urban transportation systems.

The proposed Freight Transit Tour Synthesis (FTTS) model, using fuzzy logic and entropy maximization, analyzes freight and transit systems as a multiclass category, exploring scenarios where buses and trucks share infrastructure. The experiments demonstrate that capacity and maximum cost significantly influence the solutions obtained using fuzzy parameters, with ε-values indicating the best solution. The model's robustness is evident across various subjective value of time (SVT) scenarios. The application of the FTTS model offers a novel approach to estimate tour flows, incorporating traffic counts and fuzzy parameters for immediate, relevant results. The model's multiclass formulation accurately represents real-world traffic conditions, considering congestion in traffic assignments.

Finally, the key findings of this research and recommendations for further research are given in Chapter 5.

1.3. References

Bastida, C., & Holguin-Veras, J. (2009). Freight Generation Models. Comparative analysis of regression models and multiple classification analysis. *Transportation Research Record:*

Journal of the Transportation Research Board, *2097*(1), 51–61. https://doi.org/10.3141/2097-07

Board, T. R., Engineering, of S., & Medicine. (2016). *Using Commodity Flow Survey Microdata and Other Establishment Data to Estimate the Generation of Freight, Freight Trips, and Service Trips: Guidebook* (J. Holguín-Veras, C. Lawson, C. Wang, M. Jaller, C. González-Calderón, S. Campbell, L. Kalahashti, J. Wojtowicz, & D. Ramirez, Eds.). The National Academies Press. https://doi.org/10.17226/24602

Campbell, S., Holguin-Veras, J., Ramirez-Rios, D. G., Gonzalez-Calderon, C. A., Kalahasthi, L., & Wojtowicz, J. (2018). Freight and service parking needs and the role of demand management. *European Transport Research Review*, *10*(2), 1–13. https://doi.org/http://dx.doi.org/10.1186/s12544-018-0309-5

Fan, W., & Machemehl, R. B. (2006). Optimal Transit Route Network Design Problem with Variable Transit Demand : Genetic Algorithm Approach. *Journal of Transportation Engineering*, *132*(January), 40–51. https://doi.org/10.1061/(ASCE)0733-947X(2006)132:1(40)

Gonzalez-Calderon, C. A. (2014). *Multiclass Equilibrium Demand Synthesis*. PhD dissertation. Doctor of Philosophy, Rensselaer Polytechnic Institute.

Gonzalez-Calderon, C. A., & Holguín-Veras, J. (2019). Entropy-based freight tour synthesis and the role of traffic count sampling. *Transportation Research Part E: Logistics and Transportation Review*, *121*(November 2017), 63–83. https://doi.org/10.1016/j.tre.2017.10.010

Holguin-Veras, J., Amaya-Leal, J., Wojtowicz, J., Jaller, M., Gonzalez-Calderon, C. A., Sanchez-Diaz, I., Wang, X., Haake, D. G., Rhodes, S. S. ;, Darville Hodge, S., Frazier, R. J., Nick, M. K., Dack, J., & Casinelli, L. (2015). *Improving freight system performance in metropolitan areas: a planning guide*. https://doi.org/10.17226/22159

Holguin-Veras, J., Encarnacion, T., Ramirez-Rios, D., He, X. (Sean), Kalahasthi, L., Perez-Guzman, S., Sanchez-Diaz, I., & Gonzalez-Calderon, C. A. (2020). A Multiclass Tour Flow Model and lts Role in Multiclass Freight Tour Synthesis. *Transportation Science*. https://doi.org/10.1287/trsc.2019.0936

Holguín-Veras, J., & Jaller, M. (2014). Comprehensive freight demand data collection framework for large urban areas. In *Sustainable Urban Logistics: Concepts, Methods and Information Systems* (pp. 91–112). Springer.

Holguin-Veras, J., & Patil, G. R. (2005). Observed trip chain behavior of commercial vehicles. *Transportation Research Record*, *1906*(1), 74–80.

Kurauchi, F., Bell, M. G. H., & Schmöcker, J.-D. (2003). Capacity Constrained Transit Assignment with Common Lines. *Journal of Mathematical Modelling and Algorithms*, *2*(4), 309–327. https://doi.org/10.1023/B:JMMA.0000020426.22501.c1

Lam, W. H. K., Wu, Z. X., & Chan, K. S. (2003). Estimation of Transit Origin-Destination Matrices from Passenger Counts Using a Frequency-Based Approach. *Journal of Mathematical Modelling and Algorithms*, *2*(4), 329–348. https://doi.org/10.1023/B:JMMA.0000020423.93104.14

López-Ospina, H., Cortés, C. E., Pérez, J., Peña, R., Figueroa-García, J. C., & Urrutia-Mosquera, J. (2021). A maximum entropy optimization model for origin-destination trip matrix estimation with fuzzy entropic parameters. *Transportmetrica A: Transport Science*, 1–38. https://doi.org/10.1080/23249935.2021.1913257

Moreno-Palacio, D. P., Gonzalez-Calderon, C. A., López-Ospina, H., Gil-Marin, J. K., & Posada-Henao, J. J. (2023). Freight tour synthesis based on entropy maximization with fuzzy logic constraints. *Transportation*. https://doi.org/10.1007/s11116-023-10407-y

Moreno-Palacio, D. P., Gonzalez-Calderon, C. A., Posada-Henao, J. J., Lopez-Ospina, H., & Gil-Marin, J. K. (2022). Entropy-Based Transit Tour Synthesis Using Fuzzy Logic. In *Sustainability* (Vol. 14, Issue 21). https://doi.org/10.3390/su142114564

Nagy, A., Ercsey, Z., Tick, J., & Kovács, Z. (2019). Bus transport process network synthesis. *Acta Polytechnica Hungarica*, *16*(7), 25–43. https://doi.org/10.12700/APH.16.7.2019.7.2

Sanchez-Diaz, I., Holguin-Veras, J., & Ban, X. (Jeff). (2015). A time-dependent freight tour synthesis model. *Transportation Research Part B: Methodological*, *78*, 144–168. https://doi.org/10.1016/j.trb.2015.04.007

Soehodo, S., & Koshi, M. (1999). Design of public transit network in urban area with elastic demand. *Journal of Advanced Transportation*, *33*(3), 335–369. https://doi.org/10.1002/atr.5670330306

Wirasinghe, S. C., Kattan, L., Rahman, M. M., Hubbell, J., Thilakaratne, R., & Anowar, & S. (2013). Bus rapid transit-a review. *International Journal of Urban Sciences*, *17*(1), 1–31. https://doi.org/10.1080/12265934.2013.777514

2. Freight Tour Synthesis based on Entropy Maximization with Fuzzy Logic constraints.

https://doi.org/10.1007/s11116-023-10407-y

Abstract

This paper presents an improved entropy-based freight tour synthesis (FTS) using fuzzy logic (FL). One approach used in formulating FTS models is entropy maximization, which aims to obtain the most probable freight (trucks) tour flow distribution in a network based on traffic counts. These models consider fixed parameters and constraints. However, the variations in costs, traffic counts, and truck demands depending on human behavior, are not always captured in detail in such models. FL can include such variabilities in its modeling. The flexibility FL provides to the model allows to obtain solutions where some or all the constraints do not entirely satisfy—but are close to—their expected values. Moreover, the modeling approach used based on FL theory is the membership function, specifically the triangular membership function, which is defined by three points corresponding to the vertices. This optimization problem was transformed into a bi-objective problem when the optimization variables are the membership and the entropy. The performance of the proposed formulation was assessed in the Sioux Falls network. To solve the problem, the model was run in General Algebraic Modeling System (GAMS), applying the ε approach, where ε value (ε ∈ [0, 1] with steps of 0.01) represents the level of accomplishment that at least one of the constraints (but can be more) gets. The results show that the entropy value decreased as the accomplishment level increased, and this behavior indicates a Pareto frontier, which proves that the optimization problem is bi-objective.

Keywords: *Entropy, Fuzzy Logic, Freight Tour Synthesis, Freight Transportation, Sioux Falls Network.*

2.1. Introduction

The transport of goods is a daily necessity for society. Thus, freight trips exist with the main objective of moving goods using vehicles that make the process more efficient (mostly trucks). The freight trips created by these movements of goods are called freight trip generation (FTG), which is the summation of freight trips arriving at establishments to deliver goods (called freight trip attraction—FTA) and the number of freight trips departing from establishments (called freight trip production—FTP), (Bastida & Holguin-Veras, 2009; Board et al., 2016; Campbell et al., 2018; Gonzalez-Calderon et al., 2018; Gonzalez-Feliu & Sánchez-Díaz, 2019; Holguin-Veras et al., 2011, 2014; Krisztin, 2018; Sanchez-Diaz et al., 2016; Wigan et al., 2002).

A common methodology used to estimate the FTG distribution in an urban network is the origin–destination (OD) synthesis based on the entropy maximization (EM) approach. This procedure results in the most probable OD matrix subject to any information known about the system, which could be information on the number of trips to the origins or destinations. This is an efficient and low-cost technique to obtain freight flows based on demand using traffic counts for estimations.

In the case of freight transportation, considering trip chains (tours) is more important than considering just origins and destinations. A freight tour describes the sequence that freight vehicles must follow to make the deliveries or pick-ups assigned. In such a sequence, the delivery or pick-up places are represented by nodes linked by arcs, which is a particular characteristic of urban freight transport. Thus, when considering freight tours, the process of obtaining tour flows is known as freight tour synthesis (FTS) (Gonzalez-Calderon & Holguín-Veras, 2019; Holguin-Veras et al., 2020; Sanchez-Diaz et al., 2015). The freight modeling process is a helpful tool to represent, as much as possible, the movement of goods in society from local to global. Human behavior and inherent features such as location (rural or urban) and requirements are the basic conditions to define the modeling process's approaches, inputs, and outputs.

One usual approach to developing freight tour models is EM, which aims to obtain the most probable freight (trucks) tour flow distributions in a network based on counts. These models consider fixed parameters and constraints. However, in reality, aspects such as the costs, traffic counts, and truck demands present variations, uncertainty, and even ambiguity dependent on human behavior, all of which could not be captured in detail in deterministic models. Regarding the traffic counts, which are the primary data source for the synthesis of OD and FT, it is worth clarifying that there are high-precision methods. They still obtain data just for the moment the counts are made. Then there will always be variations due to different factors such as weather, demand dependent on the commercial season, or time of the day, which means uncertainty will still be present. One way to capture such variability is to apply fuzzy logic (FL). This technique can include in the modeling variabilities, which allows for obtaining solutions where some or all the parameters do not entirely satisfy their expected value but are close to it (in a predefined range). This approximation of the optimum value within an admissible range is done by applying membership functions; in this research, this function is triangular.

FL allows incorporating uncertainties into the optimization modeling from previous data. Some initial information (preconception) about data commonly exists. Uncertainty could come from several sources, including data variability. If variability is the only source of uncertainty, other techniques could be better than FL. However, FL as an optimization technique has some unique characteristics. For instance, it could include information variability, which is critical given that, in many cases, counting on information is necessary for developing correct modeling processes. FL is also easy to apply as the data to be used mainly depend on the modeler's criteria (the same data for the static model work), including the interval or range of variation used in the membership function. Also, uncertainty may be associated with ambiguity or different sources of information. In that case, FL is a good tool, as well as when data, judgment, and expert knowledge are not available.

The inclusion of variability draws FL solutions closer to reality as the street truck counts change every day for a lot of different reasons, as it was just mentioned a few lines before. These include congestion (which could influence logistical decisions), the daily freight demand by zones (which means that the truck flow also changes depending on the type of goods moved), external reasons such as weather, and day of the week (the truck count is different during weekends and workdays). Thus, the solutions obtained using FL are dynamic, which are better approximations than those of classic static solutions. Even though decision-makers know that transportation is dynamic, data collection in this context is difficult most of the time. Thus, alternatives to static solutions are used, where information is obtained at the most critical time of the day (i.e., at peak hours) to include as many as possible scenarios. Still, even in this, it is possible to find variations between one day and the next.

This research seeks to include the impact of parameter variabilities on freight transport system planning using FL by implementing the FL theory in the EM procedure. To do so, membership functions are applied to model constraints. The novelty in the methodology is represented in developing a more flexible FTS model formulation using FL, which allows finding some solutions that achieve a certain level of accomplishment for the constraints when it is not possible to achieve complete satisfaction. The methodology presented here is an improvement to the FTS entropy-based modelling of Gonzalez-Calderon and Holguin-Veras (2019), which also included fuzzy logic parameters in order to model uncertainty of the FTS model based on EM. To the authors' best knowledge, the FL theory has not been applied to estimating tour flows in the FTS procedure, which is the goal of this paper.

The rest of the paper is organized as follows. Section 2 presents the background of FTS and FL. Section 3 proposes the modeling formulation. Section 4 shows a numerical experiment applying the proposed method. Finally, section 5 summarizes the main findings of the research.

2.2. Background

As it was mentioned this research aim is to present the entropy-based formulation of FTS with FL constraints, which allows to include the variability of the parameters since it gives flexibility to the model. Even though, the background of the research shows a general view on four topics which are the basis of FTS and FL. They are: freight synthesis for both OD and tours, and entropy-based FTS; FL concepts and membership functions and FL related to transportation modeling.

2.2.1. Freight demand synthesis

The transport of goods is a daily necessity for society. It responds to activities people do every single day. These activities are why freight trips exist with the primary objective of moving goods (Gonzalez-Calderon et al., 2018; Holguín-Veras et al., 2018) using vehicles that make transport more efficient (primarily trucks). The freight trips created by the transport of goods are called FTG. However, when analyzing FTG for rural and urban areas, the latter has more trips representing a large part of the commercial activities in an urban area. Formally, FTG corresponds to the number of freight trips that commercial establishments generate, both those they produce—the number of freight trips departing from the establishment—and those they attract—the number of freight trips arriving at the establishment to make deliveries (Bastida & Holguin-Veras, 2009; Board et al., 2016; Campbell et al., 2018; Gonzalez-Calderon et al., 2018; Gonzalez-Feliu & Sánchez-Díaz, 2019; Holguin-Veras et al., 2011, 2014; Krisztin, 2018; Sanchez-Diaz et al., 2016; Wigan et al., 2002).

In general, in demand synthesis modeling for both passenger and freight, several approaches have been considered. The one applied in this paper is Entropy Maximization (EM) which, according to

(Willumsen, 1978a), can be expressed as “a way to find the most likely origin destination matrix compatible with the available set of link counts. In other words, the idea is to "exploit" all the information contained in the matrix observed link flows to determine the most likely OD compatible with them”. Other method is Network Equilibrium Model (EQ). This is a modelling assignment approach that tries to satisfy that, traffic on a network distributes itself in such a way that the travel costs on all routes used from any origin to all or any destination are equal while unused routes have equal or greater costs (Wardrop, 1952). The Gradient-Based Solution (GBS) is another approach which allows to become a bi-level optimization program for estimating OD demand into a one-level optimization approach. Gradient based solution techniques have been proposed, to solve optimization problems obtained from traffic modelling or statistical inference-based approaches (Abrahamsson, 1998; Noriega & Florian, 2007). Genetic Algorithms (GA) work in searching globally optimum. It is highly used when the problem has multiple solutions. GA is widely applied as a probabilistic searching method. GA is a family of computational models inspired by the concept of evolution Tavasszy & Stada (1994), Al-Battaineh & Kaysi (2005). The Statistical Inference modelling approaches such as Maximum Likelihood (LH), Linear methods (LN: Generalized Least Squares (GLS) and Ordinary Least Squares OLS))), Non-linear methods (NL), Bayesian Method (BY), Logit Models (LG) are other group of methods which have been used for estimating the OD matrix from link traffic counts. In Willumsen (1978a) and Willumsen (1978b) several approaches to estimate OD Matrix from traffic counts are considered. The author discusses LH, EQ, and EM approaches. As well as how Linear and non-linear methods (Ordinary Least Squares (OLS), Non-linear (NL)) allow to find satisfactory OD matrices, even when “they fail to consider congestion as an important factor in route choice, that is why maybe these methods could be more appropriate to rural or uncongested zones”.

In statistical inference approaches as LH, GLS and BY, “the traffic volumes and the target OD matrix are assumed to be generated by some probability distributions. An estimate of the OD matrix is obtained by estimating the parameters of the probability distributions” (Abrahamsson, 1998). The same author also sustains “The target OD matrix is in traffic modelling approaches normally obtained as an old (outdated) OD matrix, while statistical approaches rely on a target OD matrix obtained from sample surveys”. Traffic modelling or statistical inference-based approaches can be solved applying optimization techniques.

As it was mentioned the EM is the method used in this paper to develop the modeling formulation. This approach inputs are traffic counts which are data of high availability, due to the cost to this kind of data is considerably lesser than surveys for instance. But a major reason to use this method in this investigation is that this is a variation on FTS previous works, so this should be an extension of them.

2.2.2. Freight OD synthesis

With the interest in estimating the destination of trips produced by a certain origin and the origin of the attracted trips, Wilson (1969) proposed a statistical theory for models of spatial distribution, which, according to him, became "*a new method of constructing distribution and modal split models, which can be called an entropy maximizing, or probability maximizing, method."* This theory was extended and applied to the theory of trip distribution, modal split, and route split in Wilson (1969), considering the cost perceived by the users as an impedance function. Also, the entropy concept was applied to urban and regional planning models by Wilson (1970).

Generally, the trip distribution OD matrix contains the number of trips from origin *i* to destination *j*, and the results depend on the data quality (Li et al., 2019; Lim & Kim, 2016; López-Ospina, 2013; López-Ospina et al., 2021; Ortuzar & Willumsen, 2011). Although surveys are sure methods to obtain the required information on trips patterns, the use of the EM approach with traffic counts for the trip distribution results in the most probable OD matrix.

An efficient and low-cost way to estimate freight flows based on the demand is using traffic counts that is, the freight demand synthesis approach, which could be based on OD. To do so, the method uses the trips between an origin *i* and a destination *j* to reproduce, in the best way possible, the traffic counts used in the calibration of the matrix, subject to any information known about the system, which could be information on the number of trips in the origins or destinations. Thus, three states of the OD matrix must be defined: 1) the macro-state, which is related to the amount of produced and attracted trips for each zone; 2) the meso-state, which defines the number of trips between every OD pair; and 3) the micro-state, where an individual's trips between every OD pair are identified. Knowing this, the most probable OD matrix is the combination of meso-states with a major number of possible micro-states. This approach has been applied in most of this style of research. The result, in this case, is an OD matrix, which shows how goods are moved in a city. In this approach, EM is considered suitable, according to Holguin-Veras et al.(2020).

2.2.3. Freight tour synthesis - FTS

Another method to obtain the freight trip distribution is based on tours. A tour (also known as a trip chain behavior) describes the sequence that a freight vehicle must follow to make the deliveries or pick-ups assigned to it. In this sequence, the delivery or pick-up places are represented by nodes, and they are linked by arcs, making this a special characteristic of urban freight transport. The origin and the destination of the freight trips are at the same point. This process is known as FTS (Gonzalez-Calderon & Holguín-Veras, 2019; Holguin-Veras et al., 2020; Sanchez-Diaz et al., 2015). FTS seeks to replicate, as best as possible, secondary data, which means the obtention of tour flows (or the number of trucks following a determined sequence of stops). This process entails significant savings in time and costs, especially in freight transportation. The based-on EM approach has been, so far, the unique optimization program used in FTS (Holguin-Veras et al., 2020).

Whereas transit tours are predesigned and stay fixed, in the freight case, the tours could change daily depending on the needed stops. This freight trip chaining behavior is relatively new (Wang & Holguin-Veras, 2009) when compared with passenger car modeling or even with freight OD, as presented by Gonzalez-Calderon (2014). According to the insights of Holguin-Veras and Thorson (2003), the average number of stops in every tour depends on the type of good that is being transported, which makes sense considering that there are high-consumption (and not) commodities. Gonzalez-Calderon (2014) expresses in his work that according to Holguin-Veras (2013), *“the number of tours depends on the characteristics of the country, city, type of truck, the number of trip chains, type of carrier, service time, and commodity transported.”*

FTS modeling research has been, so far, consider fixed parameters and constraints (deterministic). However, the variations in costs, traffic counts, and truck demands depending on human behavior, and they are not always captured in detail in such models.

2.2.4. Fuzzy Logic (FL) and transportation modeling

To effectively tackle the problem we have just elucidated, we propose the integration of fuzzy logic into FTS modeling.

Fuzzy sets were introduced to handle objects that cannot be precisely classified inside a certain class using a certain criterion, in the real world. These sets allow for partial membership in multiple classes, addressing imprecision caused by the absence of well-defined criteria. To do so, a grade of membership ranges between zero and one is assigned by a membership function. Fuzzy algorithms, unlike deterministic ones, can accommodate intermediate grades of membership between full membership and non-membership. (Zadeh, 1965, 1968). In the context of estimating freight tour demand, it becomes apparent that certain factors such as costs, traffic counts, and truck demands exhibit variations influenced by human behavior. These variations are not adequately captured by classical models. By employing fuzzy logic, this variability can be incorporated into the modeling process, enabling the derivation of solutions where constraints, whether partial or complete, closely approximate their expected values through the utilization of membership functions.

With the development of fuzzy programming, Bit et al. (1992) applied it to the multi-objective transportation problem, which is a vector minimization problem, by assigning a range to the objective function (OF). This initial application in transportation problem solving was subsequently expanded to encompass multi-objective scenarios by the same authors (Bit et al., 1993a, 1993b). Chanas and Kuchta (1996) further employed fuzzy logic (FL) not only for the OF but also for the cost coefficients within the transportation problem.

Even when human behavior plays a crucial role in transport modeling for passengers and freight, leading to parameter uncertainty, models based on Entropy Maximization (EM) still assume fixed parameters. Distribution models, which utilize the EM approach with fixed parameters and objectives, rely on probability theory. However, some studies have addressed this limitation. For instance, López-Ospina (2013) tackled the passenger trip distribution problem by aiming to obtain an Origin-Destination (OD) matrix using the EM approach and minimizing generalized cost while accounting for cost variability within a given interval. The study was expanded to propose a bi-objective model for urban passenger trips with constraints at both origins and destinations (López-Ospina et al., 2021). In this research, the model maximized entropy to derive the most probable OD distribution matrix while simultaneously minimizing costs. Fuzzy parameters with entropic membership functions were employed to better represent uncertainty due to variability in human behavior.

Fuzzy entropy, which quantifies fuzziness and ambiguity, has significant relevance in decision-making applications involving imprecise or fuzzy values (Aggarwal, 2021). This application could better represent the uncertainty due to the variability in human behavior. Furthermore, recent research by Moreno-Palacio et al. (2022) developed a transit tour synthesis (TTS) formulation using entropy maximization approach. The research oriented to buses routes, was the first formulation to construct an entropy-based transit tour synthesis. That formulation was expanded with the use of fuzzy parameters to include flexibility to the first formulation for transit.

The FL approach has found applications in various other problem domains. Majidi et al. (2017) investigated the green vehicle routing problem with simultaneous pick-up–delivery and time windows, aiming to minimize fuel consumption while incorporating a fuzzy approach for handling pick-up and delivery demands. In a similar vein, Zhang and Ye (2008) utilized the FL system for short-term traffic flow forecasting, leveraging it to enhance the accuracy of traditional forecasting methods.

This literature review affirms that EM has been one of the methods main used in freight trips analysis, in both ODS and FTS models. Nevertheless, the impact of parameter variability on the planning transport area could be estimated using different methods as FL. FL can include such variabilities in its modeling. The flexibility FL provides to the model allows to obtain solutions where some or all the constraints do not entirely satisfy—but are close to—their expected values. However, to the author's best knowledge, this has not been done yet by anyone for the specific freight tour synthesis field in transportation modelling. Thus, this paper is the first exploration of such impact using FL, taking advantage of its characteristics discussed previously.

2.3. Modeling formulation

This section presents the modeling formulation considering the traditional entropy-based FTS deterministic formulation and shows the proposed improved formulation considering fuzzy parameters.

2.3.1. Entropy-based FTS deterministic formulation

The FTS seeks to obtain the freight tour flows in a network. In this regard, EM is used to obtain the freight tour flows, calculating which could be the most probable truck flow distribution based on counts, as stated by Holguin-Veras et al. (2020): *"Freight tour synthesis seeks to infer the tour flows—the number of freight vehicles that traverse a sequence of pick-up and delivery stops—that replicate the known input data."* Moreover, these authors affirmed that FTS models developed so far have been based only on the EM approach. This is because an EM model can overcome the indeterminate nature of FTS. This indeterminate condition makes the problem unable to obtain a unique solution. However, the same problem obtains a unique solution using EM by just applying optimization techniques.

The freight demand synthesis formulation works of Wang and Holguin-Veras (2009) studied vehicle-based demand models, that is, tour flow models. Thereafter, Gonzalez-Calderon (2014) and Gonzalez-Calderon and Holguin-Veras (2019) extended this idea in their research by developing the FTS formulation, which this paper briefly shows in Eq. (1) to Eq. (6).

The OF is a freight tour (tm) optimization program using EM Eq. (1). This model considers four parameters. Oi represents the tour production constraints, where origin and destination are of the same node (Eq. (2)), C represents the total cost (Eq. (3)), and V_a^t represents the traffic counts (Eq. (4)); the fourth parameter is the nonnegativity constraint (Eq. (5)). The model is as follows:

$$Min\ Z = \sum_{m=1}^{M} t_m \ln t_m - t_m \qquad \text{Eq. (1)}$$

subject to

$$O_i = \sum_{m=1}^{M} t_m \delta_{mi} \qquad \forall i \in \{1, 2, ..., N\} \qquad (\lambda_i) \qquad \text{Eq. (2)}$$

$$C = \sum_{m=1}^{M} C_m t_m \qquad (\beta) \qquad \text{Eq. (3)}$$

$$V_a^t = \sum_{m=1}^{M} t_m \delta_{ma} \qquad \forall a \in \{1, 2, ..., Q\} \qquad (\gamma_a) \qquad \text{Eq. (4)}$$

$$t_m \geq 0 \qquad \forall m \in \{1, 2, ..., M\} \qquad \text{Eq. (5)}$$

The optimal solution (first-order condition) is as follows:

$$t_m^* = \exp\left(\sum_{i=1}^{N} \lambda_i^* \delta_{mi} + \beta^* C_m + \sum_{a=1}^{Q} \gamma_a^* \delta_{ma} \right) \qquad \text{Eq. (6)}$$

where,

m: node sequence (tour), an ordered set of the nodes visited by a truck from the start node until the end node;

t_m: number of truck journeys (tour flow) following tour m;

M: total number of possible truck tours in the system;

N: total number of nodes in the system;

Q: total number of links with traffic counts in the system;

δ_{mi}: binary variable indicating whether node i is in the truck tour m (1 if node i is in truck tour m, 0 otherwise);

C_m: impedance (cost) of truck tour m, associated with travel and handling in the truck tour m;

δ_{ma}: binary variable indicating whether truck tour m uses link a (1 if truck tour m uses link a, 0 otherwise);

V_a^t: observed truck traffic count in link a;

λ_i: Lagrange multiplier to the trip production i constraint;

β: Lagrange multiplier to the total truck tour impedance constraint;

γ_a: Lagrange multiplier to the observed bus traffic counts in link a;

**: related to the optimum solution.*

A complete FTS formulation and model development are available in Gonzalez-Calderon and Holguin-Veras (2019).

Eq. (6) expresses the solution to this model, and it is achieved using Lagrange multipliers $(t, \lambda, \beta, \gamma)$ and the consequent Lagrange function and by proving the first and second-order conditions. These mathematical proofs demonstrate the convexity and uniqueness of the solution for finding the optimal solution, which shows that the number of trucks that follow a tour (tour flows) is an exponential function of the Lagrange multipliers. See Gonzalez-Calderon and Holguin-Veras (2019) for details.

2.3.2. Entropy-based FTS formulation with fuzzy parameters

As it was mentioned before, fuzzy sets were proposed as a solution for handling objects in the real world that defy precise classification. They provide a means to account for the partial membership of objects in multiple classes, effectively addressing imprecision resulting from the lack of well-defined criteria. Unlike deterministic algorithms, fuzzy algorithms have the ability to incorporate intermediate grades of membership between complete membership and non-membership (Zadeh, 1965, 1968).

The concept of fuzzy sets has been applied in various applications, including linear programming, multi-objective linear programming (Hannan, 1981a, 1981b; Zimmermann, 1978), multi-objective nonlinear programming (Sakawa, 1983; Sakawa & Yano, 1987), decision-making involving alternatives with flexible boundaries (Bellman & Zadeh, 1970), and the integration of probability theory with *"a possibility distribution as a fuzzy restriction which acts as an elastic constraint on the values that may be assigned to a variable"* (Zadeh, 1999).

Zadeh (1965) defines the membership function for fuzzy sets as follows: *"Let X be a space of points (objects), with a generic element of X denoted by x. Thus, X = {z}. A fuzzy set (class) A in X membership is characterized by a (characteristic) function f_A (x) which associates with each point in X a real number in the interval [0, 1], with the value of f_A (x) at x representing the "grade of membership" of x in A. Thus, the nearer the value of f_A (x) to unity, the higher the grade of membership of x in A."*

Membership functions, which represent the degree of compliance of a constraint, can be graphically visualized on a plane. The x-axis corresponds to the range of values within which the constraint can vary, while the y-axis indicates the level or percentage of compliance achieved for each value on the x-axis. The compliance level of the constraint ranges from 0 (indicating no compliance) to 1 (representing 100% compliance). Membership functions can take on different geometric shapes. For example, a commonly used shape is a triangular function, defined by three points on the x-axis: the minimum value (initial point), an intermediate value or vertex (located at value 1), and the maximum value (endpoint). The minimum and maximum values have a compliance percentage of 0%, while the intermediate value corresponds to full compliance at 100%. Other shapes, such as trapezoidal or ellipsoidal, can also be employed for membership functions.

Generally, a panel of experts on the theme, according to their high grade of expertise and/or belief about the problem behavior, is what decides—for example, in cases where there are no historical data—on what type of membership function should be used in the modeling and the ranges of variation of the constraint's values. Even though, in some cases could be the data variability distribution, represented by the approximate shape of the parameter histogram, which could help to choose the type of membership function to use in the model.

Figure 2-1 provides an illustration of a triangular membership function, depicting the three mentioned points (minimum, requested, and maximum) along with their respective grades of accomplishment.

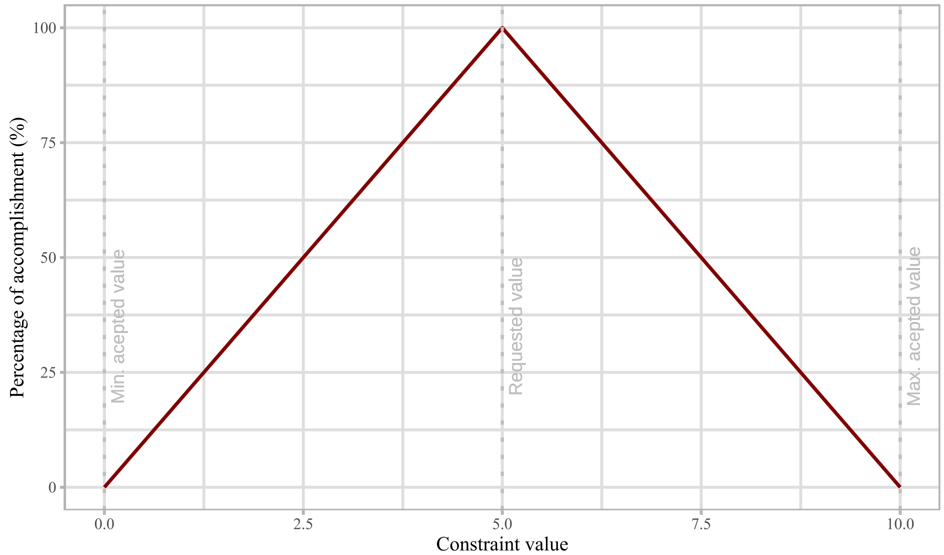


Figure 2-1 Membership function example (triangular)

The formulation of freight tour models has been developed using the EM approach and based on traffic counts. This formulation considers the parameters and objectives as fixed or deterministic. However, in reality, the costs, traffic counts, and truck demands present variations dependent on human behavior, which are not captured in classic models. Therefore, because the FL can include this variability in the modeling, solutions can be obtained where some (or all) constraints achieve values close to their expected values using membership functions. In this research, the *x*-axis components' functions form a values interval that contains the solution, which is the number of trucks in every tour and the correspondent constraint membership's level of accomplishment.

The research data in this study correspond to the ones used by Gonzalez-Calderon and Holguin-Veras (2019) in their research and consist of sequences of nodes (tours), tour lengths, costs, counts, and volume of tours. In such previous paper, the authors estimated the EM model for FTS with equality constraints, that is, fixed values. Those parameters are used in the current paper, but they are flexible in this case.

The opinion of experts was the tool to choose the membership functions' shape in this research. Here triangular membership functions are tried for the proposed model. The selection of maximum and minimum values was done by the authors, and it did not require any protocol in this case given that the greatest aim was to model data variability using FL. Nevertheless, it must be mentioned that using FL, those values in actual scenarios would correspond to the minimum and maximum values from the traffic counts obtained, whereas those of classical modeling (fixed) are taken using the traffic counts' average value. The software chosen to write and run the code was GAMS® (General Algebraic Modeling System), a program package that includes the mathematical model obtained, flexible parameters, membership functions, and input data (costs, volume in links, and tour production in nodes). The parameters that were made flexible were the tour production, the traffic counts, and the network total cost. Meanwhile, the costs/tour, which was known information, was kept fixed. The EM application to the FTS model using fuzzy parameters gives the number of trucks that made a given tour (flow/tour).

Following the formulations presented by Gonzalez-Calderon and Holguin-Veras (2019) and López-Ospina et al. (2021), this research developed a fuzzy version proposal of the EM problem with triangular parameters for the FTS model. In particular, the triangular fuzzy parameters are as follows:

- tour production/attraction ($\boldsymbol{O_i} = (\boldsymbol{O_i^1}, \boldsymbol{O_i^2}, \boldsymbol{O_i^3})$),
- total cost in the system ($\boldsymbol{C} = (\boldsymbol{C_1}, \boldsymbol{C_2}, \boldsymbol{C_3})$), and
- truck traffic count at link *a* ($\boldsymbol{V_a} = (\boldsymbol{V_a^1}, \boldsymbol{V_a^2}, \boldsymbol{V_a^3})$).

According to the parameter flexibilization, the mathematical formulation is as follows. The OF, Eq. (7), is a freight tour (t_m) optimization program using EM, similar to the one in Eq. (1), but in this case, the model includes FL parameters, which are going to be explained along the formulation.

$$max \sum_{(m=1)}^{M} t_m - t_m ln(t_m) \qquad \text{Eq. (7)}$$

subject to

$$\sum_{(m=1)}^{M} t_m \delta_{mi} = (O_i^1, O_i^2, O_i^3), \qquad i = 1, 2, ..., N \qquad \text{Eq. (8)}$$

$$\sum_{(m=1)}^{M} t_m C_m = (C_1, C_2, C_3) \qquad \text{Eq. (9)}$$

$$\sum_{(m=1)}^{M} t_m \delta_{ma} = (V_a^1, V_a^2, V_a^3), \qquad a = 1, 2, ..., Q \qquad \text{Eq. (10)}$$

where

M: number of possible tours in the system;

m: node sequence (or tour), an ordered set of nodes visited by a delivery vehicle from the start node until the end node;

N: total number of nodes in the system;

Q: total number of links with traffic counts in the system;

t_m*: number of freight vehicle journeys (tour flow) following node sequence (or tour) m (a listing of the nodes visited), that is, the number of trucks that travel along the same tour;*

C_m*: cost of tour m associated with travel and handling in the tour;*

δ_{mi}***:** a binary parameter equal to 1 if node i is in tour m but equal to 0 otherwise;*

δ_{ma}***:** a binary parameter equal to 1 if tour m uses link a but equal to 0 otherwise.*

Following Gonzalez-Calderon and Holguin-Veras (2019), Eq. (8) contains the tour production constraints. Eq. (9) represents the total cost in the system, and Eq. (10) ensures that the model replicates the observed traffic counts (the total number of links with traffic counts in the system is less than the total number of links in the network). The optimization problem with triangular fuzzy parameters (Eq. (7) to Eq. (10)) transforms into an equivalent parametric optimization model. As an example, the transform procedure is shown using a constraint (Eq. (8)). It defines:

$$m_i(t) = \sum_{m=1}^{M} t_m \delta_{mi} \qquad \text{Eq. (11)}$$

For this constraint, the membership levels (λ_i) are as follows:

- If $m_i(t) \geq O_i^3$ or $m_i(t) \leq O_i^1$, the membership level is

$$\lambda_i = 0 \qquad \text{Eq. (12)}$$

- If $m_i(t) \geq O_i^1$ and $m_i(t) \leq O_i^2$, the membership level is

$$\lambda_i = \frac{O_i^1 - m_i(t)}{O_i^1 - O_i^2} \qquad \text{Eq. (13)}$$

or equivalent to

$$m_i(t) = (1-\lambda_i) * O_i^1 + \lambda_i * O_i^2 \qquad \text{Eq. (14)}$$

- If $m_i(t) \geq O_i^2$ and $m_i(t) \leq O_i^3$, the membership level is

$$\lambda_i = \frac{O_i^3 - m_i(t)}{O_i^3 - O_i^2} \quad \text{Eq. (15)}$$

or equivalent to

$$m_i(t) = \lambda_i * O_i^2 + (1-\lambda_i) * O_i^3 \quad \text{Eq. (16)}$$

Analogously,

$$m_c(t) = \sum_{m=1}^{M} t_m C_m, \quad \lambda_C \quad \text{Eq. (17)}$$

$$m_a(t) = \sum_{m=1}^{M} t_m \delta_{ma}, \quad \text{and } \lambda_a \quad \text{Eq. (18)}$$

Where,

λ_c and λ_a are the membership levels corresponding to cost and volume in link *a*

The following optimization problem is solved to obtain the maximum fulfillment of the fuzzy constraints (Eq. (8), Eq. (9), and Eq. (10)).

$$\max \{\lambda_i(t), i = 1, \dots, N;\ \lambda_C(t);\ \lambda_a(t), a = 1, \dots., Q\} \quad \text{Eq. (19)}$$

Note that the previous problem maximized the intersection of several fuzzy membership functions. Using the definition of intersection in fuzzy sets, this problem is equivalent to:

$$\max \min \{\lambda_i(t), i = 1, \dots, N;\ \lambda_C(t);\ \lambda_a(t), a = 1, \dots., Q\} \quad \text{Eq. (20)}$$

The last problem is equivalent to

$$max\ \lambda \quad \text{Eq. (21)}$$

$$\lambda_i(t) \ \geq \lambda, i = 1, \dots, N; \quad \text{Eq. (22)}$$

$$\lambda_C(t) \geq \lambda \quad \text{Eq. (23)}$$

$$\lambda_a(t) \geq \lambda, a = 1, \dots., Q \quad \text{Eq. (24)}$$

where λ is the minimum membership level of whole fuzzy constraints.

Note that $\lambda_i(t) \geq \lambda$ is a λ-cut of the membership function $\lambda_i(t)$. For this reason, $\lambda_i(t) \geq \lambda$ is equivalent to

$$(1-\lambda) * O_i^1 + \lambda * O_i^2 \leq \sum_{m=1}^{M} t_m \delta_{mi} \leq \ \lambda * O_i^2 + (1-\lambda) * O_i^3 \quad \text{Eq. (25)}$$

Analogously, $\lambda_C(t) \geq \lambda$ is equivalent to

$$(1-\lambda)*C^1 + \lambda * C^2 \leq \sum_{m=1}^{M} t_m C_m \leq \lambda * C^2 + (1-\lambda)*C^3$$ Eq. (26)

and $\lambda_a(t) \geq \lambda$ is equivalent to

$$(1-\lambda)*V_a^1 + \lambda * V_a^2 \leq \sum_{m=1}^{M} t_m \delta_{ma} \leq \lambda * V_a^2 + (1-\lambda)*V_a^3$$ Eq. (27)

Using the previous results (Eq. (11) to Eq. (27)), the model defined by Eq. (7) to Eq. (10) is equivalent to the following bi-objective problem:

$$max(\lambda, \sum_{(m=1)}^{M} t_m - t_m ln(t_m))$$ Eq. (28)

subject to

Eq. (25) to Eq. (27)

$$t_m \geq 0 \qquad \forall m \in \{1, 2, ..., M\}, \quad \lambda \in [0,1]$$ Eq. (29)

In the optimization model described by Eq. (28) and Eq. (29), the two objectives are to maximize the entropy and λ, where λ is the minimum membership level of the flexible constraints. Compared with a deterministic model of EM (Gonzalez-Calderon & Holguín-Veras, 2019), the number of constraints doubles because of this parameter's flexibility. Thus, the epsilon (ε)-constraint method is considered to solve this problem. The mathematical formulation of the optimization problem using the ε-constraint method is as follows:

$$max \sum_{(m=1)}^{M} t_m - t_m ln(t_m)$$ Eq. (30)

subject to

$$\lambda \geq \varepsilon$$ Eq. (31)

and Eq. (25) *to* Eq. (27**)**.

According to the multi-objective optimization theory, we solve this bi-objective model for different values of $\varepsilon \in [0, 1]$ (López-Ospina et al., 2021).

In summary, the model was run in GAMS using several values for ε between 0 and 1 each time. In this way, infeasible solutions were discarded, and only the ε values that deliver feasible results are preserved. The λ parameter, in FL theory, can take values between 0 and 1 and represent the membership percentage for every solution. This is a reason for running the model several times using different ε parameter values (minimum allowed value for the λ parameter), starting from 0 with steps of 0.01. This ε value means that at least one of the constraints (but can be more) gets a membership value or accomplishment by an ε%. Lower values of ε are reflected in the model flexibility because it gives the chance to accomplish the constraints partially.

2.4. Numerical Experiment

The authors used the Sioux Falls (SF) network to assess the formulation's performance. SF is not considered a real network, although it has been used in many studies and publications to test transportation problems and is suitable for code debugging. Morlok et al. (1973) were the first ones to use the SF network as a traffic equilibrium network in their research. However, Abdulaal and LeBlanc (2018) were the ones who introduced the continuous network design problem for this network. In this section, the authors present a case study describing the zone and characteristics of the network used for the numerical experiment. Moreover, the section explains the information corresponding to the inputs, data, and conditions used to try the model. Finally, it shows the obtained results from the model and the analysis of those outputs.

2.4.1. Case study

Sioux Falls, like any other network, comprises nodes and links (24 nodes and 76 links), as shown in **Figure 2-2**. In this paper and as an academic example, the 24 nodes represent the freight generator points (i.e., the commercial establishments, not the generator zones), and they represent the freight demand as origin, stops, and ending points in the freight tours. An important aspect to consider is that when the behavior of the journey is that of a tour, the origin and destination nodes coincide and are at the same point because a tour is a haul trip. Meanwhile, the links are the lines that join the sequence of nodes that the truck follows while performing the tour. All the edges between nodes are bidirectional connectors.

The freight transportation behavior in the SF network was analyzed using the FTS optic and published by Gonzalez-Calderon (2014). Therein, aspects such as population, employment, FTG, truck trips, and tour length distribution were the main aspects that affected the cargo transportation behavior. Those aspects were studied and considered in generating the freight tours.

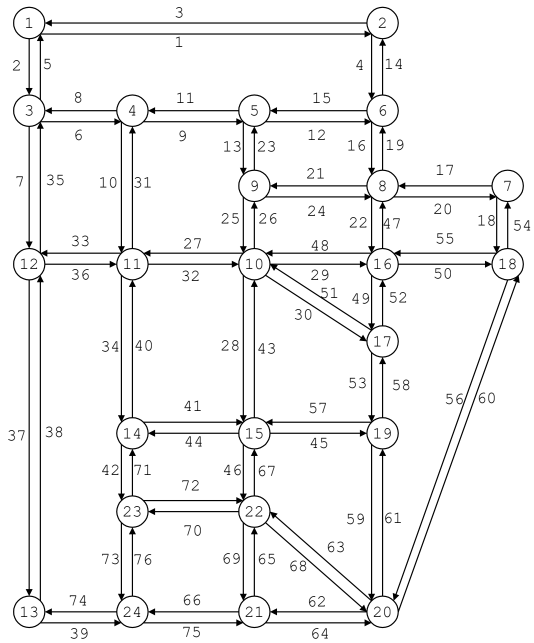


Figure 2-2 Sioux Falls network (Gonzalez-Calderon & Holguín-Veras, 2019)

2.4.2. Modeling approach

The proposed model in this research used as inputs 50 tours based on the SF network that Gonzalez-Calderon (2014) created for his numerical proof. On another note, as part of the conditions of the model, it must be defined the type of membership function to be used. In this case, both triangular and trapezoidal membership functions are useful in proving a new FTS model using FL, which allows the inclusion of uncertainty parameters into the model. Nevertheless, due to the data structure distribution and as per expert validation, the authors chose triangular membership functions for the modeling. Besides, its simplicity becomes this membership function, the best way to try the model. The triangular function should define the range of values where every constraint will oscillate between, is given by its three vertices' values, The vertices' vertical measure indicates the λ value or the accomplishment level, and it is achieved running the model and test the formulation—on entropy-based FTS using FL—proposed in this paper. To do so, GAMS® (General Algebraic Modeling System) was the software used. For practical purposes and with the interest of getting as realistic results as possible, the model was run as a continuous problem.

2.4.3. Results and analysis

As it was mentioned, the optimization process run using the ε approach. To do so, ε corresponds to an input parameter which changes in every run with steps of 0.01 for $\varepsilon \in [0, 1]$. The triangular membership function vertices were defined in this paper, taking as the minimum value 11% below, and as the maximum 33% above the requested value. Those requested values are the deterministic values which are the results of the assignment stage. In other words, those are the values for $\varepsilon=1$, which are unfeasible solutions. This is due, the flexibilization in the program allows to obtain the closest values which are "*satisfying solution*" (Luhandjula, 2015). This is a nonlinear optimization program with linear constraints. The computational time running the entropy-based model is not significant for data used in this numerical experiment (less than 5 seconds), even with the inclusion of FL constraints. The code was run in a 4-core processor 2.50GHz cache 6M.

The GAMS model obtained feasible solutions for values less than $\varepsilon = 0.18$, whereas the tries with greater values resulted in unfeasible solutions. This means that at least one of the constraints meets, at least, 18% of the target value. Furthermore, for higher values of ε, the problem became infeasible because the constraints cannot be satisfied at higher levels. This infeasibility could be a consequence of the uncertainty provided by the variability, which is a characteristic of a flexible model.

In real life, achieving 100% for all the constraints is not always possible. It is common just to have one part from any of these constraints. This is exactly what FL tries, and the existence of a limit beyond which it is not possible to take solutions is not surprising. This FL feature of robustness, in several cases, makes it much closer to reality than others could be. Thus, thanks to this model's flexibility, some solutions were found for $\varepsilon < 0.18$. However, even with such flexibility, there is no guarantee that all constraints can be fully satisfied.

To analyze the level of similarity between solutions (travel vectors) with different values of ε, the χ2 statistical test is used, following the ideas of Black (2018) and replicated in Lopez-Ospina et al. (2021). Given two solution vectors $t_{\varepsilon_1}$and t_{ε_2} for two values of ε_1 *and* ε_2, the test seeks to determine whether the vector t_{ε_1} fits the vector t_{ε_1}using the following test statistic:

$$\chi^2(t_{\varepsilon_1}, t_{\varepsilon_2}) = \sum_{m=1}^{M} \frac{(t_m(\varepsilon_1) - t_m(\varepsilon_2))^2}{t_m(\varepsilon_2)} \qquad \text{Eq. (32)}$$

Where $t_m(\varepsilon_k)$ is the m-th component of the solution vector. This test statistic $\chi^2(t_{\varepsilon_1}, t_{\varepsilon_2})$ follows a Chi-squared distribution with M − 1 degrees of freedom (49 for our numerical instance).

If the p-value = $\mathrm{P}(\chi^2 gl = M - 1 > \chi^2(t_{\varepsilon_1}, t_{\varepsilon_2}))$ is greater than a significance level (e.g., 5%), then the hypothesis that t_{ε_1} fits the vector cannot be rejected. That is, if that probability is less than the significance level, it can be concluded that the solution vectors are statistically different.

The following **Table 2-1** presents the p-values for the comparison of all feasible solutions. The highlighted the results correspond to those solution which are statistically different.

Table 2-1. Results of testing statistically differences between all feasible solutions.

ε	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
0	100%	100%	100%	99.2%	86.0%	37.6%	4.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.01		100%	100%	99.2%	86.0%	37.6%	4.1%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.02			100%	100%	100%	100%	99.6%	82.1%	23.2%	1.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.03				100%	100%	100%	100%	99.9%	89.4%	30.8%	0.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.04					100%	100%	100%	100%	99.9%	88.6%	21.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.05						100%	100%	100%	100%	99.9%	79.3%	1.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.06							100%	100%	100%	100%	99.3%	15.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.07								100%	100%	100%	100%	58.4%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.08									100%	100%	100%	90.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.09										100%	100%	100%	1.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.1											100%	100%	62.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.11												100%	100%	24.9%	0.0%	0.0%	0.0%	0.0%	0.0%
0.12													100%	100%	74.2%	0.6%	0.0%	0.0%	0.0%
0.13														100%	100%	99.3%	1.9%	0.0%	0.0%
0.14															100%	100%	87.9%	1.5%	0.0%
0.15																100%	100%	88.9%	12.2%
0.16																	100%	100%	100%
0.17																		100%	100%
0.18																			100%

Finally, after applying a chi-squared test, only five solutions were statistically different: $\varepsilon = \{0, 0.01, 0.06, 0.12, 0.15\}$. Every one of these values has its corresponding entropy value as part of the solution.

The graphic of the five statistically different solutions obtained, presented in **Figure 2-3**, shows the accomplishment level versus the entropy values. Every value from the *x*-axis corresponds to the minimum level accomplishment obtained, that is, the constraint that fared the worst. The *y*-axis corresponds to the maximum entropy in every case. This is a multi-objective optimization problem, as **Figure 2-3** shows, where the two objectives to maximize are entropy and the minimum level of accomplishment, λ. Moreover, the figure reveals that while the entropy decreases, the accomplishment level increases. This behavior indicates that there exists a Pareto frontier. Consequently, this paper's optimization problem is indeed multi-objective. Thus, if an increase in the accomplishment level is desired, the entropy should be decreased, and vice versa.

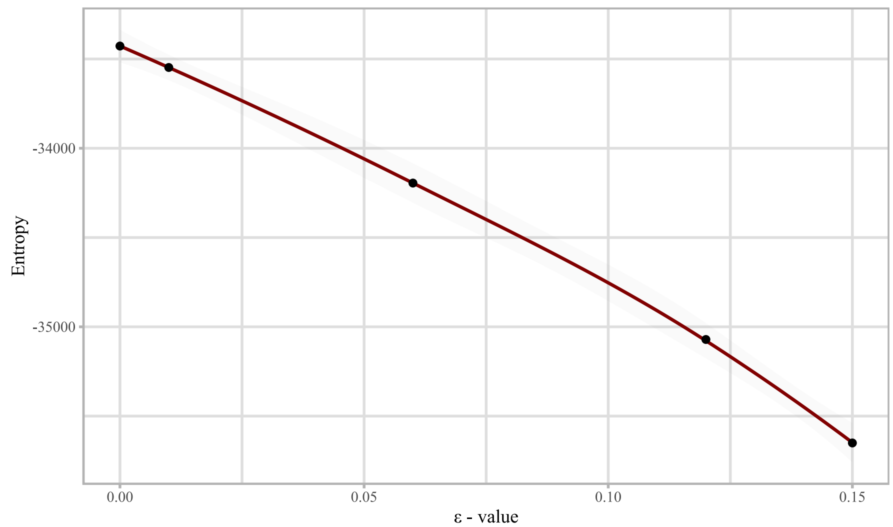


Figure 2-3. Pareto frontier. Entropy vs. accomplishment level

The entropies obtained using the proposed model were compared to observe the behavior of the OFs optimized using FL. Moreover, that comparison proves whether this paper's goal is correct and how FL is useful in solving a bi-objective optimization problem.

The Pareto frontier graphic consistently had a decrementing behavior, which is the ideal behavior in a Pareto frontier graph: it should always present the same behavior (increasing or decreasing). This numerical experiment seeks to validate the methodology, using fuzzy parameters, proposes by this research, and the results are the application of such methodology (formulation on entropy-based FTS using FL) using the authors' data. Thus, the obtained results are associated with these data. Now, if the formulation is replicated hereafter, using a different numerical experiment, the Pareto frontier will be different; it could get a better or worse λ value, in our case λ = 0.15, and these results correspond only to the case studied.

With the relaxing of the constraints (in different degrees) with every different value of ε, the number of trucks that use a specific tour change. These changes vary widely, as shown in **Figure 2-4** which represents the spread of the percentual change in the number of trucks that use a specific tour. The changes in **Figure 2-4** show the implications of the relaxation of the constraints to the assigned traffic for each tour. In this case, some tours presented changes even greater that 1000%, this occurs due to those tours moved to be not used at all to be high used, becoming into the outlier shown. Moreover, the average change of the volume of the tours are around 100%, indicating a significant influence of the epsilon value (minimum accepted percentage of accomplishment) in the model. Given that Figure 4 is a box and whisker plot, the Y-axis corresponds to the box width, and represents the category: tours.

Different thresholds for the minimum percentage of accomplishment produce different utilization (volume) in the tours. As it was noted before, this experiment used 19 thresholds, starting from 0 (which corresponds to the determinist case) to 0.18, with steps of 0.1. So, for every link 19 volumes were computed. Then the maximum observed change, in percentage, between the volumes in each link was computed and represented in **Figure 2-4**. It can be observed that the distribution of the changes in the volume of tours using different thresholds is positive skewed, with the mean (84.46%) considerably higher than the median (31.87%). Also, the 75% of the computed percentage of change in the tour volumes is less than 55.03. This indicates that around 75% of the times difference between the deterministic solution and the flexible solution will be less or equal that 55% However, there are outliers with great percentage of change which correspond to the tours that are not used in the determinist models but are used with the flexible one.

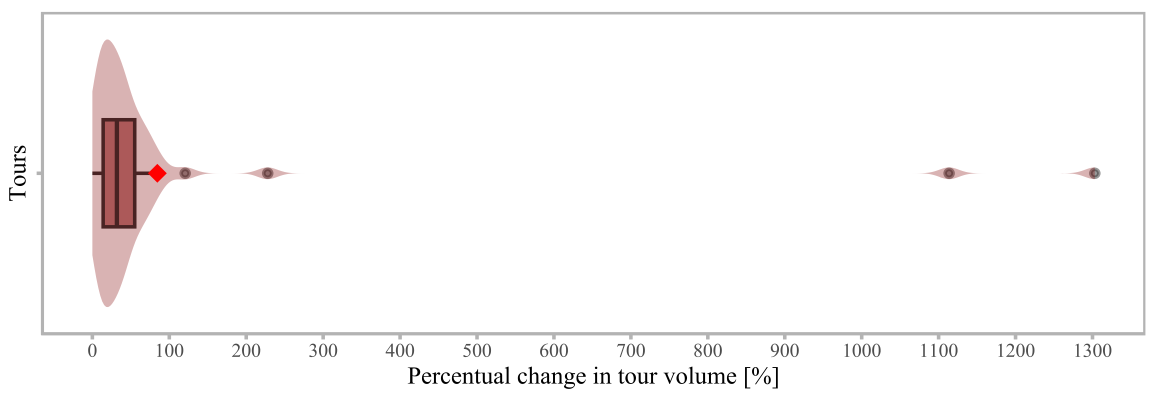
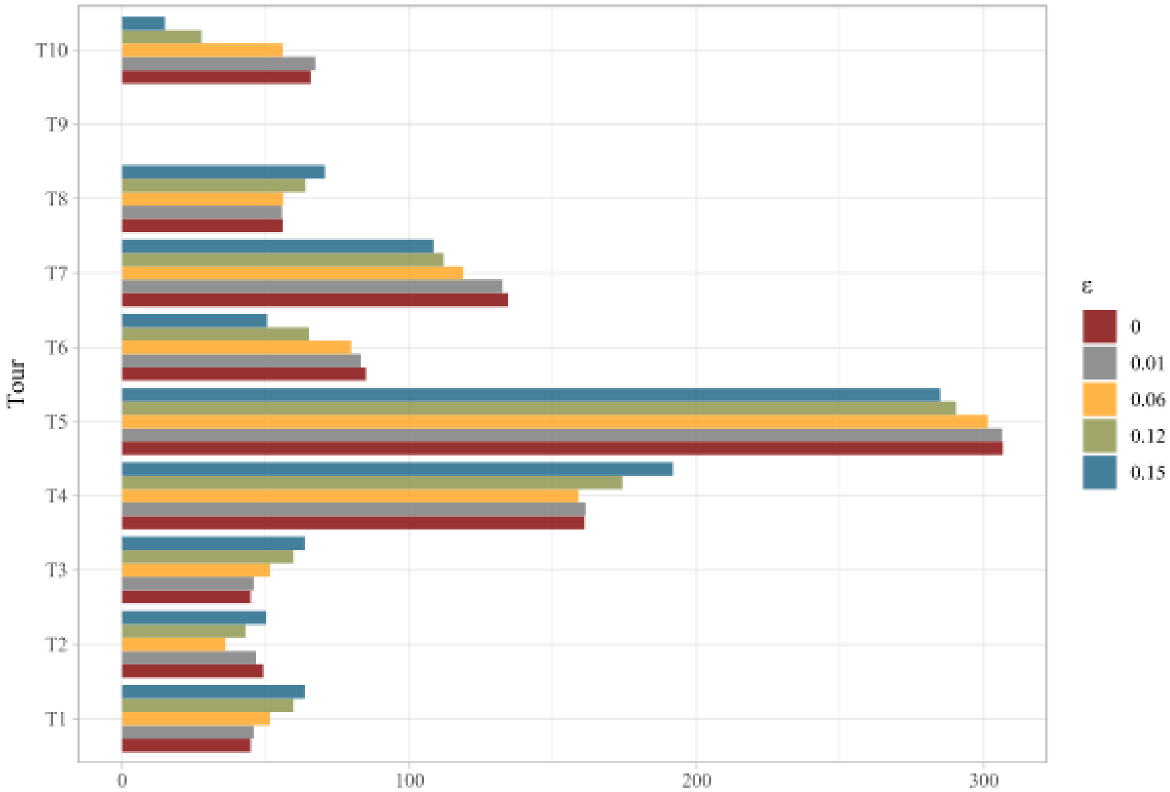


Figure 2-4. Traffic change for each solution

However, if the volumes for each tour obtained using different ε values are incomparable, an example (10 of the 50 tours) of the changes in volume with different ε values can be observed in **Tour Volume** (Number of trucks)

Figure 2-5. It can be observed that tour 9 remained unused no matter the ε value. In contrast, in tours 6 and 1, opposite effects are observed: whereas the ε value increased the volume in tour 1, that in tour 6 decreased the volume. Other tours did not have a clear linear (increasing or decreasing) tendency, as tour 2 shows. Thus, clear patterns in the volume changes of the tours cannot be established easily because the flexibilization of the constraints impacts many factors in the optimization problem.



Tour Volume (Number of trucks)

Figure 2-5. Truck tour volumes for different ε values

The values obtained for the constraints using different ε values can also be shown graphically with their respective membership functions to observe how different grades of accomplishment are obtained with different ε values. **Figure 2-6** shows the obtained values in two different nodes. It exhibits the different grades of accomplishments obtained. However, it must be noted that the same level of accomplishment can be obtained in two ways: to the left side of the membership function (the value obtained is below the requirement) or to the right side (the value obtained is over the requirement). In **Figure 2-6**, node 1 produced more tours than required, whereas node 22 produced fewer tours than required. The same analysis and representation can be done for all the nodes and for all the links where the constraint is the volume. Generally, this can be done for all the flexible constraints.

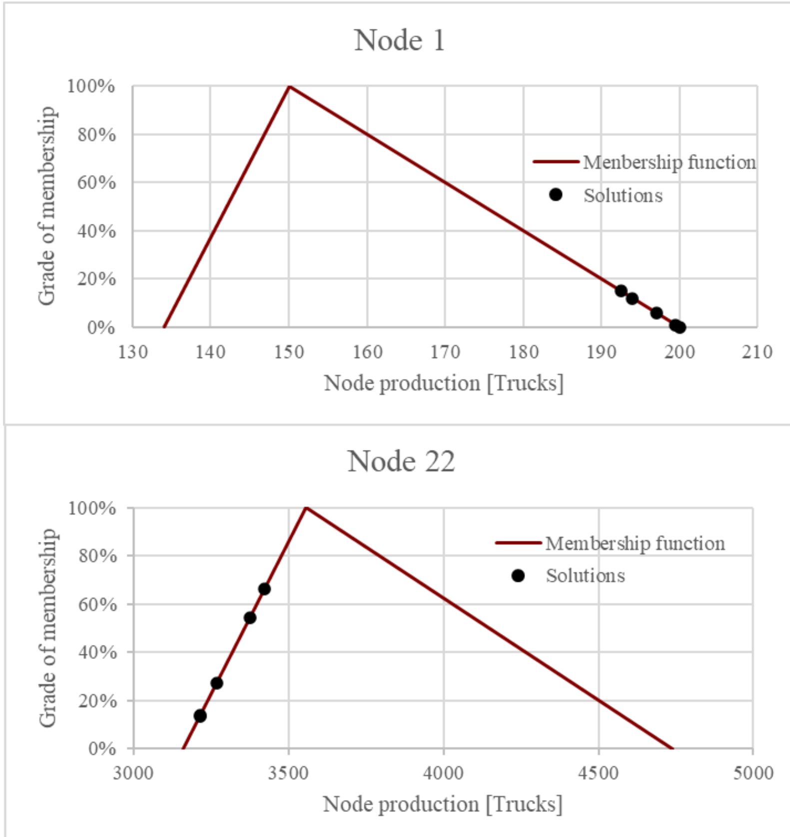


Figure 2-6. Accomplishment of the node-production

The membership function used has three points (min, requested value, and max) given that it is a triangular function. When the solution in one tour corresponds to the minimum value in the interval, with a chosen value of λ, at least one of the constraints is obtained, this being the worst case, and the others may be in the same or a better situation.

Figure 2-7 shows the percentage of accomplishment for each tour and its distribution. It must be noted that the minimum accomplishment value always corresponds to the ε-value, whereas the other constraints achieve higher values of accomplishment, even 100%.

Table 2-2 shows the number of accomplishments with the minimum and maximum values, along with other statistics for the node-production constraints.

Both **Figure 2-7** and **Table 2-2** show that the median values seem to stay constant for all the solutions (68%); besides, the third quartile has a slight increase. In contrast, **Figure 2-7** also reveals a decrease in variability. This last observation was expected because of the increase in the minimum compliance value.

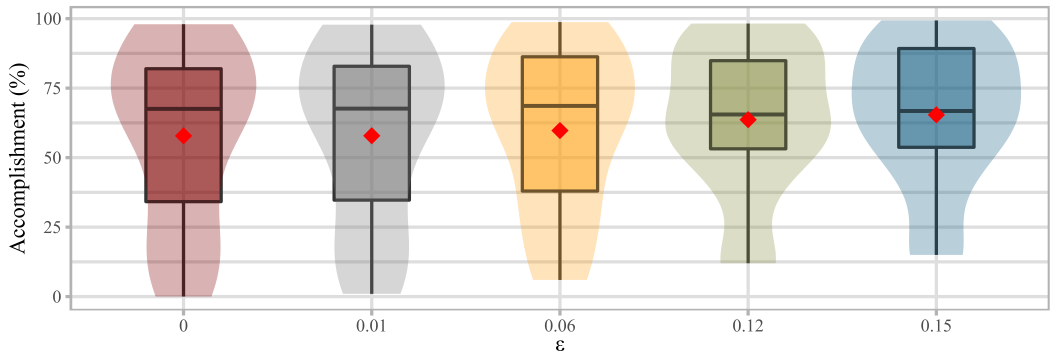


Figure 2-7. Percentage of the accomplishment node-production constraint

Table 2-2. Descriptive analysis of the accomplishment node-production constraint

Descriptions	**Epsilon (ε)**				
	0	**0.01**	**0.06**	**0.12**	**0.15**
Mean (%)	58	58	60	64	65
Median (%)	68	68	69	66	67
Standard deviation	0.32	0.31	0.29	0.26	0.25
Maximum value (%)	98	98	99	98	99
Number of maximum accomplishments	1	1	1	1	1
Minimum value	0	0.01	0.06	0.12	0.15
Number of minimum accomplishments	2	2	2	3	3

Every solution corresponds to an ε value, as already explained and shown in the above table. This also shows that, for example, for the ε = 0.01 solution, the tour's average compliance was 58% for the production constraint (taking 24 nodes). The table shows the results for the rest of the solutions. As also mentioned earlier, **Figure 2-8** proves that the accomplishment minimum value is equal to the corresponding ε-value.

In a homologous case, the accomplishment levels with respect to the volume constraint measured in every link are represented in **Figure 2-8**. The figure shows the percentage of accomplishment for each tour and its distribution. In this figure, it is clear that the variation in the averages is minimum, in contrast with the median values, which present a notable variation (from 0% to 15%). This is because the number of minimum values (see **Table 2-3**) has significant changes.

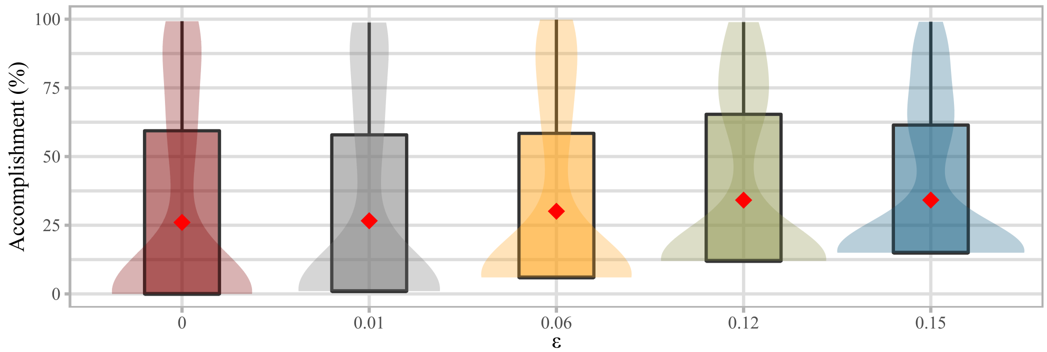


Figure 2-8. Percentage of the accomplishment link-volume constraint

Table 2-3. Descriptive analysis of the accomplishment link-volume constraint

Description	**Epsilon (*ε*)**				
	0	**0.01**	**0.06**	**0.12**	**0.15**
Mean (%)	26	27	30	34	34
Median (%)	0	1	6	12	15
Standard deviation	0.37	0.36	0.34	0.31	0.28
Maximum value (%)	99	99	100	99	99
Number of maximum accomplishments	1	1	1	1	1
Minimum value	0	0.01	0.06	0.12	0.15
Number of minimum accomplishments	44	32	26	45	42

For the case of the ε = 0.01 solution, according to the results in **Table 2-3**, the tour's average compliance relative to the volume constraint (on the 76 links) is 27%, with a standard deviation of 0.36, and the compliance minimum value is equal to the ε-value, just as expected.

The proposed method needs as input traffic counts, which after cross the model becomes into truck tours flows. An alternative to validate the model is to separate a part of the traffic counts collected and, leave them clean. That is, do not use them in the model. Those could be used after as a way to validate the results (Ortuzar & Willumsen, 2011). This is a classic method of validation. Another alternative is proposed by You & Ritchie (2019) They use GPS data, from GPS installed in trucks in Southern California, to be used in a trip/tour entropy model using a specific type of truck.

Moreover, it is essential to acknowledge that the data under examination represent samples that introduce uncertainty and require statistical analysis. Histograms are a viable tool for conducting such analysis, as they enable the creation of a membership function. Other methods, including the utilization of confidence interval tests based on the mean value, may also be employed to generate the membership function or in the absence of information, one can rely on the expertise of domain experts, as their knowledge and experience in a particular problem allow them to define the membership functions. Experts can provide insights into the relationships between input and output variables and suggest the appropriate shape for the membership functions.

An Analysis of Flexibility Models versus Deterministic Models

The model considering the constraints as equality in the target value is unfeasible. In these types of situations, it is common in the literature to make modifications to certain parameters in order to satisfy the equality constraints. To use the original parameters, a reasonable approach will be to change the constraints to inequalities expecting to obtain values close, but not equal, to the requested value. One option is to make all the constraints less or equal to the requested value. Other option is to make all the constraints to greater or equal to the requested value. Other approach could be trying combinations varying the constraints less or equal, or greater or equal than the target value. However, a boundary to determine if the solution is close enough to the requested value must be established. This is an iterative process to find a close enough solution.

Implementing FL into the model avoid the iterative process to find a close enough solution when the problem using constraints as equalities is no feasible. The model with FL gave the best possible solution within the defined boundaries of the constraints are close enough.

For the case study the problem with the equality constrains was infeasible, so the FL was implemented. However, to compare the proposed model with others without the FL, 5 cases were tested. Case 1) All the constraints are less or equal to the target value, Case 2) All the constraints are greater or equal to the target value, Case 3) All the constraints are less or equal to the upper value (maximum value) of the interval used in FL, Case 4) All the constraints are greater or equal to the lower value (minimum value) of the interval used in FL, and Case 5) All the constraints are less or equal to the upper value (maximum value) of the interval used in FL and All the constraints are greater or equal to the lower value (minimum value) of the interval used in FL, for both nodes (tour generation) and links (volume).

Table 2-4 and **Table 2-5** show the percentage of accomplishment (membership level associated with the proximity to the request value in each constraint) in all the cases for both production and volumes respectively compared with the one obtained in FL solution.

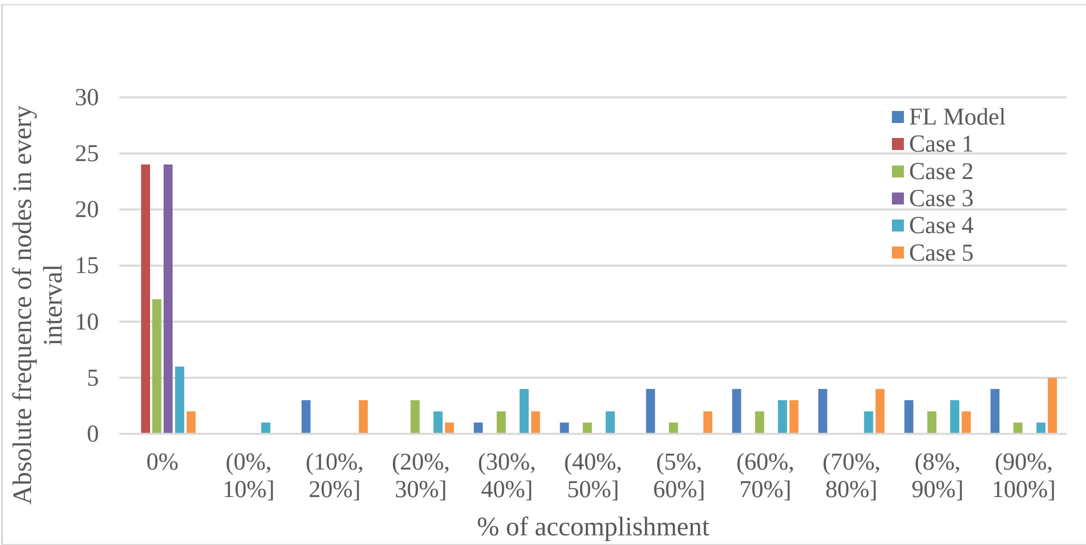
Table 2-4. Percentage of accomplishment FL model and No FL models – Nodes (production)

Node	FL ε=0.15	NO FL All constraints ≤ requested value	NO FL All constraints ≥ requested value	NO FL All constraints ≤ max value	NO FL All constraints ≥ min value	NO FL All constraints ≥ min value and ≤ max value
	FL Model	Case 1	Case 2	Case 3	Case 4	Case 5
I1	15%	0%	0%	0%	0%	0%
I2	15%	0%	0%	0%	0%	0%
I3	67%	0%	34%	0%	0%	68%
I4	92%	0%	0%	0%	75%	98%
I5	97%	0%	0%	0%	42%	95%
I6	75%	0%	0%	0%	0%	86%
I7	66%	0%	0%	0%	21%	54%
I8	59%	0%	0%	0%	37%	75%
I9	64%	0%	48%	0%	41%	76%
I10	96%	0%	27%	0%	87%	97%
I11	89%	0%	0%	0%	69%	81%
I12	79%	0%	0%	0%	0%	72%
I13	38%	0%	34%	0%	0%	27%
I14	90%	0%	86%	0%	75%	90%
I15	74%	0%	21%	0%	38%	12%
I16	56%	0%	23%	0%	63%	63%
I17	79%	0%	0%	0%	64%	67%
I18	15%	0%	61%	0%	35%	15%
I19	99%	0%	87%	0%	98%	96%
I20	90%	0%	50%	0%	36%	71%
I21	41%	0%	99%	0%	90%	38%
I22	66%	0%	67%	0%	4%	14%
I23	51%	0%	0%	0%	88%	36%
I24	54%	0%	0%	0%	30%	58%

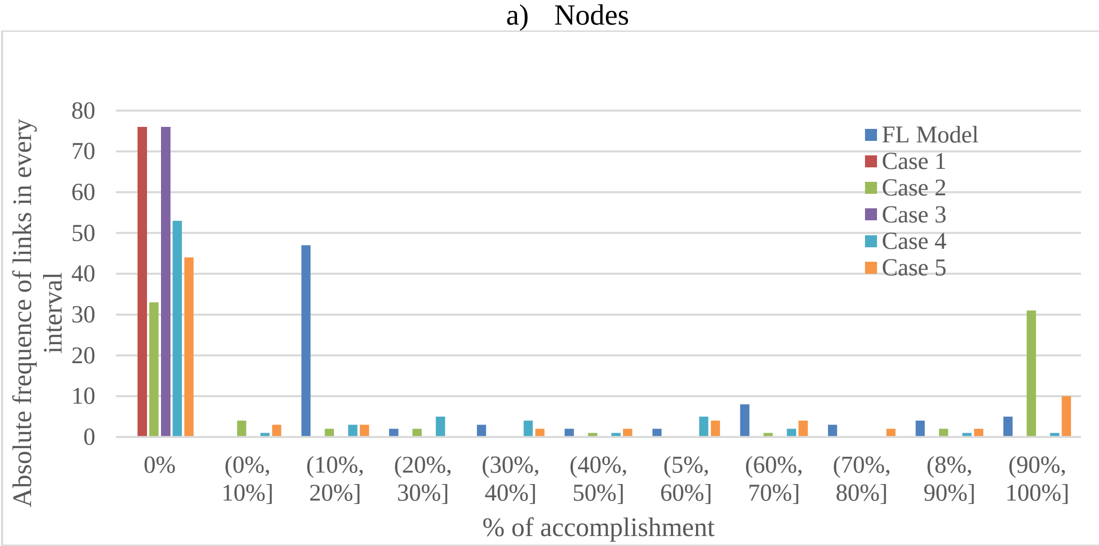
Table 2-5. Percentage of accomplishment FL model and No FL models – Links (Volumes)

Link	FL ε=0.15	NO FL All constraints ≤ requested value	NO FL All constraints ≥ requested value	NO FL All constraints ≤ max value	NO FL All constraints ≥ min value	NO FL All constraints ≥ min value and ≤ max value
	FL Model	Case 1	Case 2	Case 3	Case 4	Case 5
A1	15%	0%	0%	0%	0%	0%
A2	15%	0%	0%	0%	0%	0%
A3	15%	0%	0%	0%	0%	0%
A4	15%	0%	0%	0%	0%	0%
A5	15%	0%	0%	0%	0%	0%
A6	15%	0%	100%	0%	0%	0%
A7	31%	0%	0%	0%	0%	33%
A8	62%	0%	0%	0%	38%	78%
A9	15%	0%	0%	0%	0%	0%
A10	15%	0%	100%	0%	0%	0%
A11	63%	0%	0%	0%	27%	69%
A12	15%	0%	0%	0%	0%	73%
A13	15%	0%	87%	0%	35%	0%
A14	15%	0%	0%	0%	0%	0%
A15	99%	0%	100%	0%	0%	12%
A16	30%	0%	0%	0%	0%	0%
A17	15%	0%	0%	0%	0%	0%
A18	61%	0%	100%	0%	0%	93%
A19	68%	0%	45%	0%	86%	64%
A20	61%	0%	100%	0%	0%	93%
A21	15%	0%	0%	0%	0%	0%
A22	30%	0%	0%	0%	35%	59%
A23	64%	0%	0%	0%	7%	91%
A24	50%	0%	100%	0%	0%	99%
A25	15%	0%	0%	0%	0%	0%
A26	47%	0%	26%	0%	68%	67%
A27	15%	0%	100%	0%	0%	47%
A28	15%	0%	0%	0%	22%	96%
A29	15%	0%	0%	0%	0%	0%
A30	15%	0%	100%	0%	0%	0%
A31	15%	0%	100%	0%	0%	0%
A32	90%	0%	6%	0%	51%	91%
A33	73%	0%	0%	0%	14%	97%
A34	69%	0%	63%	0%	100%	52%
A35	82%	0%	0%	0%	0%	67%
A36	15%	0%	0%	0%	0%	0%
A37	15%	0%	0%	0%	0%	0%
A38	15%	0%	0%	0%	0%	0%
A39	15%	0%	0%	0%	0%	0%
A40	15%	0%	0%	0%	10%	0%
A41	15%	0%	100%	0%	0%	0%
A42	94%	0%	6%	0%	50%	99%
A43	21%	0%	100%	0%	0%	0%
A44	54%	0%	24%	0%	66%	0%
A45	15%	0%	100%	0%	0%	0%
A46	56%	0%	94%	0%	18%	53%
A47	15%	0%	100%	0%	0%	0%
A48	15%	0%	100%	0%	0%	0%
A49	15%	0%	100%	0%	0%	0%
A50	15%	0%	100%	0%	0%	0%
A51	15%	0%	7%	0%	50%	2%
A52	15%	0%	0%	0%	0%	0%
A53	15%	0%	0%	0%	0%	0%
A54	17%	0%	0%	0%	0%	2%
A55	15%	0%	100%	0%	0%	40%
A56	16%	0%	17%	0%	59%	0%
A57	85%	0%	100%	0%	0%	0%
A58	84%	0%	0%	0%	35%	43%
A59	15%	0%	100%	0%	0%	0%
A60	15%	0%	9%	0%	53%	15%
A61	93%	0%	90%	0%	28%	87%
A62	26%	0%	100%	0%	0%	13%
A63	15%	0%	100%	0%	0%	0%
A64	15%	0%	100%	0%	0%	0%
A65	15%	0%	100%	0%	0%	0%
A66	78%	0%	0%	0%	0%	60%
A67	90%	0%	100%	0%	0%	0%
A68	62%	0%	89%	0%	30%	98%
A69	15%	0%	100%	0%	0%	0%
A70	15%	0%	100%	0%	0%	0%
A71	95%	0%	10%	0%	54%	92%
A72	15%	0%	100%	0%	0%	9%
A73	15%	0%	100%	0%	0%	0%
A74	15%	0%	0%	0%	0%	0%
A75	71%	0%	0%	0%	28%	84%
A76	15%	0%	100%	0%	0%	0%

The **Table 2-4** and **Table 2-5** reveal that the deterministic solutions evaluated in the five cases have several results where the values obtained are out of the interval proposed, and those are expressed as 0% of accomplishment. This means that those results are far than those obtained in the FL solution, where all the results are inside of the interval. In case 5, the results with 0% of accomplishment means that the result is inside the interval even when it is in the limit, which means that even when all the results are inside of the range, those in the limit continue be far than the ones of FL. Figure 2-9 a) and b), are a summary of the latest tables, and they are presented as a way to be clearer about the results obtained. in Both nodes and links, it reveals that all five cases have elements in 0% value, while FL does not have anyone.



a) Nodes



a) Links

Figure 2-9. Histogram of % of accomplishment in comparison FL and No FL cases

Moreover, Table 2-6 allows to check that FL model had the highest % of accomplishment mean. This result join with the already explained confirm that the behavior of FL model is better than the deterministic cases. This means that the level of accomplishment for fuzzy model is better even in the case 5.

Table 2-6. Percentage of accomplishment - Descriptives

Descriptives	FL Model	Case 1	Case 2	Case 3	Case 4	Case 5
Min	15%	0%	0%	0%	0%	0%
|Min|	15%	0%	0%	0%	0%	0%
|Mean|	65%	0%	26%	0%	41%	58%
Max	99%	0%	99%	0%	98%	98%
|Max|	99%	0%	99%	0%	98%	98%

a) Nodes

Descriptives	FL Model	Case 1	Case 2	Case 3	Case 4	Case 5
Min	15%	0%	0%	0%	0%	0%
|Min|	15%	0%	0%	0%	0%	0%
|Mean|	65%	0%	26%	0%	41%	58%
Max	99%	0%	99%	0%	98%	98%
|Max|	99%	0%	99%	0%	98%	98%

b) Links

Finally, Table 2-7 presents a summary about the quantity of nodes and links that presented solutions out of the interval in every case. We see again that FL did not have any result out of the range, as well as case 5, which explanation was already given.

Table 2-7. Quantity of solutions inside and outside the range

	FL	**NO FL**				
	ε=0.15	**All constraints ≤ requested value**	**All constraints ≥ requested value**	**All constraints ≤ max value**	**All constraints ≥ min value**	**All constraints ≥ min value and ≤ max value**
	FL Model	**Case 1**	**Case 2**	**Case 3**	**Case 4**	**Case 5**
Production in nodes (24 in total)						
Out of the range	0	24	12	24	6	0
In the range	24	0	12	0	18	24
Volume in links (76 in total)						
Out of the range	0	76	33	76	53	0
In the range	76	0	43	0	23	76

2.5. Concluding Remarks and Planning Implications

Trucks are the primary vehicles used for freight transportation as they have great capacities, making distributions more efficient. However, trucks are usually larger than standard vehicles, which means they occupy more space on roads, contributing to congestion. Also, trucks must stop in establishments to deliver or pick-up packages, and a lot of these stops frequently do not have parking places. Thus, these trucks must stop on one side of the road, usually in front of the establishment, reducing the capacity of the roads. Thus, planners must know what routes are more prone to truck traffic to plan an infrastructure that meets their necessities without affecting other users on the road.

Because of finite resources, decision-makers must make the best possible decisions with the resources available. Thus, the use of flexible models allows for considering cases where it is impossible to accomplish all constraints. Moreover, those accomplished are just part of such constraints most of the time but are still helpful in decision-making. This is a significant advantage of FL over models with fixed parameters where no solution is generated to help in the decision-making process when all constraints cannot be accomplished.

The FL is a powerful tool when a problem implies variability and uncertainty. The reason behind this is that in optimization with the fuzzy constraints technique, the obtained results are more reliable because of the more realistic shape of the models. Accordingly, FL modeling has great importance in freight transport planning (among others).

On another note, the FL modeling allows for estimating relevant aspects in planning in greater detail, such as congestion behaviors, diagnostics related to emissions levels, and accidentality. This methodology can generate helpful information for decision-makers. The solutions obtained can well represent the natural behavior of transport or any other problem. FL widens the picture for planning professionals and allows, for example, for deciding how the behavior of trucks would be if they had exclusive lanes or if the delivery time improves at a specific time in a specific zone.

Using FL in a freight tour synthesis (FTS) model allows the incorporation of uncertainty in the input data, such as tour production, traffic counts, and total cost in the system, which is helpful in calibrating the FTS.

The FL aids in addressing the problem brought on by data with varying degrees of precision and unpredictability, data acquired at various times of the day, etc. With the system's total cost being invisible, this is a particularly frustrating issue. In order to do this, FL constraints were added to the FTS formulation to replace the equality constraints.

For decision-maker the Pareto frontier could be a guide to choose the right FTS model. As the Pareto frontier shows the accomplishment level versus the entropy values, it is useful to see that while the entropy decreases, the accomplishment level increases and vice versa, which means if an increase in the accomplishment level is desired, the entropy should be decreased. This face to the decision-makers to the question of which is their desired or need. For more certain in the entropy the model says that the level of accomplishment of the constraints must be low.

The initial model was deterministic, and it is infeasible in our case, that means that a variation in the parameters is necessary. That variation means a try and error process. The inclusion of fuzzy parameters is a way to avoid that try and error process, because in a certain way it is part of the model now, and that is what the flexibility sums.

An additional relevant aspect pertains to the model's capability to estimate the minimum and maximum number of trucks required for each tour. This feature may potentially facilitate the redistribution or management of the fleet, particularly in the event that additional trucks need to be procured. In such cases, analyses of low demand versus high demand scenarios may be conducted. Moreover, the bi-objective model permits the integration of various scenarios into a singular case, with statistical tests serving to examine the robustness of the chosen solution. The degree of similarity between the selected solution and other solutions determines the robustness of the model. For the planner, the ability to select robust solutions that are minimally impacted by changes to the model's parameters mitigates the effects of data ambiguity.

It is important to emphasize the flexibility that the fuzzy logic (FL) model offers in terms of modeling uncertainty. In light of this, a rigorous examination was conducted to compare the flexible FL model with the deterministic models. Five diverse deterministic scenarios were utilized for this purpose. The results demonstrate that the solution obtained with the fuzzy model exhibits superior performance, as the percentage of accomplishment it achieves surpasses that of the deterministic cases presented in all aspects.

Clearly, the FTS with the FL model is a flexible model, which is ideal as reality requires some flexibility. This is a key in the modeling process, which primarily aims to make the best representation possible of reality. FL could contribute to building better transportation planning for agencies to improve mobility in cities and urban areas. The more accurate the results, the better the ensuing decisions will be. FL helps include the parameters variability inherent in daily life into the input data, the same that should be reflected in the outputs.

Governmental offices in charge of mobility must plan aspects such as freight routes, traffic improvement initiatives, or truck tours that allow for increasing/decreasing truck flow in certain areas. The more accurate the data, the more realistic the decisions, such as planning new loading/unloading bays or more parking spots.

Some of the conclusions of the research are as follows:

• Comparing graphically the entropies for the system, for each minimum value of the membership function (the ε value), allows to observe that the Pareto frontier has the ideal behavior when, in this case, it decreases consistently, since it should always present the same behavior (increasing or decreasing). The existence of a Pareto frontier proves that the optimization problem is multi-objective. Therefore, to obtain comparable entropy values, it is necessary to take into account the number of vehicles using the links.

In each system (where to change the ε value), the solution implies different traffic assignments for each link. However, the total value is still optimum.

• When the minimum value of the membership (the lowest value for ε) is a feasible solution, it means the nonflexible problem does not have a solution with the constraints applied. This result implies that the problem has a partial solution which, for practitioners, is better than nothing. The proposed model precisely works on that option. It helps to get the best possible solution when it is not possible to satisfy each constraint fully.

• The experiment tested minimum membership values starting from 0 with steps of 0.01 until feasible solutions exist (in this case, at 0.18). From these 19 solutions, only five were statistically different. These solutions correspond to the ones used in the numerical analysis.

• This research used FL to make the corresponding constraints to node-production and link-volume flexible. However, the authors think that the link cost can also be flexible in further research. This research may be furthered by studying better forms for the membership function applied to constraint flexibilization.

2.6. References

Abdulaal, M., & LeBlanc, L. J. (1979). Continuous equilibrium network design models. *Transportation Research Part B: Methodological*, *13*(1), 19–32. https://doi.org/10.1016/0191-2615(79)90004-3

Abrahamsson, T. (1998). *Estimation of Origin-Destination Matrices Using Traffic Counts - A Literature Survey*. IIASA, Laxenburg, Austria: IR-98-021. Retrieved from http://pure.iiasa.ac.at/id/eprint/5627/

Aggarwal, M. (2021). Redefining fuzzy entropy with a general framework. *Expert Systems with Applications*, *164*, 113671. https://doi.org/10.1016/j.eswa.2020.113671

Al-Battaineh, O., & Kaysi, I. A. (2005). Commodity-based truck origin--destination matrix estimation using input--output data and genetic algorithms. *Transportation Research Record*, *1923*(1), 37–45.

Bastida, C., & Holguin-Veras, J. (2009). Freight Generation Models. Comparative analysis of regression models and multiple classification analysis. *Transportation Research Record: Journal of the Transportation Research Board*, *2097*(1), 51–61. https://doi.org/10.3141/2097-07

Bellman, R., & Zadeh, L. A. (1970). Decision-Making in a Fuzzy Environment. *Management Science*, *17*(4), B141–B164. Retrieved from https://www.jstor.org/stable/2629367

Bit, A. K., Biswal, M. P., & Alam, S. S. (1992). Fuzzy programming approach to multicriteria decision making transportation problem. *Fuzzy Sets and Systems*, *50*(2), 135–141. https://doi.org/10.1016/0165-0114(92)90212-M

Bit, A. K., Biswal, M. P., & Alam, S. S. (1993a). An additive fuzzy programming model for multiobjective transportation problem. *Fuzzy Sets and Systems*, *57*(3), 313–319. https://doi.org/10.1016/0165-0114(93)90026-E

Bit, A. K., Biswal, M. P., & Alam, S. S. (1993b). Fuzzy programming approach to multiobjective solid transportation problem. *Fuzzy Sets and Systems*, *57*(2), 183–194. https://doi.org/10.1016/0165-0114(93)90158-E

Black, J. (2018). *Urban transport planning: Theory and practice* (Vol. 4). Routledge.

Board, T. R., Engineering, of S., & Medicine. (2016). *Using Commodity Flow Survey Microdata and Other Establishment Data to Estimate the Generation of Freight, Freight Trips, and Service Trips: Guidebook*. (J. Holguín-Veras, C. Lawson, C. Wang, M. Jaller, C. González-Calderón, S. Campbell, … D. Ramirez, Eds.). Washington, DC, USA: The National Academies Press. https://doi.org/10.17226/24602

Campbell, S., Holguin-Veras, J., Ramirez-Rios, D. G., Gonzalez-Calderon, C. A., Kalahasthi, L., & Wojtowicz, J. (2018). Freight and service parking needs and the role of demand management. *European Transport Research Review*, *10*(2), 1–13. https://doi.org/http://dx.doi.org/10.1186/s12544-018-0309-5

Chanas, S., & Kuchta, D. (1996). A concept of the optimal solution of the transportation problem with fuzzy cost coefficients. *Fuzzy Sets and Systems*, *82*(3), 299–305. https://doi.org/10.1016/0165-0114(95)00278-2

Gonzalez-Calderon, C. A. (2014). *Multiclass Equilibrium Demand Synthesis*. PhD dissertation. Doctor of Philosophy, Rensselaer Polytechnic Institute.

Gonzalez-Calderon, C. A., & Holguín-Veras, J. (2019). Entropy-based freight tour synthesis and the role of traffic count sampling. *Transportation Research Part E: Logistics and Transportation Review*, *121*(November 2017), 63–83. https://doi.org/10.1016/j.tre.2017.10.010

Gonzalez-Calderon, C. A., Sanchez-Diaz, I., Sarmiento-Ordosgoitia, I., & Holguin-Veras, J.

(2018). Characterization and analysis of metropolitan freight patterns in Medellin, Colombia. *European Transport Research Review*, *10*(2), 23. https://doi.org/10.1186/s12544-018-0290-z

Gonzalez-Feliu, J., & Sánchez-Díaz, I. (2019). The influence of aggregation level and category construction on estimation quality for freight trip generation models. *Transportation Research Part E: Logistics and Transportation Review*, *121*, 134–148. https://doi.org/10.1016/j.tre.2018.07.007

Hannan, E. L. (1981a). Linear programming with multiple fuzzy goals. *Fuzzy Sets and Systems*, *6*(3), 235–248. https://doi.org/10.1016/0165-0114(81)90002-6

Hannan, E. L. (1981b). On Fuzzy Goal Programming. *Decision Sciences*, *12*(3), 522–531. https://doi.org/10.1111/j.1540-5915.1981.tb00102.x

Holguin-Veras, J. (2013). Freight demand modeling: state of the art and practice. In *Onlinepubs Trb*. Washington, D.C.: paper presented to Adapting freight models and traditional freight data programs for performance measurement Workshop. Retrieved from http://onlinepubs.trb.org/onlinepubs/conferences/2013/Freight/Holguin-Veras.pdf

Holguin-Veras, J., Encarnacion, T., Ramirez-Rios, D., He, X. (Sean), Kalahasthi, L., Perez-Guzman, S., … Gonzalez-Calderon, C. A. (2020). A Multiclass Tour Flow Model and lts Role in Multiclass Freight Tour Synthesis. *Transportation Science*. https://doi.org/10.1287/trsc.2019.0936

Holguín-Veras, J., Hodge, S., Wojtowicz, J., Singh, C., Wang, C., Jaller, M., … Cruz, B. (2018). The New York city off-hour delivery program: a business and community-friendly sustainability program. *Interfaces*, *48*(1), 70–86. https://doi.org/10.1287/inte.2017.0929

Holguin-Veras, J., Jaller, M., Destro, L., Ban, X., Lawson, C., & Levinson, H. (2011). Freight generation, freight trip generation, and perils of using constant trip rates. *Transportation Research Record: Journal of the Transportation Research Board*, *2224*, 68–81. https://doi.org/10.3141/2224-09

Holguin-Veras, J., Jaller, M., Sanchez-Diaz, I., Campbell, S., & Lawson, C. (2014). Freight Generation and Freight Trip Generation Models. In *Modelling Freight Transport* (pp. 43–63). Elsevier Inc. https://doi.org/10.1016/B978-0-12-410400-6.00003-3

Holguin-Veras, J., & Thorson, E. (2003). Practical implications of modeling commercial vehicle empty trips. *Transportation Research Record*, (1833), 87–94. https://doi.org/10.3141/1833-12

Krisztin, T. (2018). Semi-parametric spatial autoregressive models in freight generation modeling. *Transportation Research Part E: Logistics and Transportation Review*, *114*, 121–143. https://doi.org/10.1016/j.tre.2018.03.003

Li, T., Sun, H., Wu, J., Gao, Z., Ge, Y., & Ding, R. (2019). Optimal urban expressway system in a transportation and land use interaction equilibrium framework. *Transportmetrica A: Transport Science*, *15*(2), 1247–1277. https://doi.org/10.1080/23249935.2019.1576798

Lim, Y., & Kim, H. (2016). A combined model of trip distribution and route choice problem. *Transportmetrica A: Transport Science*, *12*(8), 721–735. https://doi.org/10.1080/23249935.2016.1166171

López-Ospina, H. (2013). Modelo de maximización de la entropía y costos generalizados intervalares para la distribución de viajes urbanos. *Ingenieria y Universidad*, *17*(2), 390–407. Retrieved from https://www.redalyc.org/articulo.oa?id=47728826008

López-Ospina, H., Cortés, C. E., Pérez, J., Peña, R., Figueroa-García, J. C., & Urrutia-Mosquera, J. (2021). A maximum entropy optimization model for origin-destination trip matrix estimation with fuzzy entropic parameters. *Transportmetrica A: Transport Science*, 1–38. https://doi.org/10.1080/23249935.2021.1913257

Luhandjula, M. K. (2015). Fuzzy optimization: Milestones and perspectives. *Fuzzy Sets and Systems*, *274*, 4–11. https://doi.org/10.1016/j.fss.2014.01.004

Majidi, S., Hosseini-Motlagh, S. M., Yaghoubi, S., & Jokar, A. (2017). Fuzzy green vehicle routing problem with simultaneous pickup-delivery and time windows. *RAIRO - Operations Research*, *51*(4), 1151–1176. https://doi.org/10.1051/ro/2017007

Moreno-Palacio, D. P., Gonzalez-Calderon, C. A., Posada-Henao, J. J., Lopez-Ospina, H., & Gil-Marin, J. K. (2022). Entropy-Based Transit Tour Synthesis Using Fuzzy Logic. *Sustainability*. https://doi.org/10.3390/su142114564

Morlok, E. K., Schofer, J. L., Pierskalla, W. P., Marsten, R. E., Agarwal, S. K., Stoner, J. W., … Spacek, D. T. (1973). *Development and Application of a Highway Network Design Model* (Vol. 1,2).

Noriega, Y., & Florian, M. (2007). *Multi-Class Demand Matrix Adjustment*. Canada: CIRRELT.

Ortuzar, J. de D., & Willumsen, L. G. (2011). *Modelling Transport* (Fourth Edi). John Wiley & Sons, Ltd. https://doi.org/10.1002/9781119993308

Sakawa, M. (1983). Interactive Fuzzy Decision Making for Multiobjective Linear Programming Problems and its Application. *IFAC Proceedings Volumes*, *16*(13), 295–300. https://doi.org/10.1016/S1474-6670(17)62049-4

Sakawa, M., & Yano, H. (1987). An Interactive Fuzzy Satisficing Method for Multiobjective Linear Programming Problems with Fuzzy Parameters. *IFAC Proceedings Volumes*, *20*(9), 437–442. https://doi.org/10.1016/S1474-6670(17)55745-6

Sanchez-Diaz, I., Holguin-Veras, J., & Ban, X. (Jeff). (2015). A time-dependent freight tour synthesis model. *Transportation Research Part B: Methodological*, *78*, 144–168. https://doi.org/10.1016/j.trb.2015.04.007

Sanchez-Diaz, I., Holguin-Veras, J., & Wang, X. (2016). An Exploratory Analysis of Spatial Effects on Freight Trip Attraction. *Transportation*, *43*(1), 177–196. https://doi.org/10.1007/s11116-014-9570-1

Tavasszy, L. A., & Stada, J. E. (1994). THE IMPACT OF DECREASING BORDER BARRIERS IN EUROPE ON FREIGHT TRANSPORT BY ROAD. In *Transportation Research Forum, 36th Annual Conference, 2 volumesTransportation Research Forum*.

Wang, Q., & Holguin-Veras, J. (2009). Tour-Based Entropy Maximization Formulations of Urban Commercial Vehicle Movements. In *2009 Annual Meeting of the Transportation Research Board* (pp. 1–22). New York: 2009 Annual Meeting of the Transportation Research Board. Retrieved from https://abstracts.aetransport.org/

Wardrop, J. G. (1952). Some Theoretical Aspects of Road Traffic Research. *ROAD ENGINEERING DIVISION MEETING*.

Wigan, M., Browne, M., Allen, J., & Anderson, S. (2002). Understanding the growth in service trips and developing transport modelling approaches to commercial, service and light goods movements. *Association for European Transport*, 15. Retrieved from http://abstracts.aetransport.org/paper/index/id/1370/confid/8

Willumsen, L. G. (1978a). *Estimation of an OD Matrix from Traffic Counts - A Review*. Leeds, UK: Institute of Transport Studies, University of Leeds. Retrieved from http://eprints.whiterose.ac.uk/2415/

Willumsen, L. G. (1978b). OD matrices from network data: a comparison of alternative methods for their estimation. In *Proc. of PTRC Summer Annual Meeting 1978 Seminar in Transport Models*.

Wilson, A. G. (1969). The use of entropy maximising models, in the theory of trip distribution, mode split and route split. *Journal of Transport Economics and Policy*, *3*(1), 108–126. Retrieved from https://www.jstor.org/stable/20052128

Wilson, A. G. (1970). Inter-regional Commodity Flows: Entropy Maximizing Approaches. *Geographical Analysis*, *2*(3), 255–282. https://doi.org/10.1111/j.1538-4632.1970.tb00859.x

You, S. I., & Ritchie, S. G. (2019). Tour-Based Truck Demand Modeling with Entropy Maximization Using GPS Data. *Journal of Advanced Transportation*, *2019*. https://doi.org/10.1155/2019/5021026

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

Zadeh, L. A. (1968). Fuzzy algorithms. *Information and Control*, *12*(2), 94–102. https://doi.org/10.1016/S0019-9958(68)90211-8

Zadeh, L. A. (1999). Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*, *100*(6), 9–34. https://doi.org/10.1016/S0165-0114(99)80004-9

Zhang, Y., & Ye, Z. (2008). Short-term traffic flow forecasting using fuzzy logic system methods. *Journal of Intelligent Transportation Systems: Technology, Planning, and Operations*, *12*(3), 102–112. https://doi.org/10.1080/15472450802262281

Zimmermann, H.-J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, *1*(1), 45–55. https://doi.org/10.1016/0165-0114(78)90031-3

3. Entropy-based Transit Tour Synthesis using Fuzzy Logic

https://doi.org/10.3390/su142114564

Abstract

This paper presents an entropy-based Transit Tour Synthesis (TTS) using Fuzzy Logic (FL) based on Entropy Maximization (EM). The objective is to obtain the most probable transit (bus) tour flow distribution in the network based on traffic counts. These models consider fixed parameters and constraints. The costs, traffic counts, and demand for buses vary depending on different aspects (e.g., congestion), which are not captured in detail in the models. Then, as the FL can be included in modeling that variability, it allows obtaining solutions where some or all the constraints do not entirely satisfy their expected value but are close to it due to the flexibility this method provides to the model. This optimization problem was transformed into a bi-objective problem when the optimization variables were the membership and entropy. The performance of the proposed formulation was assessed in the Sioux Falls Network. It was created an indicator (Δ) which measure the distance between the model's obtained solution and the requested value or target value. It was calculated for both production and volume constraints. The indicator allowed to observe that the flexible problem (*FL Mode*) had smaller Δ values, than the ones obtained in the No FL models. These results prove that the inclusion of the FL and EM approaches to estimate bus tour flow, applying the synthesis method (traffic counts), improves the quality of the tour estimation.

Keywords: *Transit tour synthesis; entropy maximization; fuzzy logic; Transit tours; bi-objective optimization, traffic counts.*

3.1. Introduction

People move through the city's streets using various types of transportation. Collective public transportation is one of the key players in doing this (buses). However, the allocation of the urban transportation network in a city is decided mainly by planning decision-makers and is based on both passenger demand and street capacity.

The importance of public transportation is undeniable. It impacts people's daily lives. When the public transportation system works, the city work, and people's quality of life grows when, for instance, it improves access to jobs, making it faster (Nagy et al., 2019). The transit system is useful and necessary for transporting people in urban areas. This covers an important segment of public transportation. The concept is wide, and "*it ranges from several bus modes to tram, light rail transit (LRT), commuter rail and metropolitan rail (metro) systems."* (Wirasinghe et al., 2013). According to Wirasinghe et al. (2013), the planification process for urban public buses—and transit in general—seeks to provide a good level of service and maintain a fair, cost-effective factor for both the transit operator and the passengers.

In the race to alleviate congestion, several strategies have been proposed and executed (i.e., the extension of road capacity by road construction, optimization of traffic control, etc.), all of them addressed to optimize completely, or even partially, the total flow of vehicles in the system. However, those actions could have consequences as the flow increases, precisely for improving the system. When the traffic reaches its capacity, the problem starts again, which indicates that another strategy is necessary. Public transportation appears to hold the key—or at least part of it—to changing demand behavior, which would help with the congestion issue. It is a better means of transportation. The assignment problem in the metropolitan public transportation system is then given priority. To do this, it is necessary to develop solutions and create an effective bus transit system for urban areas in terms of their physical, social, and economic structures, (Fan & Machemehl, 2006; Soehodo & Koshi, 1999).

There are two main approaches in transit modeling: the frequency-based approach and the schedule-based approach (De Cea & Fernández, 1993; Lam et al., 1999; Spiess & Florian, 1989; Wu et al., 1994). In that sense, Lam et al. (2003) state the different conditions for using each approach. In the case of the frequency-based approach, intracity bus services fix better because high frequencies and low punctuality characterize them. In contrast, the schedule-based approach is especially appropriate for intercity train services with low frequency and very high punctuality.

Transit networks have specific characteristics such as points or nodes, which have different natures: origin nodes represent the trip start nodes, stop nodes represent stations, and destination nodes represent trip end nodes. Then, it makes sense to recognize that maximizing street usage is essential, and the two systems boarded significantly impact the overall system. To do so, the entropy maximization technique is quite useful and can be practiced in transportation modeling. The entropy maximization (EM) approach has been widely used, as an optimization technique, in passenger and freight analysis, both on OD synthesis and tour synthesis distribution modeling. *"While the problem of estimating OD matrices from traffic counts in road networks has been extensively studied in literature, little attention has been given to the transit passenger OD estimation problem"* (Lam et al., 2003). This statement explains why the literature misses more papers on this transit research area, which could contribute to solving the transit OD estimation problem. Even though, to Lam et al., (2003) there is some remarkable research on the theme.

To the authors' best knowledge, there is a gap in analyzing bus tour flows (transit tour flows) using EM. Even though in transit (buses) systems, EM has been used to obtain OD matrices, it would allow finding the most likely bus tour flows in an urban network using traffic counts. Therefore, the first goal of this paper is to present, for the first time, the entropy-based TTS formulation as a tool that can help estimate tour flow for this system.

The interest in obtaining models also has concerns that the models are a good representation of the real world. Maybe it never will be but make shorter the difference has good chances. The estimation of the impact of the parameters' variability on modeling and the planning transport area could be performed using different methods. Fuzzy logic is a method studied in the context of transportation, for instance, in the estimation of the OD synthesis distribution matrix for passenger cars (López-Ospina et al., 2021), using EM and fuzzy parameters in the constraints, which enables the inclusion of uncertainty brought on by data variability into the model. This implies that constraints have some flexibility, and the results demonstrate that not all constraints are always satisfied, let alone they achieve the 100% of every one of them.

This paper brings a proposed entropy-based transit tour synthesis (TTS) formulation using fuzzy parameter formulation, which, to the author's best knowledge, does not seem to have been developed before. Then, this is the first time developing and presenting it to shed light on this literature gap. Additionally, this research presents a numerical experiment seeking to taste the behavior of the formulation, as a methodology for the respective transport system, beyond the numbers. The formulation was tested in Sioux Falls, and the outcomes allow for the observation of how the variability affects each case.

The paper has 5 sections, being this the first one. Section 2 corresponds to the background of public transportation, EM in transportation modeling, and FL in transportation modeling. Section 3 presents the EM and FL approaches in tour estimation for transit (buses). Section 4 develops the TTS's numerical experiments, using entropy maximization and fuzzy parameters. Finally, Section 5 presents the conclusions and remarks about the research.

3.2. Background

This section provides background information about relevant research literature on transit demand, the characteristics of transit in the transportation system, and methodologies to estimate transportation demand, such as entropy maximization and fuzzy logic.

3.2.1. Transit demand synthesis

Two sets of elements constitute a transit network. On one side are the nodes, which are the stops or stations where passengers can board or leave the system, make transborder, or change vehicles. In addition to nodes, the network has transit lines, which are the segments that join the stations (Lam et al., 1999). A sequence of nodes defines a transit tour or transit route joined by arcs or links that a transit passenger can follow to travel from node *i* to node *j*. The sequence of nodes describes the transit tour or path, where the first node corresponds to the origin node of the trip, the final node corresponds to the destination, and all the intermediate nodes are the stop or the transfer nodes. The network links that join two consecutive nodes represent the route sections. Thus, the path or transit route can also be defined as a sequence of those route sections (Lam et al., 2003). After these definitions, the transit tour flows can be defined as the number of buses that follow every tour in the network.

Buses route network design focuses on optimizing several objectives representing the efficiency of public transportation networks under operational and resource constraints, such as the number and length of public transportation routes, allowable service frequencies, and the number of available buses (Konstantinos & Matthew, 2009) s. The passenger flows guide route layout design: routes are established to provide a direct or indirect connection between locations and areas that produce and attract demand for transit travel, such as residential and activity-related centers. Calculations are based on expected passenger volumes along routes empirically estimated or by applying transit assignment techniques under frequency requirement constraints (minimum and maximum allowed frequencies guaranteeing safety and tolerable waiting times, respectively), desired load factors, fleet size, and availability. The transit tour flows are defined by the frequency and fleet size, as mentioned. They change if the passenger demand changes. The coincidence of many routes in a route section (or link) connecting a certain set of stations is another aspect that affects the tour flows. When the individual flows are combined, this causes the flow in those particular parts to grow (Lam et al., 1999, 2002, 2003; Nielsen, 2000).

Abdulaal and LeBlanc (1979), proposed analyzing transit tours in Sioux Falls (SF), a network used as a reference in many transport studies before. They proposed 5 bus lines and located the stop points at regular intervals of 600 m to evaluate methods related to combining modal split and equilibrium assignment. Those lines were also used by Chakirov & Fourie (2014), who introduced some modifications to fix it better to their objective, which was to present a scenario with dynamic demand and an integrated public transportation system.

Usually, the transit demand synthesis has been estimated using the OD pair method, obtaining the OD trips distribution matrix. Although tours, rather than OD trips, may provide a more detailed account of the bus rides, that is precisely what they do. They follow a list of nodes that correspond to the stations of the bus system in real life, with each segment connecting one station to the next. Because bus tours estimation could record more aspects of transportation systems' regular operations, they may be advantageous. Additionally, the number of buses that follow each tour equals the flow of transit or bus tours. As a result, tour flows could be used to estimate transit demand synthesis.

3.2.2. EM in transportation modeling

A very popular approach used to estimate OD matrixes is EM, which is based on traffic counts, and the first one who used it was (Nielsen, 2000). According to Willumsen (1978a) "*EM is a way to find the most likely origin destination matrix compatible with the available set of link counts. In other words, the idea is to 'exploit' all the information contained in the matrix observed link flows to determine the most likely OD compatible with them.*"

EM is not the only approach used in OD estimation. There are some others like maximum likelihood or generalized least squares methods, just to mention some of them. Although the entropy-based technique offers better solution properties like convexity and uniqueness and requires less data because it does not require statistical data like variance-covariance matrixes, which are typically challenging to obtain, it has been widely used in practice (Wong et al., 2005).

EM has been used as a method in passenger car demand synthesis because of the estimation of OD matrixes based on traffic counts (Willumsen, 1978a, 1982, 1984). These models, based on an EM framework, must be validated using a comprehensive data set. Willumsen (1982, 1984), show the validation of models where congestion plays a role in route choice. These models make possible the use of relatively inexpensive traffic counts to update and estimate trip matrixes under several conditions.

Abrahamsson (1998) considered that the OD matrix can be estimated using traffic counts on links in the transport network and other available information, given that the EM method requires minimum information. EM is important as a modeling approach in several vehicle classes that demand synthesis.

Beyond passenger car demand modeling, EM has been used for estimating freight demand OD synthesis (ODS) (Ortuzar & Willumsen, 2011; Wong et al., 2005) the estimation of freight demand tour synthesis (Gonzalez-Calderon, 2014; Gonzalez-Calderon & Holguín-Veras, 2019; Holguin-Veras et al., 2015; Wang & Holguin-Veras, 2009); multi-class demand ODS including passengers and freight, represented into five major vehicle classes, by Wong et al. (2005), and multi-class demand tour synthesis for private cars and trucks with a static time approach (Gonzalez-Calderon, 2014) and with a dynamic time approach (Sanchez-Diaz et al., 2015). All these studies consider the treatment of congestion effects in the network. The optimization model applying EM for origin-destination trip matrix estimation with fuzzy entropic (López-Ospina et al., 2021), the estimation of freight tour flows using fuzzy entropic (Moreno-Palacio et al., 2022), estimating OD matrixes under travel demand constraints (Sun et al., 2019), modeling interregional transportation (Velichko, 2016), modeling taxi trip distributions (Tang et al., 2018), input-output analysis (Hewings & Fernandez-Vazquez, 2019), and modeling highway traffic flows (Hu et al., 2020) are some other works developed using EM.

In terms of transit, Lam et al. (2003) and Nguyen & Pallottino (1988) , proposed a maximum entropy estimator from the frequency-based approach and used "*a simple constrained maximum likelihood model with Poisson distributions*" to finally obtain *"a model for updating passenger OD matrix on a transit network."* Lam et al. (2003) , Wong & Tong (1998), and Nuzzolo & Crisalli (2001), used a schedule-based approach to obtain a maximum entropy estimator of the time-dependent O–D matrix in a transit network, in the former case, and generalized least square (GLS) estimator for time-varying O-D matrixes derived from time-varying onboard passenger counts, in the later. The methods used in the cited publications are better suited for uncongested transit networks, assuming that the route selections are independent of O-D needs, even though the conclusions are relevant to the research area. Although, the congestion effects must be included in the O–D estimation because route choices depend on route travel times, which, simultaneously, are affected by the O–D demands (Fisk, 1989). In that sense, Lam et al. (2003) estimated the transit passenger O–D matrixes using a frequency-based transit assignment model with a bi-level program. In the upper level, the objective function used the GLS expression, and in the lower level, it could use a Logit or Probit model to define the transit assignment. To the authors´ best knowledge, EM has not been used to estimate transit tours, i.e., TTS formulation developed in this paper.

3.2.3. Fuzzy logic and membership functions in transportation modeling

Often, it is difficult or impossible to classify certain world objects in the precise class they belong to. This is because an object may partially belong to several classes. This condition implies a grade of imprecision or uncertainty related to the variable's nature. Using a membership function, the *fuzzy sets* are tools that enable the definition of a continuum grade of membership—a range between zero and one—of the objects to a particular class of objects (constraints) (Zadeh, 1965, 1968). So, far, fuzzy logic (FL) has been applied in linear programming, multi-objective linear programming, multi-objective nonlinear programming, optimization processes where the objective function and/or the constraints are not exactly bounded, (Bellman & Zadeh, 1970; Hannan, 1981a; Sakawa, 1983; Sakawa & Yano, 1987; Zadeh, 1999; Zimmermann, 1978), and *"a possibility distribution as a fuzzy restriction, which acts as an elastic constraint on the values that may be assigned to a variable"*(Zadeh, 1999).

Fuzzy programming allowed the application of fuzzy parameters to transportation problems as a multi-objective problem (Bellman & Zadeh, 1970; Bit et al., 1992; Zadeh, 1999), and later Chanas & Kuchta (1996) included the use of fuzzy parameters for cost coefficients. *"Fuzzy entropy gives a measure of fuzziness (ambiguity). Hence, it has got significance in decision-making applications with imprecise (fuzzy) values"*(Aggarwal, 2021).

The entropy-based modeling approach in the distribution problem assumes fixed parameters to constraints and objective function. For example, Lopez-Ospina (2013) found the O-D matrix based on the EM but considered a variation interval to the cost. Later, López-Ospina et al. (2021) extended this model to a bi-objective model for urban trip passengers, with double constraints (in the origins and destinations), maximizing the entropy to find the distribution matrix of urban trip passengers, which minimizes the cost through fuzzy parameters and entropic membership functions.

Recently, Moreno-Palacio et al. (2022), proposed a freight tour synthesis model using fuzzy parameters with an entropy maximization technique, which estimates the most probable freight tour flows in an urban network (SF). With fuzzy parameters, it is possible to incorporate human behavior-related variability in costs, traffic counts, and truck demand into modeling.

Some other problems have been analyzed using the FL approach. For instance, Zhang & Ye (2008) used the FL system to improve the accuracy of forecasting traffic flows. In contrast, Majidi et al. (2017) minimized the fuel consumption of the vehicles and used fuzzy approach for the pickup and delivery to solve the green vehicle routing problem with simultaneous pick-up–delivery and time windows. Graphically, membership functions represent a plane in which the independent variable on the x-axis contains the ranges within which the value of the constraint can move, and the dependent variable on the y-axis shows the level or percentage of compliance reached dependent on the constraint's value on the x-axis. The accomplishment range is [0,1], that is 0%–100% respectively (Zadeh, 1965). The simplest membership function geometric shape is a triangular function, which is defined by three points that correspond to the triangle's three vertexes: the minimum value of the constraint or start point, the intermediate value or requested value (correspondent to the upper vertex of the triangle), and the maximum value of the constraint, also known as the endpoint. The membership functions could also respond to the approximate shape of the parameter distribution observed in a histogram, representing the variability in data. Thus, it could also have a trapezoidal shape with four points on the x-axis (minimum value, value 1, value 2, maximum value) and even an ellipsoidal shape.

To the authors' best knowledge, there is a gap in the transit demand synthesis, and that is the TTS model formulation, which is the most probable distribution of the tour's flows. In other words, the number of buses on every tour in the network. The current research presents the TTS formulation development using EM in the next sections. Moreover, the authors described the TTS using fuzzy logic to obtain the most probable tour flows with fuzzy parameters, which gives flexibility to some constraints.

3.3. Estimation of transit tours based on entropy maximization and fuzzy logic

As mentioned in the previous section, transit demand synthesis is an efficient and inexpensive method for estimating transit flows. This technique uses traffic counts for the demand estimations instead of traditional surveys that result in expensive and disruptive methods (Ortuzar & Willumsen, 2011; Tamin & Willumsem, 1992; Willumsen, 1978b). Transit demand synthesis could be analyzed under two different approaches, and in any case, is the modeler, the one who chooses which approach is the best to be used, according to the purpose of the model. One could be the trip generation from an origin to a destination, resulting in a transit OD synthesis matrix reproducing, in the best way possible, the traffic counts used in the calibration (Holguin-Veras et al., 2020). The other method to analyze the flow is the TTS, where a public transportation vehicle follows a sequence of nodes (stops) to pick up and drop off people.

The transit demand has been modeled by applying the OD synthesis technique to get the OD distribution matrixes, which has been useful. Even though transit behavior could also be described as a sequence of nodes visited, which are the stations or stops and are joined by links, this behavior corresponds to a tour behavior. Then, transit distribution can be estimated as the TTS, developed by applying the EM approach. To the authors' best knowledge, the TTS formulation developed in this paper is first that estimates transit tours using the EM approach. In TTS and FTS, the main result is the most probable distribution of the buses in the different tours (tours flows).

Thus far, the entropy-based modeling approach used in the distribution problem has assumed fixed parameters in both constraints and objective functions. The use of fuzzy logic in these optimization problems allows for estimating the parameters' variability. This information could be a way to include the impact of the uncertainty caused by that variability on the planning transport area. In the case of passenger cars studied by López-Ospina (2013), who found the O-D matrix based on the EM but considering a variation interval to the cost. This model was later extended to a bi-objective model for urban trip passengers with double constraints—in the origins and destinations—(López-Ospina et al., 2021), maximizing the entropy to find the distribution matrix, which minimizes the cost through fuzzy parameters and entropic membership functions. The FTS is just already presented by Moreno-Palacio et al. (2022) and has been novel in the freight tour synthesis modeling process. Since this study developed the TTS formulation and is the first to construct the entropy-based TTS using FL to include the uncertainty created by the variability of the parameters, if it exists, it makes sense that TTS has not yet been developed using fuzzy parameters.

This section is dedicated to transit demand synthesis, which exposes the entropy-based TTS formulation. Then, it presents the development of the entropy-based TTS using fuzzy parameters formulation. To the authors' best knowledge, the last two formulations (the entropy-based TTS formulation with fixed and fuzzy parameters) are developed for the first time in the literature.

3.3.1. Transit tour synthesis formulation (TTS) using EM

This formulation seeks to obtain, from traffic counts, the most probable bus flow distribution, in other words, the number of buses that use the tour r (tour flows). To do so, this paper develops an EM-based formulation for TTS. The EM approach is used to go on with this optimization program. The formulation addresses the flow of transit vehicles (buses) that follow the same sequences of points (stations or stops). The reality is that some segments of those sequences will overlap several tours, and the TTS formulation works for knowing how the most probable distribution of the bus's flows is. Here, entropy-based TTS formulation must prove the convexity and uniqueness of the solution to find the optimal solution. The formulation development is presented below:

$$Max\ \mathrm{S}(t_r) = \frac{T_b\,!}{t_1\,!(T_b - t_1)!} \cdot \frac{(T_b - t_1)!}{t_2\,!(T_b - t_1 - t_2)!} \cdots = \frac{T_b\,!}{\prod_{r=1}^{R} t_r\,!} \tag{1}$$

Subject to:

$$Or_i = \sum_{(r=1)}^{R} t_r \delta_{ri} \qquad \forall i \in \{1, 2, ..., N\} \tag{2}$$

$$Cr = \sum_{(r=1)}^{R} C_r t_r \tag{3}$$

$$Vr_a^b = \sum_{(r=1)}^{R} t_r \delta_{ra} \qquad \forall a \in \{1, 2, ..., Q\} \tag{4}$$

$$t_r \geq 0 \qquad \forall r \in \{1, 2, ..., R\} \tag{5}$$

Where,

S: Number of ways that bus tours could be arranged

T_b*: Total number of bus tour flows in the network*

r: Node sequence (tour), an ordered set of the nodes visited by a bus, from the start node until the end node;

t_r*: Number of bus journeys following tour r*

R: Total number of possible bus tours in the system

O_r*: Bus Trips production constraint, including origin and destination (the same point);*

Cr: Total cost in bus travel time (impedance of the system);

N: Total number of nodes in the System

Q: Total number of links with traffic counts in the system

δ_{ri}*: Binary variable indicating whether node i is in bus tour r. Is equal to 1 if node i is in bus tour m, 0 otherwise;*

C_r*: Impedance of bus tour r, associated with travel and handling in the bus tour r.*

δ_{ra}*: Binary variable indicating whether bus tour r uses link a. Is equal to 1 if link a is in bus tour r, 0 otherwise;*

Vr_a^b*: Observed bus traffic count in link a*

The objective function (OF) aim is to maximize the entropy, which allows knowing the most likely way to distribute bus tours. In constraints, there are four groups. The first one (Equation (2)) is related to bus tour generation, which occupies just one point in the case of tours, given that *O* and *D* are the same nodes. The second group of constraints expresses the total impedance in the system (Equation (3)), in this case, the impedance of bus tours. The third group (Equation (4)) refers to the traffic counts or the volume of buses. The last group (Equation (5)) declares the non-negativity constraints. The OF was rewritten based on the Wilson's work (1969). Knowing that the logarithm function is a crescent function, it is enough to take the logarithm on both sides and maximize the logarithm of the function $S(t_r)$, which is equivalent to maximizing the origin function. The rewritten OF is:

$$Max\ Z' = \ln(S) = \ln(T_b!) - \sum_{r=1}^{R} \ln t_r! \tag{6}$$

Given that the first term of the function $\ln(T_b!)$ is constant, if the function is derived, it will be zero, so it can be eliminated from the OF. So now it is possible to state that maximizing $\ln(S)$ is the same as minimizing $-\ln(S)$, this is $MaxZ' = \ln(S) = MinZ'' = -\ln(S)$; and the OF obtained is:

$$Min\ Z'' = \sum_{r=1}^{R} \ln t_r\,! \tag{7}$$

Stirling's approximation is used, where $Ln(x!) \simeq xLnx - x$, permitting expressing $Ln(t_r!)$ as follows:

$$ln(t_r!) \simeq t_r\, lnt_r - t_r \tag{8}$$

Thus, the OF and the constraints are presented below:

$$Min\ Z = \sum_{r=1}^{R} (t_r \ln t_r - t_r) \tag{9}$$

Subject to

$$Or_i = \sum_{r=1}^{R} t_r \delta_{ri} \qquad \forall i \in \{1,2,\dots,N\} \qquad (\lambda_i) \tag{10}$$

$$Cr = \sum_{r=1}^{R} C_r t_r \qquad (\beta) \tag{11}$$

$$Vr_a^b = \sum_{r=1}^{R} t_r \delta_{ra} \qquad \forall a \in \{1,2,\dots,Q\} \qquad (\gamma_a) \tag{12}$$

$$t_r \geq 0 \qquad \forall r \in \{1,2,\dots,R\} \tag{13}$$

where,

λ_i*: Lagrange multiplier to trip production ith constraint*

β*: Lagrange multiplier to the total bus tours impedance constraint*

γ_a*: Lagrange multiplier to the observed bus traffic counts in link a*

First/Second-order conditions

The formulation should demonstrate that it accomplishes the first-order conditions—KKT conditions (Karush-Kuhn-Tucker)—and the second-order conditions, which prove that the mathematical problem is convex with a unique solution. *"The evaluation of the First-Order conditions begins with the Lagrangeans. Because the constraints are linear, it is sufficiently prove that the objective function is convex for the solution from the KKT first-order conditions to be global optimal"* (Wang & Holguin-Veras, 2009).

a. Proof of first-order conditions

The Lagrange function formulation is shown in Eq. (14):

$$L(t,\lambda,\beta,\gamma)=\sum_{r=1}^{R}(t_r \ln t_r - t_r)+\sum_{i=1}^{N}\lambda_i\left(Or_i-\sum_{r=1}^{R}t_r\delta_{ri}\right)+\beta\left(Cr-\sum_{r=1}^{R}C_r t_r\right)+\sum_{a=1}^{Q}\gamma_a\left(Vr_a^b-\sum_{r=1}^{R}t_r\delta_{ra}\right) \quad (14)$$

Taking the partial derivative with respect to the number of bus tours t_r, is:

$$\frac{\partial L(t,\lambda,\beta,\gamma)}{\partial t_r}=\ln t_r-\sum_{i=1}^{N}\lambda_i\delta_{ri}-\beta C_r-\sum_{a=1}^{Q}\gamma_a\delta_{ra} \quad ; \quad \forall r \quad (15)$$

And $\ln S(t_r)$ takes its maximum value when $\frac{\partial L}{\partial t_r}=0$

Taking the partial derivative with respect to the number of bus tours t_r, is $(t^*,\lambda^*,\beta^*,\gamma^*)$ represents the optimal solution for the model. The solution is as follows below:

$$t_r^*\frac{\partial L(t^*,\lambda^*,\beta^*,\gamma^*)}{\partial t_r^*}=0, \qquad \forall r\in\{1,2,...,R\} \quad (16)$$

$$\frac{\partial L(t^*,\lambda^*,\beta^*,\gamma^*)}{\partial t_r^*}\geq 0 \qquad \forall r\in\{1,2,...,R\} \quad (17)$$

$$t_r\geq 0 \qquad \forall r\in\{1,2,...,R\} \quad (18)$$

Replacing Equation (15) in Equation (16) and Equation (17), then, the first-order conditions are as follows:

$$t_r^*\left(\ln t_r^* - \sum_{i=1}^{N}\lambda_i^*\delta_{ri} - \beta^* C_r - \sum_{a=1}^{Q}\gamma_a^*\delta_{ra}\right) = 0 \quad ; \quad \forall r \in \{1,2,...,R\} \tag{19}$$

$$\ln t_r^* - \sum_{i=1}^{N}\lambda_i^*\delta_{ri} - \beta^* C_r - \sum_{a=1}^{Q}\gamma_a^*\delta_{ra} \geq 0 \quad ; \quad \forall r \in \{1,2,...,R\} \tag{20}$$

$$Or_i = \sum_{r=1}^{R} t_r^*\delta_{ri} \quad ; \quad \forall i \in \{1,2,...,N\} \tag{21}$$

$$Cr = \sum_{r=1}^{R} C_r t_r^* \tag{22}$$

$$Vr_a^b = \sum_{r=1}^{R} t_r\delta_{ra} \quad ; \quad \forall a \in \{1,2,...,Q\} \tag{23}$$

$$t_r \geq 0 \quad \forall r \in \{1,2,...,R\} \tag{24}$$

Now the Equation (19) is rewritten as its equivalent:

$$t_r^* = \begin{cases} 0 \text{ if } \ln t_r^* - \sum_{i=1}^{N}\lambda_i^*\delta_{ri} - \beta^* C_r - \sum_{a=1}^{Q}\gamma_a^*\delta_{ra} \neq 0 \text{ or if } t_r^* \neq \exp\left(\sum_{i=1}^{N}\lambda_i^*\delta_{ri} - \beta^* C_r - \sum_{a=1}^{Q}\gamma_a^*\delta_{ra}\right) \\ \text{or} \\ \exp\left(\sum_{i=1}^{N}\lambda_i^*\delta_{ri} - \beta^* C_r - \sum_{a=1}^{Q}\gamma_a^*\delta_{ra}\right) \text{ if } t_r^* \neq 0 \end{cases} \tag{25}$$

Using Equation (25) and Equation (20) is obtained the Equation (26):

$$t_r^* \geq \exp\left(\sum_{i=1}^{N}\lambda_i^*\delta_{ri} + \beta^* C_r + \sum_{a=1}^{Q}\gamma_a^*\delta_{ra}\right) > 0 \tag{26}$$

The latest equation shows that the optimal bus flows are greater than zero or positive. This means that Equation (27) represents the optimal solution, and it is expressed as

$$t_r^* = \exp\left(\sum_{i=1}^{N} \lambda_i^* \delta_{ri} + \beta^* C_r + \sum_{a=1}^{Q} \gamma_a^* \delta_{ra}\right) \quad (27)$$

This equation means that the quantity of bus tour flows following a tour is an exponential function of the Lagrange multipliers related to the trip generation of all nodes that comprise the bus tour, the bus tour cost, and the bus traffic counts observed in the links.

λ_i represents the Lagrange multiplier associated with node *i*, which belongs to a tour, and quantifies the effects of tour production at that node. Lagrange multiplier β quantifies the effects of the impedance or cost variable and γ_a is the Lagrange multiplier related to the effects of the observed bus traffic counts in link *a*.

b. <u>Second-order conditions</u>

Accomplishing this condition ensures the convexity of the OF and the uniqueness of the optimal bus tour flow solution. Calculating the Hessian of the OF is necessary, which corresponds to the second derivative' as presented in Equation (28).

$$\frac{\partial Z^2(t)}{\partial(t_i)\partial(t_j)} = \frac{1}{t_i} \quad \forall i, j \in \{1, 2, ...N\} \qquad \text{for } i = j, \quad 0 \text{ otherwise} \quad (28)$$

In this problem, the Hessian obtained is positive definite, then the second-order condition is also satisfied; therefore, the function is convex and has a unique solution.

3.3.2. TTS formulation using the fuzzy logic formulation

This paper proposes a formulation for entropy-based TTS with fixed parameters (the previous one) and entropy-based TTS with fuzzy parameters (developed in the current section). Both formulations in transit modeling are novel developments in the tour flow synthesis field, and to the authors' best knowledge, this is the first time they have been proposed.

The proposed formulation objective is to optimize the entropy to obtain the most likely tour flows for buses—transit—applying fuzzy parameters with entropic membership functions, specifically triangular membership function, which is used in the formulation. The x-axis represents values interval containing the solution, which corresponds to the number of buses by the tour in each scenario, whereas the y-axis displays the success of the constraint membership. The membership functions' shape selection, and the maximum and minimum values obeyed the experts' opinions. However, it is irrelevant for this research because the important thing is to include data variability using FL in the model process. It is worth clarifying that in real life data, the intervals used in the membership function are defined by the minimum, and maximum values observed in the traffic counts, whereas those of deterministic modeling correspond to the traffic counts' average value. This entropy-based TTS model with fuzzy parameters follows the Gonzalez-Calderon and Holguin-Veras (2019), López-Ospina et al. (2021), and Moreno-Palacio et al. (2022), formulations.

The fuzzy parameters, in this formulation, are bus tour production, the total cost or impedance in the transit system, and the volume or bus traffic counts; instead, the cost of bus tour continues to be fixed. The result expected from this model is the bus flow by tour (number of buses that follow the tour a).

Then, the triangular parameters are the next:

- Buses Tour production/attraction at node *i*: ($\boldsymbol{Or_i} = (\boldsymbol{Or_i^1}, \boldsymbol{Or_i^2}, \boldsymbol{Or_i^3})$),
- Total cost in the transit system: ($\boldsymbol{Cr} = (\boldsymbol{Cr_1}, \boldsymbol{Cr_2}, \boldsymbol{Cr_3})$), and
- Bus traffic counts at link *a:* ($\boldsymbol{Vr_a} = (\boldsymbol{Vr_a^1}, \boldsymbol{Vr_a^2}, \boldsymbol{Vr_a^3})$).

Being: $\boldsymbol{Or_i^1}, \boldsymbol{Cr_1}, \boldsymbol{Vr_a^1}$ the parameters corresponding to the vertex 1 (the minimum value of the interval in x-axle, and a grade of accomplishment of zero) for production, cost, and volume respectively.$\boldsymbol{Or_i^2}, \boldsymbol{Cr_2}, \boldsymbol{Vr_a^2}$ the vertex 2 or the middle vertex which represents the target value (the requested or target value in x-axle and a grade of accomplishment of 100%), $\boldsymbol{Or_i^3}, \boldsymbol{Cr_3}, \boldsymbol{Vr_a^3}$and the vertex 3 (the minimum value of the interval in x-axle, and a grade of accomplishment of zero).

The model development is below:

$$max \sum_{(r=1)}^{R} t_r - t_r ln(t_r) \tag{29}$$

subject to:

$$\sum_{(r=1)}^{R} t_r \delta_{ri} = (Or_i^1, Or_i^2, Or_i^3), \qquad \forall i \in \{1, 2, ..., N\} \tag{30}$$

$$\sum_{(r=1)}^{R} t_r C_r = (Cr_1, Cr_2, Cr_3) \tag{31}$$

$$\sum_{(r=1)}^{R} t_r \delta_{ra} = (Vr_a^1, Vr_a^2, Vr_a^3), \qquad \forall r \in \{1, 2, ..., R\} \tag{32}$$

where,

R: number of possible routes (tours) in the bus system;

r: Node sequence (or tour), an ordered set of the nodes visited by a bus, from the start node until the end node;

N: total number of nodes in the system;

Q: total number of links with traffic counts in the system;

t_r*: number of buses journeys (tour flow) following node sequence (or tour) r (a listing of the nodes visited), i.e., the number of buses that travel along the same tour;*

Or_i^1, Or_i^2, Or_i^3*: triangular parameters for Bus Tours production*

Cr_1, Cr_2, Cr_3*: triangular parameters for Total cost in the transit system*

Vr_a^1, Vr_a^2, Vr_a^3*: triangular parameters for Bus Traffic counts (volume)*

C_r*: Cost of tour r, associated with travel on the tour;*

δ_{ri}*: a binary parameter equal to 1 if node i is in tour r, equal to 0 otherwise;*

δ_{ra}*: a binary parameter equal to 1 if the tour r uses link a, equal to 0 otherwise.*

The model above, presented in Equation (29) to Equation (32), has been rewritten in Equation (33) to Equation (40). The constraints are represented in the Equation (30) to Equation (32). The Equation (30), representing the buses tour production constraints, is expressed in Equation (34) and Equation (35). Equation (31) is the total cost in the transit system, and is rewritten in Equation (36) and Equation (37). Finally, Equation (32), which is responsible for the model replicating the observed bus traffic counts in the most probable way—the total number of links with traffic counts in the transit system will be less than the total number of links in the network—is rewritten as Equation (38), and Equation (39).

The redefined model is a bi-objective problem, and it is as follows:

$$max(\lambda, \sum_{(r=1)}^{R} t_r - t_r ln(t_r)) \tag{33}$$

subject to:

$$\sum_{(r=1)}^{R} t_r \delta_{ri} \le \lambda Or_i^2 + (1-\lambda) Or_i^3, \qquad \forall i \in \{1,2,...,N\} \tag{34}$$

$$\sum_{(r=1)}^{R} t_r \delta_{ri} \ge (1-\lambda) Or_i^1 + \lambda Or_i^2, \qquad \forall i \in \{1,2,...,N\} \tag{35}$$

$$\sum_{(r=1)}^{R} t_r C_r \le \lambda Cr_2 + (1-\lambda) Cr_3 \tag{36}$$

$$\sum_{(r=1)}^{R} t_r C_r \ge (1-\lambda) Cr_1 + \lambda Cr_2 \tag{37}$$

$$\sum_{(r=1)}^{R} t_r \delta_{ra} \le \lambda Vr_a^2 + (1-\lambda) Vr_a^3, \qquad \forall a \in \{1,2,...,Q\} \tag{38}$$

$$\sum_{(r=1)}^{R} t_r \delta_{ra} \ge (1-\lambda) V_a^1 + \lambda V_a^2, \qquad \forall a \in \{1,2,...,Q\} \tag{39}$$

$$t_r \ge 0 \qquad \forall \mathrm{r} \in \{1,2,...,R\}\ ,\ \ \lambda \in [0,1] \tag{40}$$

The two objectives to optimize in the redefined model described above are to maximize the entropy and lambda (λ). Remember that λ is the minimum membership level of the flexible constraints. The use of fuzzy entropy automatically increases the number of equations because every constraint equation in a deterministic model is replaced by at least three in the fuzzy logic (the three triangular vertices in this case study). The ε - constraint method is used to solve this problem. Then, the mathematical formulation is as follows:

$$max \sum_{(r=1)}^{R} t_r - t_r ln(t_r) \tag{41}$$

subject to:

$$\boldsymbol{\lambda} \geq \boldsymbol{\varepsilon} \quad (42)$$

Following López-Ospina et al., (2021), since it is a bi-objective model, the problem is solved for different values of ε in [0, 1].

3.4. Numerical experiments: TTS using EM-FL

This section estimates transit tours with fuzzy parameters, applying an entropy maximization approach, which can be obtained from secondary data sources (depending on availability). Public transportation tours can be obtained by secondary information or existing information provided, for example, by the Department of Transportation or transit operators. Data used in the model must include, among others, the network specifics as nodes and links, the tour generation for transit, the zoning system, etc.

This numerical research experiment includes the entropy-based fuzzy parameter models. The case study is the known SF network, which was used as a reference in many transport studies. This is not considered a realistic network. However, LeBlanc et al. (1975) introduced it in transportation research, mostly in traffic assignment. Later, Abdulaal & LeBlanc (1979), used this network to introduce the continuous network design problem. The network became a reference for algorithms that work with continuous design problems in the Suwansirikul et al. (1987) work (see, e.g., (Bar-Gera et al., 2013; Friesz et al., 1992; Josefsson & Patriksson, 2007; Lee & Lim, 2002; Luathep et al., 2011; Marcotte & Marquis, 1992; Meng et al., 2001).) The SF network is small-scale, and as Figure 3-1 shows, it comprises 76 bi-directional links and 24 nodes. The network provides a suitable test case, mixing demand with socio-economic and demographic features and becoming a useful tool for transportation research (Chakirov & Fourie, 2014). This experiment seeks to obtain the most probable tour flows for buses.

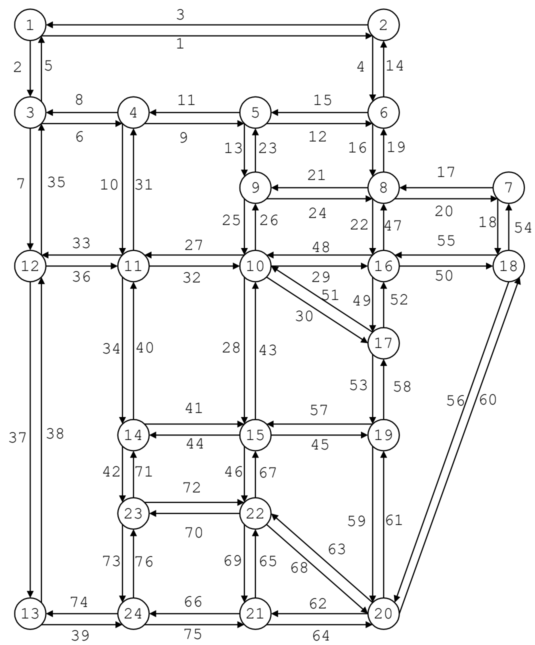


Figure 3-1. **Sioux Falls Network.**

Source: LeBlanc et al., (1975)

The kind of membership function used in this FL model must be specified. Data structure distribution and the expert's validation are considered in this process. The authors chose triangular membership functions to prove their models. Thus, this membership function is more than enough, and it is the best way to try the model because of its simplicity. Due to the function's shape, it should define the three vertices' values, which define the limit values for every constraint, where they can oscillate. The vertical measure indicates the λ value or the accomplishment level achieved using specialized software to run the models and to taste the formulations proposed in this paper. The models were run as continuous problems. The code was written using GAMS® (General Algebraic Modeling System). This program package includes the mathematical model obtained, the flexible parameters, the membership functions, and the input data (costs, volume in links, tours production in nodes).

The TTS using fuzzy parameter formulation was developed in section 3. To test the model, the numerical application also used the SF network. Data were created for this research, which are 15 transit tour sequences (routes). The tours were randomly generated, using 3 nodes as origin nodes, and each node served as the origin of 5 tours. The bus tours seek to cover different types of routes, including rounded, to all the perimeter of the network and other more partitioned routes for specific areas, but be sure that all nodes are in at least one of the bus tours.

In transit tours, the 24 nodes of the SF network represent the stops made in the stations by the buses during the route, except when a node is the departure and arrival points (tour start and end point), which is considered the tour producer node.

This optimization problem was run using the ε approach. Remember that ε values are in the interval [0,1], with steps of 0.05. In the beginning, GAMS results showed ε = 0.4 as the maximum value to obtain feasible solutions. All were unfeasible solutions when the model ran with ε = 0.45 and greater. However, the authors tried using a smaller step size (0.01), from ε = 0.4, just to be closer to the value where the feasible solution stopped. Then the model would be run again for {ε = 0.41, ε = 0.42, ε = 0.43, ε = 0.44}, when was used ε = 0.42, the solution becomes unfeasible, and the same for greater values. Then, the model obtained feasible solutions for values less than ε = 0.42. In other words, as a minimum, a constraint obtains at least 42% of the objective value. This result for transit formulation shows a λ value, λ = 0.42. The results are associated with these data in any case.

Figure 3-2 shows a graphic of entropy values vs. the level of accomplishment obtained. Remember that in the x-axis is the minimum accomplishment level obtained, that is, the constraint that fared the worst. Conversely, the y-axis corresponds to the maximum entropy obtained in each solution.

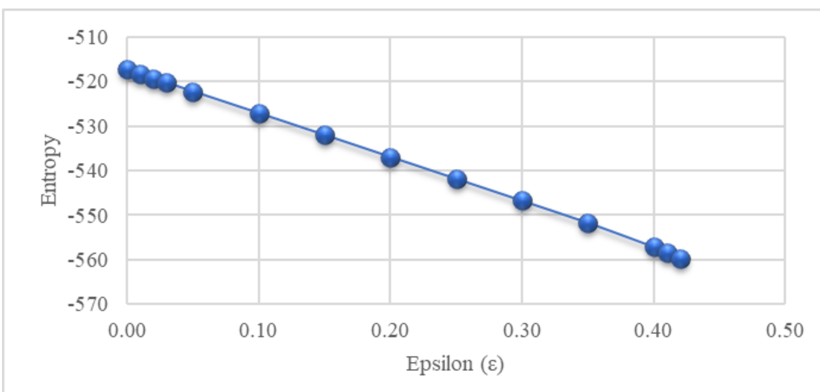


Figure 3-2. Transit Pareto frontier (Entropy vs. Accomplishment level).

The two objectives to maximize in the formulation are entropy and the minimum level of accomplishment, λ. Figure 3-2 shows that the entropy-based TTS formulation using FL is a multi-objective optimization problem since both variables are opposite each other as a battle. With each increment in the accomplishment level, the entropy decreases. That is the proof that exists, in this case, the Pareto frontier for the data used. This result is relevant to this paper's main objective: that the formulations using fuzzy parameters are useful methodologies and results always depend on the input data. When the variability data is high, the λ value is low, and if that variability is smaller, the uncertainty in the model also is, and the λ value increase, that is, the accomplishment level also increases.

Every feasible solution gives the number of buses that use every tour, that is, the buses tour flows. Thus, the flow changes in the tours and for every ε-value. Figure 3-3 shows these changes in the traffic, from the minimum ε-value, until the λ value. In this experiment, it seems that most of the tours show increments between 5% and 10% of the flow, not widely changed, and the data variability could not be very strong. The results also present moderate variability.

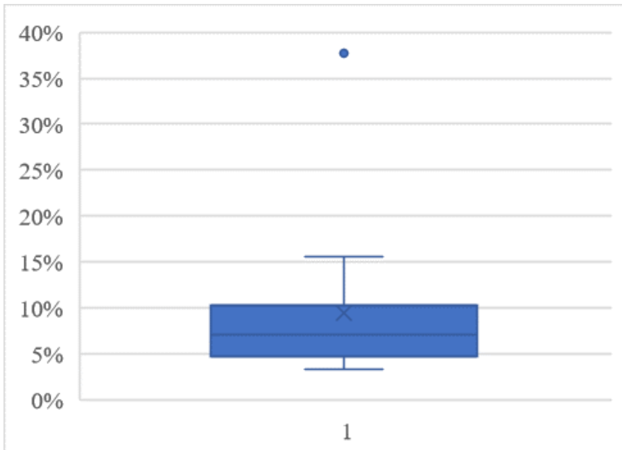


Figure 3-3. Bus traffic changes for each different solution.

Figure 3-4. Shows the changes in the flow in every tour.

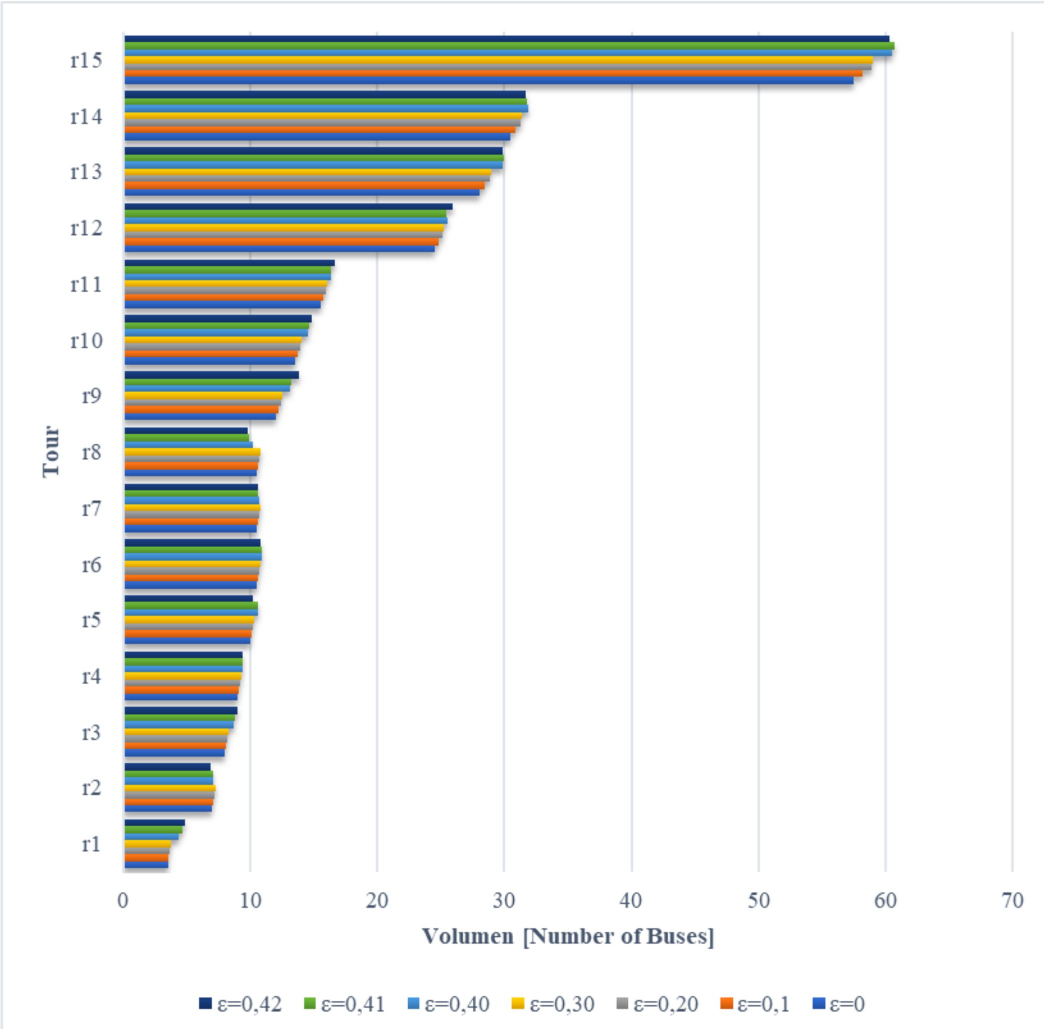


Figure 3-4. Bus tour volumes for different ε values.

The graphic exhibits the number of buses per tour for different ε values. Due to the small amount of transit data used in this experiment, the graphic includes all tours. We can see the changes in every tour, in every solution. It is possible to visualize that the changes are soft, and most keep the tendency in every tour. However, tours 1 and 15 exhibited greater changes than the rest, mainly tour 15, which has increased in bus flow as the level of compliance increases. Tour 8 illustrates the opposite circumstance; when compliance levels rise, the number of buses on tour tends to decline. Including fuzzy parameters means flexibility to the constraints; this influences the results of the solution related to the volume pattern changes of the tours since it prevents identifying those changes tendency.

The results for the different solutions give as output the percentage or level of accomplishment per tour at every ε value evaluated in the model. This can be better understood with a graphic like in Figure 3-5.

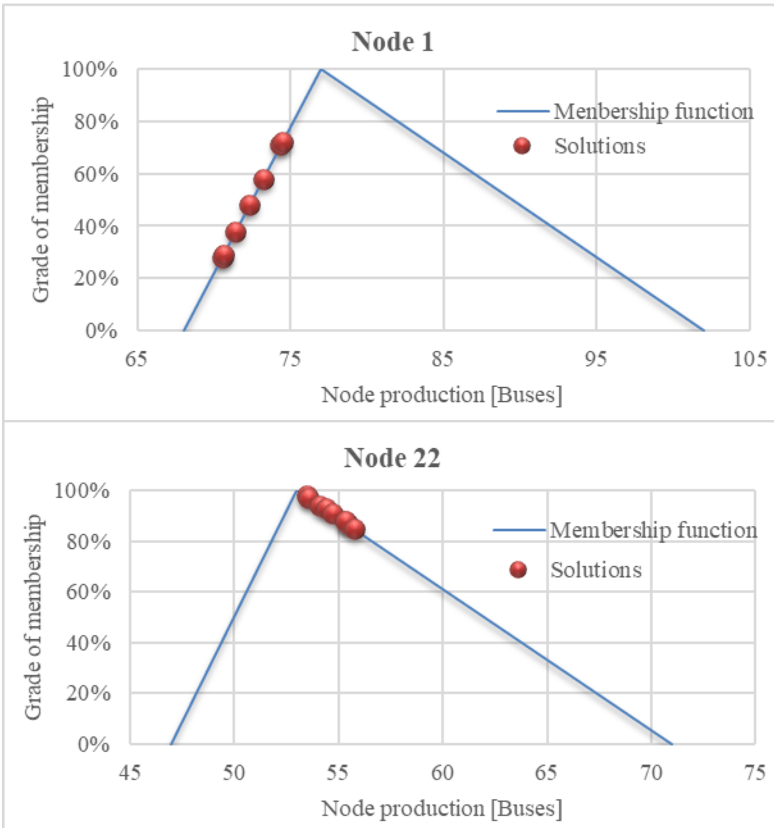


Figure 3-5. TTS - Level (%) of accomplishment in nodes production.

It shows two nodes, Node 1 and Node 22, and their respective triangular membership functions with the solution obtained at every ε value tasted. That is, on the x-axis is the number of buses using the node every tour, and on the y-axis is the respective grades of accomplishment. The results show that levels of accomplishment are related to the node-production twice in every tour. This must be understood in relation to the triangle side on which the solution is. The left side shows that the number of buses obtained is lower than the needed value, while the right side shows that the number of buses obtained is more than the required value. Following this, Figure 3-5 shows that while node 1 tour production is less than required, node 22 produces more bus tours than required. This analysis allows comparing the production per node per tour. In the case of links, the same analysis shows the volume per link per tour and every constraint using fuzzy parameters.

The sider vertices of the triangular membership functions correspond to the extreme production and grade of accomplishment values. This is due, in both cases, the grade of accomplishment is 0% (zero). If the solution is the minimum interval value, the left extreme means that at least a constraint is 0%, and this is the worst case. Others may be in the same condition or better. The solution is specific to the constraint evaluated, independent of the others.

Figure 3-6 and Figure 3-7 represent he percentage of accomplishment for each tour and its distribution for both node-production and link-volume, respectively. Note that the minimum accomplishment value is equal to the ε-value in every solution, whereas the other constraints achieve higher values of accomplishment, even 100%.

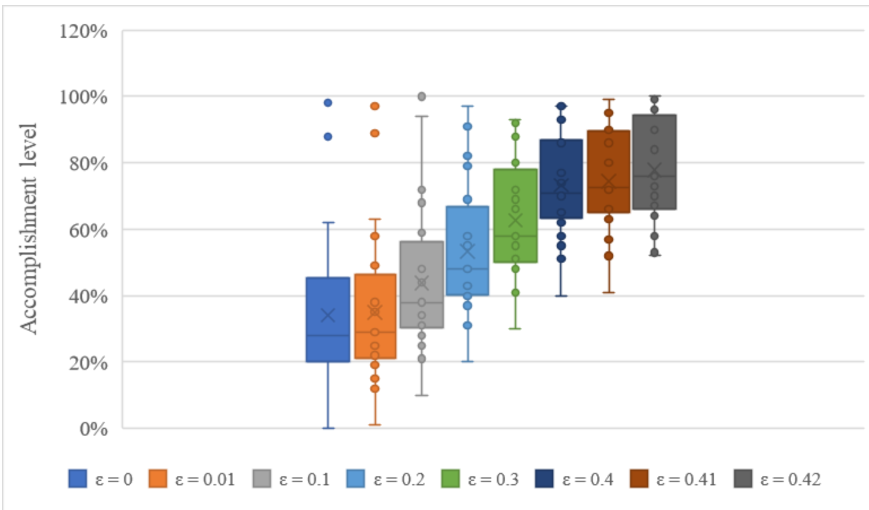


Figure 3-6. TTS - Percentage accomplishment in node-production constraint.

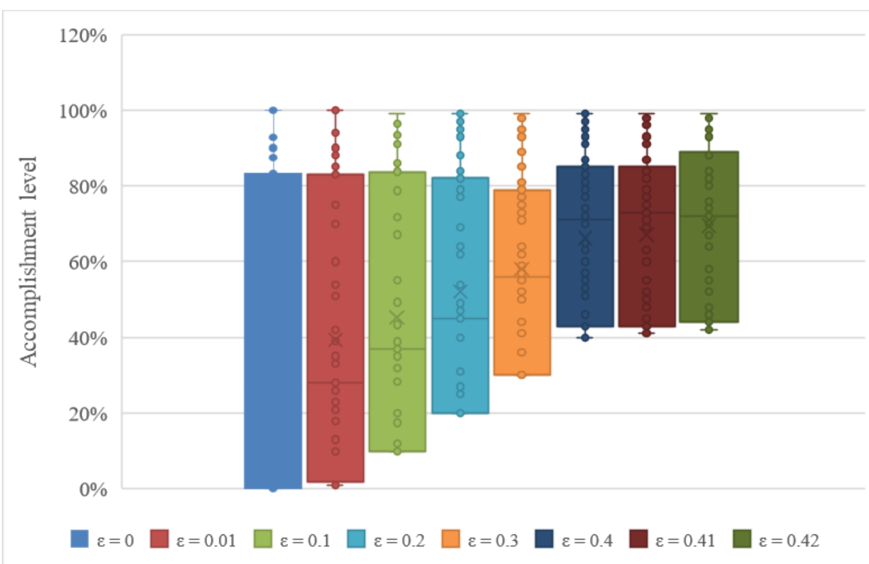


Figure 3-7. TTS - Percentage accomplishment in link-volume constraint.

Despite the tours' fluctuation in both figures, there was a significant difference between each solution, and none of the solutions exhibited this tendency.

Table 3-1 shows the number of accomplishments with minimum and maximum values and other statistics for node-production and link-volume constraints. The table shows the observations of the statistics values. While the variation per solution (every ε-value) is strong, the results of the tours inside the same solution have a homogeneous tendency. For the case of the ε = 0.3 solutions, the tour's average compliance relative to the production constraint (on the 24 nodes) was 63%, with a standard deviation of 0.17, in the case of the volume constraint (on the 76 links) the tour's average compliance was 58%, with a standard deviation of 0.24. In both production and volume, the minimum compliance value is equal to the respective ε-value, just as expected.

Table 3-1. TTS–Descriptive analysis of accomplishment.

Node-production constraint								
Descriptions	**Epsilon (*ε*)**							
	0	**0.01**	**0.1**	**0.2**	**0.3**	**0.4**	**0.41**	**0.42**
Mean (%)	34	35	44	53	63	73	75	77
Median (%)	28	29	38	48	58	71	73	76
Standard deviation	0.23	0.23	0.22	0.19	0.17	0.15	0.15	0.15
Maximum value (%)	98	97	100	97	93	97	99	100
Number of maximum accomplishments	1	1	1	1	1	2	1	2
Minimum value	0	0.01	0.1	0.2	0.3	0.4	0.41	0.42
Number of minimum accomplishments	1	1	1	1	1	1	1	0
Link-volume constraint								
Descriptions	**Epsilon (*ε*)**							
	0	**0.01**	**0.1**	**0.2**	**0.3**	**0.4**	**0.41**	**0.42**
Mean (%)	38	39	45	52	58	66	67	70
Median (%)	27	28	37	45	56	71	73	72
Standard deviation	0.36	0.36	0.32	0.29	0.24	0.21	0.20	0.21
Maximum value (%)	100	100	100	97	100	100	100	100
Number of maximum accomplishments	1	3	1	4	1	2	2	1
Minimum value	0.00	0.01	0.10	0.20	0.30	0.40	0.41	0.42
Number of minimum accomplishments	22	14	8	21	18	16	16	12

This numerical experiment objective is to validate the methodology proposed in this research, applying the formulation to estimate transit (buses) tours based on entropy maximization using fuzzy parameters. The Pareto frontier is specific to every case study, given that depends on every data set. That is, every replication of the experiment will have its own Pareto frontier; it could get a better or worse λ value. In this case, λ = 0.42 for transit, corresponds only to the specific case study.

In a few words, the numerical experiments consisted of running the model using several values for ε between 0 and 1 each time. What is common is that, at a certain point, the run stops the feasible solutions and starts giving infeasible solutions from then on. In the experiment, those solutions were discarded, and only the ε values that deliver feasible results were preserved. The λ parameter, in FL theory, can take values between 0 and 1 and represents the membership percentage for every solution. This is the reason for running the model several times using different ε parameter values (minimum allowed value for the λ parameter), starting from 0 with steps of 0.01 in FTS and 0.05 in TTS. The ε value means that at least a constraint (but can be more) gets a membership value or accomplishment by ε%. Lower values of ε are reflected in the model flexibility because it gives the chance to partially accomplish the constraints.

The running optimization process, as explained earlier, worked using the *ε* approach, where *ε* is an input parameter where $\varepsilon \in [0, 1]$, using a triangular membership function. The upper vertex of the triangle represents the requested (goal or target) value corresponding to the deterministic values obtained after assignment stage. This means that the requested values are those values giving when *ε=1,* since those are unfeasible solutions, *"the term "satisfying solution" suits better than "optimal solution"* (Luhandjula, 2015)*;* having in the flexibilization of the program a *"satisfying solution",* instead of a No solution.

Comparison FL Model and No FL models solutions

To demonstrate if the flexible solution really is better than the deterministic solution of the problem, the EM-based model was run using a deterministic form of the model, using, instead of flexibility constraints (for both production and volume) using FL, fixed parameters in constraints, which constitutes deterministic model. In such cases, the solution was not feasible, becoming the formulation with FL better than deterministic. Additionally, the problem was run several times seeking to know the resultant solutions of relaxing the parameters productions and volume, one by one and simultaneously. To do so, it was created an indicator (Δ) which measure the distance between the model's obtained solution and the requested value or target value (the upper value signed by the middle triangle vertex), corresponding to λ=1. We called this indicator "*Distance from the target value* (Δ)*",* and it was calculated for both production and volume constraints.

The mentioned relaxations consider different scenarios. 1) Taking only production constraint and relaxing it as a greater than-inequality and as a lower than-inequalities; 2) Taking only volume constraint and relaxing it as a greater than-inequality and as a lower than-inequalities; 3) Taking both production and volume constraints and relaxing them as greater than-inequalities, and as lower than-inequalities, for a total of six different scenarios in the case of the nodes (tours generation) and six in the case of the links (volume). All those cases were compared with this paper model proposed solution, which used FL parameters, which best performance was for ε=0.42.

For production constraint Δ is defined as follows:

$$\Delta_{O_i} = \left| \frac{Or_{eval} - Or_i^2}{Or_i^2} \right| \qquad ; \qquad \forall Or_i^2 \neq 0 \qquad (43)$$

where;

Δ_{O_i} *: Difference from the requested value for production (nodes)*

Or_{eval} *: Obtained production value from solution*

Or_i^2 *: the Production requested value*

Analogously Δ for volume is defined as follows:

$$\Delta_{V_a} = \left| \frac{Vr_{eval} - Vr_a^2}{Vr_a^2} \right| \qquad ; \qquad \forall Vr_a^2 \neq 0 \qquad (43)$$

where;

Δ_{V_a} *: Difference from the requested value for volume (links)*

Vr_{eval} *: Obtained volume value from solution*

Vr_a^2 *: The Volume requested value*

In all the models, the proposed flexible model, and the six described comparison scenarios, the percentage distance of the solution was measured, in each case, respect to the target value of production and volumes. Table 3-2 presents those differences on the flow in every link, and Table 3-3 contains descriptive information for every scenery. This comparison allows us to observe how all scenarios compared with the flexibilizated problem (named *FL Model* in the tables) have higher percentages distance concerning the target solution (Δ), than those distances for the FL Model. We can observe for instance Link 1 (A1), Table 3-2 tell us that none of the relaxed problems (case 1 to case 6 in the table), gives a smaller distance from the target value, than the one obtained in the flexible model (FL model in the table), which means that the use of the fuzzy parameters undoubtedly improve the performance of the model with better solutions.

Table 3-2. Percentage Distance from the target value (Δ_{V_a}) of the flow per link

Link	FL ε=0.42	No FL Constraint: production (≥)	No FL Constraint: production (≤)	No FL Constraint: flow (≥)	No FL Constraint: flow (≤)	NO FL Constraints: production and flow (≥)	NO FL Constraints: production and flow (≤)
	FL Model	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
A1	4%	46%	90%	14%	90%	14%	90%
A2	3%	30%	88%	10%	88%	10%	88%
A3	3%	30%	88%	10%	88%	10%	88%
A4	4%	31%	90%	2%	90%	2%	90%
A5	4%	46%	90%	14%	90%	14%	90%
A6	1%	6%	94%	7%	94%	7%	94%
A7	1%	41%	93%	8%	93%	8%	93%
A8	6%	40%	94%	0%	94%	0%	94%
A9	2%	15%	95%	8%	95%	8%	95%
A10	6%	24%	95%	0%	95%	0%	95%
A11	2%	9%	95%	5%	95%	5%	95%
A12	4%	24%	96%	2%	96%	2%	96%
A13	1%	16%	95%	5%	95%	5%	95%
A14	1%	37%	89%	4%	89%	4%	89%
A15	7%	2%	94%	0%	94%	0%	94%
A16	6%	9%	95%	0%	95%	0%	95%
A17	3%	38%	93%	3%	93%	3%	93%
A18	6%	31%	98%	0%	98%	0%	98%
A19	7%	30%	92%	0%	92%	0%	92%
A20	6%	31%	98%	0%	98%	0%	98%
A21	17%	58%	91%	23%	91%	23%	91%
A22	7%	17%	93%	0%	93%	0%	93%
A23	3%	15%	97%	3%	97%	3%	97%
A24	7%	21%	97%	0%	97%	0%	97%
A25	1%	17%	93%	0%	93%		93%
A26	6%	20%	97%	0%	97%	0%	97%
A27	2%	53%	90%	5%	90%	5%	90%
A28	5%	117%	90%	13%	90%	13%	90%
A29	7%	39%	96%	0%	96%	0%	96%
A30	1%	17%	93%	0%	93%		93%
A31	9%	13%	93%	13%	93%	13%	93%
A32	5%	20%	94%	2%	94%	2%	94%
A33	7%	31%	96%	0%	96%	0%	96%
A34	1%	35%	93%	3%	93%	3%	93%
A35	20%	18%	93%	30%	93%	30%	93%
A36	3%	23%	95%	4%	95%	4%	95%
A37	1%	77%	91%	9%	91%	9%	91%
A38	2%	10%	90%	15%	90%	15%	90%
A39	1%	77%	91%	9%	91%	9%	91%
A40	2%	28%	94%	4%	94%	4%	94%
A41	8%	2%	91%	11%	91%	11%	91%
A42	1%	52%	94%	3%	94%	3%	94%
A43	5%	25%	97%	1%	97%	1%	97%
A44	1%	16%	95%	5%	95%	5%	95%
A45	4%	63%	92%	11%	92%	11%	92%
A46	6%	34%	89%	14%	89%	14%	89%
A47	7%	15%	88%	0%	88%	0%	88%
A48	1%	52%	94%	3%	94%	3%	94%
A49	20%	18%	93%	30%	93%	30%	93%
A50	5%	8%	95%	3%	95%	3%	95%
A51	7%	2%	92%	14%	92%	14%	92%
A52	4%	76%	94%	11%	94%	11%	94%
A53	3%	8%	94%	4%	94%	4%	94%
A54	3%	38%	93%	3%	93%	3%	93%
A55	7%	15%	88%	0%	88%	0%	88%
A56	6%	31%	98%	0%	98%	0%	98%
A57	7%	25%	97%	0%	97%	0%	97%
A58	4%	41%	93%	11%	93%	11%	93%
A59	6%	20%	90%	0%	90%		90%
A60	2%	66%	88%	3%	88%	3%	88%
A61	6%	23%	98%	0%	98%	0%	98%
A63	4%	1%	92%	10%	92%	10%	92%
A64	6%	35%	93%	0%	93%	0%	93%
A67	11%	15%	93%	17%	93%	17%	93%
A68	17%	56%	88%	21%	88%	21%	88%
A70	8%	9%	90%	2%	90%	2%	90%
A71	6%	86%	91%	12%	91%	12%	91%
A73	1%	41%	93%	6%	93%	6%	93%
A74	2%	10%	90%	15%	90%	15%	90%
A75	6%	35%	93%	0%	93%	0%	93%
A76	11%	115%	91%	17%	91%	17%	91%

**Note: links 62, 65, 66, 69, 72 are unused so the change doesn't apply*

The descriptive information presented in Table 3-3 shows that FL model's average, for volumes, is closer to the requested value than any other case for the volumes. In other words, in the FL Model $\boldsymbol{\Delta_{Va}}$ average is the smallest in the comparison process, and in other cases, for example Case 2, that $\boldsymbol{\Delta_{Va}}$ achieved 91%. Moreover, P_{75} exhibit the same behavior, and again the FL Model got a $\boldsymbol{\Delta_{Va}}$ =7%, while the rest of the scenarios are over 11%.

Table 3-3. Percentage Distance (Δ_{Va}) Descriptive - Links (volume)

Descriptives	FL Model	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Min	1%	1%	88%	0%	88%	0%	88%
P_{25}	2%	16%	91%	0%	91%	0%	91%
Mean	5%	32%	93%	6%	93%	6%	93%
P_{75}	7%	41%	95%	11%	95%	11%	95%
Max	20%	117%	98%	30%	98%	30%	98%

The situation in the case of the nodes (production) analysis preserves a similar structure. As in the case of the links, in the nodes, the indicator $\boldsymbol{\Delta_{Oi}}$ were greater in the evaluated scenarios than in the flexible problem. This result is noted in Table 3-4 and Table 3-5.

Table 3-4. Percentage Distance from the target value (Δ_{Oi}) of the production per node

Node	FL ε=0.42	NO FL Constraint production (≥)	NO FL Constraint: production (≤)	NO FL Constraint: flow (≥)	NO FL Constraint: flow (≤)	NO FL Constraints: production and flow (≥)	NO FL Constraints: production and flow (≤)
	FL Model	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
I1	3%	29%	90%	3%	90%	3%	90%
I2	3%	29%	90%	3%	90%	3%	90%
I3	2%	11%	93%	5%	93%	5%	93%
I4	0%	12%	94%	6%	94%	6%	94%
I5	3%	0%	95%	3%	95%	3%	95%
I6	5%	0%	94%	1%	94%	1%	94%
I7	4%	2%	96%	2%	96%	2%	96%
I8	4%	0%	95%	3%	95%	3%	95%
I9	2%	0%	95%	4%	95%	4%	95%
I10	3%	2%	95%	3%	95%	3%	95%
I11	3%	6%	94%	3%	94%	3%	94%
I12	3%	1%	94%	5%	94%	5%	94%
I13	0%	61%	90%	11%	90%	11%	90%
I14	0%	27%	93%	5%	93%	5%	93%
I15	1%	3%	95%	6%	95%	6%	95%
I16	5%	0%	94%	3%	94%	3%	94%
I17	1%	20%	94%	6%	94%	6%	94%
I18	5%	0%	95%	1%	95%	1%	95%
I19	4%	0%	95%	3%	95%	3%	95%
I20	4%	0%	95%	2%	95%	2%	95%
I21	6%	35%	93%	0%	93%	0%	93%
I22	5%	18%	91%	12%	91%	12%	91%
I23	1%	59%	92%	7%	92%	7%	92%
I24	0%	57%	92%	8%	92%	8%	92%

Table 3-5. Percentage Distance (ΔOi) Descriptive - Nodes (production)

Descriptives	FL Model	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Min	0%	0%	90%	0%	90%	0%	90%
P_{25}	1%	0%	92%	3%	92%	3%	92%
Mean	3%	15%	93%	4%	93%	4%	93%
P_{75}	4%	28%	95%	6%	95%	6%	95%
Max	6%	61%	96%	12%	96%	12%	96%

The percentage distance for production Δ_{Oi} obtained using this research proposed model, corresponds in average to 3% respect to the target or requested value, and that is the smaller value obtained related to the cases 1 to 6. As this result is consistent with the hypothesis of this investigation, it is possible to affirm than the flexible solution obtained from the EM based model using FL parameters, is strongly better than deterministic cases.

3.5. Conclusions and discussion

One of the main worries in planning is the daily uncertainty. Because capturing it is a very difficult issue. When decision-makers can control the uncertainty levels, the probability of success increases. The inclusion of FL in the transport modeling process is barely known and used, even when it has shown its goodness by reducing the gap between modeling and reality, to reduce, not to eliminate.

This research proposes using fuzzy parameters for estimating tour flows, using traffic counts to be solved with entropy maximization techniques for transit (buses). The paper presents and tests the entropy-based TTS using FL, which is a new-proposal formulation for modeling transit (buses) using the same technique, which, to the author's best knowledge, is a contribution to the literature. The FL applied to the formulation is based on membership functions and the ε approach. In this research, triangular membership functions, which is the simplest and very common, given that it is closest to the most common distribution in data, even the choice of this function is frequently made by experts' opinion. This paper shows a numerical experiment in the Sioux Falls network.

Some cases exist where a feasible solution corresponds to the lowest values for ε in the minimum value of membership. This condition implies that the nonflexible problem does not have a feasible solution with the used constraints. Nevertheless, a partial solution is better than nothing for practitioners. The proposed method allows for obtaining the best solution possible when a total accomplishment of each constraint is not possible.

The indicator "*Distance from the target value* (Δ)*"* was created to measure the distance between the model's obtained solution and the requested value or target value, which corresponds to $\lambda=1$. It was calculated for both production and volume constraints. The problem was run several times seeking to know the resultant solutions of relaxing the parameters productions and volume, one by one and simultaneously. All those cases were compared with this paper model proposed solution, which best performance was for $\varepsilon=0.42$. For both production and volume, the FL model had smaller Δ values, than the ones obtained in the No FL models, which means that the use of the fuzzy parameters undoubtedly improve the performance of the model with better solutions. In a few of links or nodes, some of the comparative cases scenarios could have a better Δ, but those are not relevant when in general most of them present strongly differences in the indicator. When considering what is real, it is not always possible to fully comply with the constraints. Planners and decision-makers must try finding an equilibrium, and these formulations can contribute to being closer to that point, decreasing the level of uncertainty. This is due, the FL model allows to obtain "satisfying solutions" instead of No solutions, when deterministic problem is unfeasible. The inclusion of fuzzy logic with the entropy maximization approach to estimate bus tour flows significantly improves the quality of the results.

This methodology can generate helpful information for decision-makers. The solutions obtained can well represent the natural behavior of transit or any other problem. The FL is a powerful tool when a problem implies variability and uncertainty. It could build better transportation planning for agencies to improve mobility in cities and urban areas. Moreover, FL help include the parameter variability inherent in daily life into the input data, the same that should be reflected in the outputs. The obtained results are more reliable because of the more realistic shape of the models.

The Pareto frontier, obtained from the comparison of the entropy, shows that while entropy decreases, the ε value increases, which means that the problem is multi-objective. Thus, considering the number of vehicles using the links to obtain comparable entropy values is necessary.

This research used FL to make the corresponding constraints to node-production and link-volume flexible. However, the authors think the link cost can also be flexible in further research (i.e., considering congestion). Future research should include better forms for the membership function applied to constraint flexibilization.

3.6. References

Abdulaal, Mustafa, and Larry J. LeBlanc. 1979. "Continuous Equilibrium Network Design Models." Transportation Research Part B: Methodological 13(1):19–32. doi: 10.1016/0191-2615(79)90004-3.

Abrahamsson, T. 1998. Estimation of Origin-Destination Matrices Using Traffic Counts - A Literature Survey. IIASA, Laxenburg, Austria: IR-98-021.

Aggarwal, Manish. 2021. "Redefining Fuzzy Entropy with a General Framework." Expert Systems with Applications 164:113671. doi: 10.1016/j.eswa.2020.113671.

Bar-Gera, Hillel, Fredrik Hellman, and Michael Patriksson. 2013. "Computational Precision of Traffic Equilibria Sensitivities in Automatic Network Design and Road Pricing." Procedia-Social and Behavioral Sciences 80:41–60.

Bellman, R., and L. A. Zadeh. 1970. "Decision-Making in a Fuzzy Environment." Management Science 17(4):B141–64.

Bit, A. K., M. P. Biswal, and S. S. Alam. 1992. "Fuzzy Programming Approach to Multicriteria Decision Making Transportation Problem." Fuzzy Sets and Systems 50(2):135–41. doi: 10.1016/0165-0114(92)90212-M.

De Cea, Joaquin, and Enrique Fernández. 1993. "Transit Assignment for Congested Public Transport Systems: An Equilibrium Model." Transportation Science 27(2):133–47.

Chakirov, Artem, and Pieter J. Fourie. 2014. Enriched Sioux Falls Scenario with Dynamic and Disaggregate Demand. Singapore.

Chanas, Stefan, and Dorota Kuchta. 1996. "A Concept of the Optimal Solution of the Transportation Problem with Fuzzy Cost Coefficients." Fuzzy Sets and Systems 82(3):299–305. doi: 10.1016/0165-0114(95)00278-2.

Fan, Wei, and Randy B. Machemehl. 2006. "Optimal Transit Route Network Design Problem with Variable Transit Demand: Genetic Algorithm Approach." Journal of Transportation Engineering 132(1):40–51. doi: 10.1061/(ASCE)0733-947X(2006)132:1(40).

Fisk, C. S. 1989. "Trip Matrix Estimation from Link Traffic Counts: The Congested Network Case." Transportation Research Part B 23(5):331–36. doi: 10.1016/0191-2615(89)90009-X.

Friesz, Terry L., Hsun-Jung Cho, Nihal J. Mehta, Roger L. Tobin, and G. Anandalingam. 1992. "A Simulated Anneal-ing Approach to the Network Design Problem with Variational Inequality Constraints." Transportation Science 26(1):18–26.

Gonzalez-Calderon, Carlos A. 2014. "Multiclass Equilibrium Demand Synthesis." Rensselaer Polytechnic Institute.

Gonzalez-Calderon, Carlos A., and José Holguín-Veras. 2019. "Entropy-Based Freight Tour Synthesis and the Role of Traffic Count Sampling." Transportation Research Part E: Logistics and Transportation Review 121(November 2017):63–83. doi: 10.1016/j.tre.2017.10.010.

Hannan, Edward L. 1981a. "Linear Programming with Multiple Fuzzy Goals." Fuzzy Sets and Systems 6(3):235–48. doi: 10.1016/0165-0114(81)90002-6.

Hewings, Geoffrey J. D., and Esteban Fernandez-Vazquez. 2019. "Entropy Maximization and Input–Output Analy-sis." Interdisciplinary Science Reviews 44(3–4):272–85.

Holguin-Veras, Jose, Johanna Amaya-Leal, Jeffrey Wojtowicz, Miguel Jaller, Carlos A. Gonzalez-Calderon, Ivan Sanchez-Diaz, Xiaokun Wang, Daniel G. Haake, Suzann S. ;. Rhodes, Stacey Darville Hodge, Robert J. Frazier, Molly K. Nick, Joseph Dack, and Luigi Casinelli. 2015. Improving Freight System Performance in Metropolitan Ar-eas: A Planning Guide. Washington, D.C. doi: 10.17226/22159.

Holguin-Veras, Jose, Trilce Encarnacion, Diana Ramirez-Rios, Xiaozheng (Sean) He, Lokesh Kalahasthi, Sofia Perez-Guzman, Ivan Sanchez-Diaz, and Carlos A. Gonzalez-Calderon. 2020. "A Multiclass Tour Flow Model and Lts Role in Multiclass Freight Tour Synthesis." Transportation Science. doi: 10.1287/trsc.2019.0936.

Hu, Baoyu, Yanli Ma, Yulong Pei, and Wei Gao. 2020. "Statistical Analysis and Predictability of Inter-Urban High-way Traffic Flows: A Case Study in Heilongjiang Province, China." Transportmetrica A: Transport Science 16(3):1062–78.

Josefsson, Magnus, and Michael Patriksson. 2007. "Sensitivity Analysis of Separable Traffic Equilibrium Equilibria with Application to Bilevel Optimization in Network Design." Transportation Research Part B: Methodological 41(1):4–31.

Konstantinos, Kepaptsoglou, and Karlaftis Matthew. 2009. "Transit Route Network Design Problem: Review." Jour-nal of Transportation Engineering 135(8):491–505. doi: 10.1061/(ASCE)0733-947X(2009)135:8(491).

Lam, W. H. K., Z. Y. Gao, K. S. Chan, and H. Yang. 1999. "A Stochastic User Equilibrium Assignment Model for Congested Transit Networks." Transportation Research Part B: Methodological 33(5):351–68. doi: 10.1016/S0191-2615(98)00040-X.

Lam, W. H. K., Z. X. Wu, and K. S. Chan. 2003. "Estimation of Transit Origin-Destination Matrices from Passenger Counts Using a Frequency-Based Approach." Journal of Mathematical Modelling and Algorithms 2(4):329–48. doi: 10.1023/B:JMMA.0000020423.93104.14.

Lam, William H. K., Jing Zhou, and Zhao Han Sheng. 2002. "A Capacity Restraint Transit Assignment with Elastic Line Frequency." Transportation Research Part B: Methodological 36(10):919–38. doi: 10.1016/S0191-2615(01)00042-X.

LeBlanc, Larry J., Edward K. Morlok, and William P. Pierskalla. 1975. "An Efficient Approach to Solving the Road Network Equilibrium Traffic Assignment Problem." Transportation Research 9(5):309–18. doi: 10.1016/0041-1647(75)90030-1.

Lee, S., and J. Lim. 2002. "Effectiveness And Macroeconomic Impact Analysis Of Policy Instruments For Sustainable Transport In Korea: A CGE Modelling Approach." WIT Transactions on the Built Environment 60:1–12.

López-Ospina, Héctor. 2013. "Modelo de Maximización de La Entropía y Costos Generalizados

Intervalares Para La Distribución de Viajes Urbanos.” Ingenieria y Universidad 17(2):390–407.

López-Ospina, Héctor, Cristián E. Cortés, Juan Pérez, Romario Peña, Juan Carlos Figueroa-García, and Jorge Urru-tia-Mosquera. 2021. “A Maximum Entropy Optimization Model for Origin-Destination Trip Matrix Estima-tion with Fuzzy Entropic Parameters.” Transportmetrica A: Transport Science 1–38. doi: 10.1080/23249935.2021.1913257.

Luathep, Paramet, Agachai Sumalee, William H. K. Lam, Zhi-Chun Li, and Hong K. Lo. 2011. “Global Optimization Method for Mixed Transportation Network Design Problem: A Mixed-Integer Linear Programming Approach.” Transportation Research Part B: Methodological 45(5):808–27.

Luhandjula, M. K. 2015. “Fuzzy Optimization: Milestones and Perspectives.” Fuzzy Sets and Systems 274:4–11. doi: 10.1016/j.fss.2014.01.004.

Majidi, Setareh, Seyyed Mahdi Hosseini-Motlagh, Saeed Yaghoubi, and Abbas Jokar. 2017. “Fuzzy Green Vehicle Routing Problem with Simultaneous Pickup-Delivery and Time Windows.” RAIRO - Operations Research 51(4):1151–76. doi: 10.1051/ro/2017007.

Marcotte, Patrice, and Gerald Marquis. 1992. “Efficient Implementation of Heuristics for the Continuous Network Design Problem.” Annal of Operations Research 34:163–76.

Meng, Qiang, Hai Yang, and Michael G. H. Bell. 2001. “An Equivalent Continuously Differentiable Model and a Locally Convergent Algorithm for the Continuous Network Design Problem.” Transportation Research Part B: Methodological 35(1):83–105.

Moreno-Palacio, Diana Patricia, Carlos Alberto Gonzalez-Calderon, Héctor López-Ospina, Jhan Gil-Marin, and John Jairo Posada-Henao. 2022. “POMS-2022-Conference Program Book.” P. 147 in Freight Tour Synthesis based on Entropy - Fuzzy Logic, edited by T. Dhakar, R. Tewari, and S. Singhania. PRODUCTION AND OPERA-TIONS MANAGEMENT SOCIETY.

Nagy, Albert, Zsolt Ercsey, József Tick, and Zoltán Kovács. 2019. “Bus Transport Process Network Synthesis.” Acta Polytechnica Hungarica 16(7):25–43. doi: 10.12700/APH.16.7.2019.7.2.

Nguyen, Sang, and Stefano Pallottino. 1988. “Equilibrium Traffic Assignment for Large Scale Transit Networks.” European Journal of Operational Research 37(2):176–86.

Nielsen, Otto Anker. 2000. “A Stochastic Transit Assignment Model Considering Differences in Passengers Utility Functions.” Transportation Research Part B: Methodological 34(5):377–402. doi: 10.1016/S0191-2615(99)00029-6.

Nuzzolo, Agostino, and Umberto Crisalli. 2001. “Estimation of Transit Origin/Destination Matrices from Traffic Counts Using a Scheduled-Based Approach.” in European Transport Forum 2001. Cambridge, UK: Association for European Transport.

Ortuzar, Juan de D., and Luis G. Willumsen. 2011. Modelling Transport. Fourth Edi. John Wiley & Sons, Ltd.

Sakawa, Masatoshi. 1983. “Interactive Fuzzy Decision Making for Multiobjective Linear Programming Problems and Its Application.” IFAC Proceedings Volumes 16(13):295–300. doi: 10.1016/S1474-6670(17)62049-4.

Sakawa, Masatoshi, and Hitoshi Yano. 1987. “An Interactive Fuzzy Satisficing Method for Multiobjective Linear Programming Problems with Fuzzy Parameters.” IFAC Proceedings Volumes 20(9):437–42. doi: 10.1016/S1474-6670(17)55745-6.

Sanchez-Diaz, Ivan, Jose Holguin-Veras, and Xuegang (Jeff) Ban. 2015. “A Time-Dependent Freight Tour Synthesis Model.” Transportation Research Part B: Methodological 78:144–68. doi: 10.1016/j.trb.2015.04.007.

Soehodo, Sutanto, and Masaki Koshi. 1999. “Design of Public Transit Network in Urban Area with Elastic Demand.” Journal of Advanced Transportation 33(3):335–69. doi: 10.1002/atr.5670330306.

Spiess, Heinz, and Michael Florian. 1989. “Optimal Strategies: A New Assignment Model for Transit Networks.” Transportation Research Part B: Methodological 23(2):83–102.

Sun, Chao, Yulin Chang, Yuji Shi, Lin Cheng, and Jie Ma. 2019. “Subnetwork Origin-Destination Matrix Estimation Under Travel Demand Constraints.” Networks and Spatial Economics 19(4):1123–42. doi: 10.1007/s11067-019-09449-6.

Suwansirikul, Chaisak, Terry L. Friesz, and Roger L. Tobin. 1987. “Equilibrium Decomposed Optimization: A Heu-ristic for the Continuous Equilibrium Network Design Problem.” Transportation Science 21(4):254–63.

Tamin, and LG Willumsem. 1992. “Freight Demand Model Estimation From Traffic Counts.” Simplified Transport Demand Modelling (Perspectives 2) 75–86.

Tang, Jinjun, Shen Zhang, Xinqiang Chen, Fang Liu, and Yajie Zou. 2018. “Taxi Trips Distribution Modeling Based on Entropy-Maximizing Theory: A Case Study in Harbin City—China.” Physica A: Statistical Mechanics and Its Applications 493:430–43.

Velichko, Andrey. 2016. “Interregional Transportation Modeling for the Far East of Russia Macro-Region.” Pp. 394–403 in DOOR (Supplement).

Wang, Qian, and Jose Holguin-Veras. 2009. “Tour-Based Entropy Maximization Formulations of Urban Commercial Vehicle Movements.” Pp. 1–22 in 2009 Annual Meeting of the Transportation Research Board. New York: 2009 Annual Meeting of the Transportation Research Board.

Wei, Xueyan, Weijie Yu, Wei Wang, De Zhao, and Xuedong Hua. n.d. “Optimization and Comparative Analysis of Traffic Restriction Policy by Jointly Considering Carpool Exemptions.” doi: 10.3390/su12187734.

Willumsen, L. G. 1982. Estimation of Trip Matrices from Volume Counts. Validation of a Model under Congested Condi-tions.

Willumsen, L. G. 1984. “Estimating Time-Dependent Trip Matrices from Traffic Counts.” Pp. 397–412 in Proceedings of the Ninth International Symposium on Transportation and Traffic

Theory, edited by J. Volmuller and R. Hamers-lag. Utrecht, Netherlands: VNU Science Press.

Willumsen, Luis G. 1978a. Estimation of an OD Matrix from Traffic Counts - A Review. Leeds, UK: Institute of Transport Studies, University of Leeds.

Willumsen, Luis G. 1978b. “OD Matrices from Network Data: A Comparison of Alternative Methods for Their Es-timation.” in Proc. of PTRC Summer Annual Meeting 1978 Seminar in Transport Models.

Wilson, A. G. 1969. “The Use of Entropy Maximising Models, in the Theory of Trip Distribution, Mode Split and Route Split.” Journal of Transport Economics and Policy 3(1):108–26.

Wirasinghe, S. C., L. Kattan, M. M. Rahman, J. Hubbell, R. Thilakaratne, and &. S. Anowar. 2013. “Bus Rapid Trans-it-a Review.” International Journal of Urban Sciences 17(1):1–31. doi: 10.1080/12265934.2013.777514.

Wong, S. C., and C. O. Tong. 1998. “Estimation of Time-Dependent Origin-Destination Matrices for Transit Net-works.” Transportation Research Part B: Methodological 32(1):35–48. doi: 10.1016/S0191-2615(97)00011-8.

Wong, S. C., C. O. Tong, K. I. Wong, W. H. K. Lam, H. K. Lo, H. Yang, and H. P. Lo. 2005. “Estimation of Multiclass Origin-Destination Matrices from Traffic Counts.” Journal of Urban Planning and Development 131(1):19–29. doi: 10.1061/(ASCE)0733-9488(2005)131:1(19).

Wu, Jia Hao, Michael Florian, and Patrice Marcotte. 1994. “Transit Equilibrium Assignment: A Model and Solution Algorithms.” Transportation Science 28(3):193–203.

Zadeh, L. A. 1965. “Fuzzy Sets.” Information and Control 8(3):338–53. doi: 10.1016/S0019-9958(65)90241-X.

Zadeh, L. A. 1968. “Fuzzy Algorithms.” Information and Control 12(2):94–102. doi: 10.1016/S0019-9958(68)90211-8.

Zadeh, L. A. 1999. “Fuzzy Sets as a Basis for a Theory of Possibility.” Fuzzy Sets and Systems 100(6):9–34. doi: 10.1016/S0165-0114(99)80004-9.

Zhang, Yunlong, and Zhirui Ye. 2008. “Short-Term Traffic Flow Forecasting Using Fuzzy Logic System Methods.” Journal of Intelligent Transportation Systems: Technology, Planning, and Operations 12(3):102–12. doi: 10.1080/15472450802262281.

Zimmermann, H. J. 1978. “Fuzzy Programming and Linear Programming with Several Objective Functions.” Fuzzy Sets and Systems 1(1):45–55. doi: 10.1016/0165-0114(78)90031-3.

4. Freight-Transit Tour Synthesis Entropy-Based Formulation: Sharing Infrastructure for Buses and Trucks

Abstract

The freight system's complexity and its significant impact on urban areas necessitate careful consideration for sustainable transportation options. The proposed Freight Transit Tour Synthesis (FTTS) model, using fuzzy logic and entropy maximization, analyzes freight and transit systems as a multiclass category, exploring scenarios where buses and trucks share infrastructure. The experiments demonstrate that capacity and maximum cost significantly influence the solutions obtained using fuzzy parameters, with ε-values indicating the best solution. Results may vary depending on available data, highlighting the need to explore solutions for different capacity levels if exceeded. The impact of the maximum cost constraint on tour flows is significant, emphasizing the importance of considering cost in optimizing tour flows. The model's robustness is evident across various subjective value of time (SVT) scenarios. The application of the FTTS model offers a novel approach to estimate tour flows, incorporating traffic counts and fuzzy parameters for immediate, relevant results. The model's multiclass formulation accurately represents real-world traffic conditions, considering congestion in traffic assignments. Overall, the FTTS model holds promise for optimizing tour flows and shared infrastructure between freight and transit systems, aiding decision-makers in urban transportation planning and resource allocation, ultimately leading to improved traffic management and infrastructure usage efficiency.

Keywords: *Entropy, Fuzzy Logic, Freight Tour Synthesis, Transit Tour Synthesis, Freight Transportation, Tour and Transit Tour Synthesis, Sioux Falls Network*

4.1. Introduction

The freight system is extensive and complex, requiring consideration of various freight modes and their respective infrastructure and operations. Freight flows are vital for modern economies (Holguin-Veras et al., 2015), but they also generate negative external effects, such as congestion, pollution, and infrastructure damage. Public policy aims to maximize the social benefits of freight activity while minimizing its negative impacts.

Freight planning seeks to understand freight patterns and behavior in urban areas (Holguin-Veras & Patil, 2005), especially in Freight Intensive Sectors (FIS) like construction and manufacturing, etc., according to North American Industry Classification System (NAICS). These sectors play a significant role in the production and consumption of goods (Campbell et al., 2018; Holguín-Veras et al., 2018). Trucks are widely used for transporting goods, and they have specific requirements for loading, unloading, and parking. They contribute both positive and negative externalities, making it crucial to address their impact. Trucks often make multiple deliveries in a single journey, forming tours that need to be studied, especially in urban areas.

The paper introduces the innovative Freight Transit Tour Synthesis (FTTS) model using fuzzy logic (FL) based on entropy maximization (EM). The model was developed by combining elements from various sources: the deterministic formulation of truck tour flow estimation (Gonzalez-Calderon, 2014), a formulation for transit tour flows based on EM and using FL (Moreno-Palacio et al., 2022), and a freight tour flow estimation approach also based on EM and using FL (Moreno-Palacio et al., 2023).This model explores scenarios where buses and trucks share infrastructure like dedicated lanes. This novel approach fills a gap in tour flow estimation and provides valuable insights for sustainable transportation planning. The paper focuses on freight and transit systems, enabling comparisons of flows, routes, and stop points between trucks and buses.

The paper includes five sections, starting with an introduction to the freight system and its complexities. Section 2 delves into background information on freight and transit tour transportation, including shared infrastructure between buses and trucks. Section 3 presents the formulation of FTTS and FTTS with FL. The numerical application of the FL model on the Sioux Falls network is discussed in Section 4, providing in-depth results analysis. Finally, the concluding remarks in the last section summarize the paper's contributions and findings.

4.2. Literature review

This section provides background information on relevant research literature concerning freight and transit demand characteristics and systems. It covers freight tour demand elements, transit tour demand, and the integration of both systems through shared infrastructure. Additionally, it presents the modeling approaches for FTS and TTS.

4.2.1. Freight Tours Transportation

Freight trips primarily involve moving goods using trucks (Gonzalez-Calderon et al., 2018; Holguín-Veras et al., 2018), which constitute Freight Trip Generation (FTG) (Bastida & Holguin-Veras, 2009; Board et al., 2016; Campbell et al., 2018; Gonzalez-Feliu & Sánchez-Díaz, 2019; Holguin-Veras et al., 2011, 2014; Krisztin, 2018; Sánchez-Díaz et al., 2016; Wigan et al., 2002). FTG is the number of freight vehicle trips generated by commercial establishments in an urban area, consisting of two components: Freight Trip Attraction (FTA) and Freight Trip Production (FTP). FTA represents the number of freight vehicle trips arriving at commercial locations for goods delivery, while FTP denotes the number of freight vehicle trips departing from establishments to transport cargo to other destinations.

Freight flows are a material representation of the manufacturing and consumer economies (Holguin-Veras et al., 2015). Freight demand synthesis techniques, such as entropy maximization, efficiently estimate freight flows using traffic counts, producing a Freight OD Synthesis (FODS) matrix. FODS is used to estimate freight demand and reproduce traffic counts for calibration, supporting transportation planning (Holguin-Veras et al., 2020; Wang & Holguin-Veras, 2009). Freight transport involves delivering goods from producers to receivers, sometimes to unique destinations or through a sequence of stops forming a tour. Freight Tour Synthesis (FTS) analyzes freight flows by inferring the number of trucks following specific node sequences, achieving the most probable truck flow distribution based on counts using optimization techniques like EM (Gonzalez-Calderon, 2014; Gonzalez-Calderon & Holguín-Veras, 2019).

Urban freight transport involves freight vehicles, mostly trucks, following a sequence of stops for deliveries and pickups, known as a trip chain or tour. The average number of stops in each tour depends on the transported goods, while the number of tours varies based on truck type, service time, city characteristics, among other factors (Gonzalez-Calderon, 2014; Holguin-Veras, 2013; Holguin-Veras & Thorson, 2003).

4.2.2. Transit tours transportation

Public transportation is crucial for urban life, improving access to jobs and enhancing the overall quality of life (Nagy et al., 2019). Various bus modes constitute public transit, each with its design and service aspects tailored to specific conditions and the urban environment (Wirasinghe et al., 2013). Bus Rapid Transit (BRT) systems have a significant advantage in passenger capacity compared to regular buses, making them a strong pillar of public transport. However, BRTs are less flexible and require dedicated infrastructure, including special lanes, stations, and advanced technology (Trubia et al., 2020).

BRT lanes are strategically positioned for efficiency and safety, often located in the center of the roadway. Station platforms can be in the center or on the side, and various strategies, like painted lines or physical separators, are used to separate BRT lanes from general-purpose lanes (Wirasinghe et al., 2013).

Bus route networks aim to create efficient public transportation systems despite limited resources, considering factors like the size and number of buses in service. Route layout is optimized based on passenger flows and expected volumes along routes, connecting areas with high demand for public transportation (Konstantinos & Matthew, 2009).

Public transit covers various systems, ranging from buses to trams, light rail transit (LRT), commuter rail, and metro systems (Wirasinghe et al., 2013). The planning process seeks to provide a good level of service, ensuring accessibility, reliability, minimal transfers, and affordability, while also considering environmental impact (2013).

4.2.3. Exclusive or dedicated lanes (Bus and trucks)

In many urban centers, mixed traffic consisting of transit (buses), private cars, and freight vehicles (vans and trucks) sharing the same road space can lead to congestion and delays, making public transportation less attractive for some travelers (Ben-Dor et al., 2018). Implementing dedicated lanes, such as bus lanes or truck lanes, can be a solution to reduce negative effects and incentivize the use of public transport. While the idea of dedicated truck lanes has mainly been considered for inter-urban roads, it can also be applied to urban networks to improve safety and efficiency.

To achieve positive changes and clarity in the implementation of dedicated lanes, it is crucial to communicate the purposes clearly to the community. Confusion can arise when terminologies like "truck lanes" are used differently in different places, causing misunderstanding (Mcleod &

Cherrett, 2009). Report 33 (2015) has presented initiatives like "Exclusive Truck Lanes" to improve freight transport system performance, proposing dedicated truck lanes as a viable alternative.

Various transit modes require specific infrastructure to operate efficiently, such as transitways for BRT systems (Figure 4-1) or dedicated and preferential bus lanes. Establishing dedicated bus lanes is an effective approach to prioritize public transportation, but their success depends on proper implementation and planning. The qualitative effects of dedicated bus lanes were studied in Sioux Falls to understand their impact on traffic congestion, modal split, and trip durations using the MATSim environment (Ben-Dor et al., 2018).

Transit lanes, which can be preferential or exclusive for transit vehicles, are not physically separated from other traffic like transitways (National Association of City Transportation Officials, 2016). FTTS method is specifically applied on side transit lanes, both dedicated and preferential bus lanes (Figure 4-2), to optimize tour flows in urban areas.

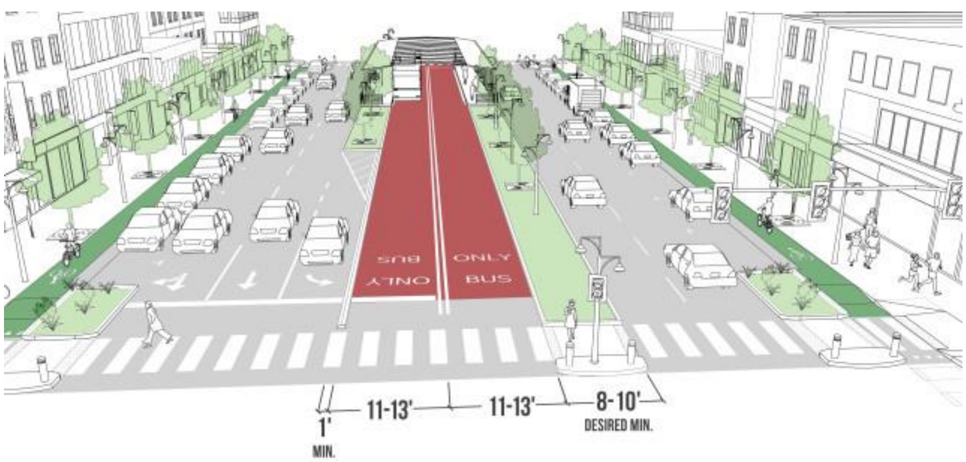


Figure 4-1. Central transitway diagram (BRT)
Source: National Association of City Transportation Officials (2016)

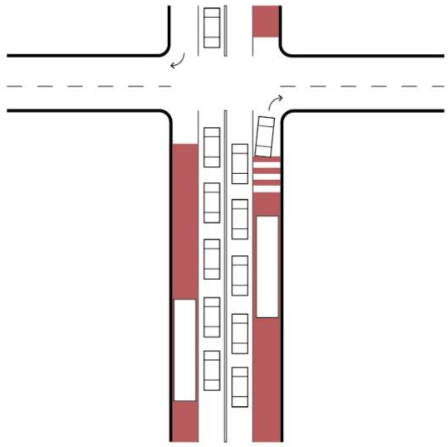


Figure 4-2. Curb side bus lane diagram
Source: National Association of City Transportation Officials (2016)

4.2.4. Integration between freight and transit

When implementing new transport systems like BRTs or making urban modifications, it's crucial to consider the wider impact on the surrounding area. Land use, commercial activity, and other transport systems for both passengers and freight will be affected. Cruz-Daraviña et al. (2021) point to achieve sustainability, addressing urban freight transport challenges is essential, as congestion has significant consequences such as lower productivity and environmental impacts. To find solutions, a holistic approach is needed, including land use planning, industry, commerce, and community needs.

Bus lane traffic management is extensively researched and vital for decision-makers and mobility offices. Dedicated bus-only lanes are commonly used in many cities, providing high-quality transit services, especially in congested areas (Agrawal et al., 2013; Gunes et al., 2021). However, freight transport poses environmental and congestion challenges that require cost-effective and easily implementable solutions. Shared transportation infrastructure, like multi-class lanes where trucks and buses share the same lanes, offers a potential solution. Studies using entropy-based formulation examine the congestion behavior in both multi-class and truck-only lanes, providing valuable insights for effective traffic management. Complementary usage of bus lanes by trucks and buses has been explored and found to be efficient without compromising the transit experience (Gunes et al., 2021). This approach presents an opportunity to enhance traffic flow and congestion control.

It is essential to emphasize the significance of bays in the model's development to ensure smooth traffic flow and prevent any disruptions when the lane is shared by both trucks and buses. Bays play a crucial role in maintaining the lane's capacity and efficiency, allowing for seamless integration of these two types of vehicles without compromising the overall performance.

In different locations, the idea of sharing bus lanes with freight vehicles has been explored. Initiatives to incentivize low-emission and environment-friendly trucks to use bus lanes have been implemented in Gothenburg and Bristol (2021). The concept of 'intermittent bus lanes,' where bus lanes are repurposed for general traffic when not in use, has been successful in Lisbon (Mcleod & Cherrett, 2009; Viegas & Lu, 1996) .

Overall, considering shared infrastructure and creative lane management strategies can be effective in reducing congestion and improving the efficiency of both freight and passenger transport systems (Gunes et al., 2021).

4.2.5. Entropy maximization (EM) in tour transportation modeling

EM is a widely used approach for estimating OD matrices using traffic counts, making it a cost-effective and agile method. Wilson (1967) pioneered its use for spatial distribution models and later applied it to trip distribution, modal split, and route split in Wilson (1969), where besides applied the concept of cost perceived by the users as an impedance function. In urban freight and transit, tours are more suitable than OD matrices as they describe the sequence of stops for passenger transit or pick-ups/deliveries for freight. EM in tour transportation modeling replicates secondary data to obtain tour flows, saving time and costs. It has been the primary optimization program used for FTS (Holguin-Veras et al., 2020) and was also used in TTS by Moreno-Palacio et al. (2022). Holguin-Veras et al. (2020) emphasize the importance of these formulations based on EM theory for estimating the flow of freight vehicles.

4.2.5.1. FTS - Deterministic and Flexible Modeling

The FTS model utilizes the EM approach to determine the most probable distribution of truck flows based on traffic counts, effectively estimating freight tour flows. EM resolves the inherent uncertainty of FTS, offering a unique solution through optimization techniques (Holguin-Veras et al., 2020). Gonzalez-Calderon et al. (2019) propose a deterministic formulation of FTS and introduce a multiclass approach for trucks and passenger cars OD trips, a significant contribution to this research. Additionally, Moreno-Palacio et al. extend the FTS problem by incorporating fuzzy parameters, adding flexibility and modeling parameter uncertainty. For further details on these models, refer to Gonzalez-Calderon et al. (2019) and Moreno-Palacio et al. (2023).

4.2.5.2. TTS - Deterministic and Flexible Modeling

Moreno-Palacio et al. (2022) initially developed the EM formulation as a deterministic problem and later introduced the TTS using fuzzy logic. This enhancement yields values close to the target value, simplifying the management of data variability. Unlike the deterministic approach, the fuzzy logic model provides a feasible solution. The flexibilization allows obtaining an interval that includes the target value, accommodating data variability. The formulation guarantees convexity and uniqueness for an optimal solution. For a complete formulation, refer to Moreno-Palacio et al. (2022).

4.3. Freight transit tour synthesis (FTTS) modelling: Deterministic and Flexible models

The Freight-Transit Tour Synthesis (FTTS) analysis aims to study the behavior of trucks and buses when sharing infrastructure like lanes and bays in congested urban areas. The objective is to maximize flow for both modes. This study focuses on analyzing exclusive or preferential side or external bus lanes without physical separation from other traffic (National Association of City Transportation Officials, 2016). BRT systems operating in central lanes are not included. The FTTS approach, based on EM, is novel and further enhanced by incorporating fuzzy logic (FL) and traffic counts, making it cost-effective and efficient with immediate results. The inclusion of fuzzy parameters simplifies modeling data variability. Sensitivity analysis will determine the optimal operational scheme.

Noriega & Florian (2007) emphasized the common use of multi-class assignments to predict transportation infrastructure usage, considering various classes of traffic. This paper considers trucks and buses as interacting vehicles, taking into account their impact on travel time and congestion. FTTS results using FL provide valuable insights into congested links, aiding congestion reduction and network service improvement. FTTS is a pioneering approach using EM and FL to estimate flow for freight and transit tours, providing valuable information for decision-makers in infrastructure usage. The model was implemented using the General Algebraic Modeling System (GAMS).

4.3.1. Link performance function estimation

The sharing infrastructure issue must include condition of congestion in the network. To do so, trucks and buses flows are mixed in the lane. This interaction might achieve using functions that interrelate flow and travel cost (travel time) for a given link. Due to congestion being part of this formulation, it is necessary to include in constraints the capacity to get the relation between the travel cost and the flow on a link a (Gonzalez-Calderon, 2014).

The function can be expressed as follows:

$$t_a = t_a\left(X_a\right) \quad \text{(Eq.1)}$$

Where,

t_a : *the travel cost or travel time on a link a;*
X_a : *the flow on link a.*

The Bureau of Public Roads (BPR) link performance function is commonly used in passenger travel (Bureau of Public Roads, 1964), but not usually applied in cases involving other modes, such as this research. However, an equivalent function to the BPR can be obtained, considering the flow of both trucks and buses in link a. This function, proposed by Holguin-Veras & Cetin (2009), utilizes a Second-Order Taylor series expansion to calculate the travel time in link for different vehicle classes (passenger cars, and small and large trucks). It was later used in Gonzalez-Calderon's research (2014) and in the present study, where the travel time depends on traffic flow for both trucks and buses in the link a. Then, expanding (Eq.1) the travel time in the link is as follows:

$$t_a\left(X_t, X_b\right) = \alpha_0 + \alpha_1 X_b + \alpha_2 X_t + \alpha_3 X_b^2 + \alpha_4 X_t^2 + \alpha_5 X_b X_t \quad \text{(Eq.2)}$$

where,

X_t : *Number of truck traffic (truck flow) in link a;*
X_b : *Number of bus traffic (bus flow) in link a.*

It should be note that bus flow, X_b is function of the frequency of the buses in the link, and it could be estimated once frequency bus routes (tours) Fr_{b_m} be known.

$$X_b = f\left(Fr_{xb}\right) \quad \text{(Eq.3)}$$

The function represents the total bus flow in link a, obtained by summing the number of buses from each route using the link, multiplied by their respective frequencies. The number of buses on a route depends on its frequency. Then the number of buses using a link *a*, could be expressed as follows:

$$X_b = \sum_m Fr_{bm} * X_{bm} \tag{Eq.4}$$

$$X_{bm} = f\left(Fr_{bm}\right) \tag{Eq.5}$$

where,

X_{bm} *: Number of buses in tour m*

Fr_{bm} *: Frequency of route m*

The frequency determination allows us to find the most probable distribution of the counts for each route on the link.

Without loss of generality, in this study, travel time in link a is assumed equal for both trucks and buses. However, the travel cost may vary between the two modes, depending on the subjective value of time (SVT) for each vehicle class. SVT is multiplied by the travel time to obtain the travel cost per link for trucks and buses. The following equations express the cost for each class:

$$C_a^t = \alpha_t t_a\left(x_t, x_b\right) \tag{Eq.6}$$

$$C_a^b = \alpha_b t_a\left(x_t, x_b\right) \tag{Eq.7}$$

where,

C_a^t : *Cost for trucks in link a;*

C_a^b : *Cost for buses in link a;*

α_t : *Value of time for trucks*;*

α_b : *Value of time for buses *.*

Values in USD.

Then, the cost functions for truck tours and for bus tours are given by (Eq.8) and (Eq.9):

$$C_m^t = \sum_a C_a^t \delta_{ma} \tag{Eq.8}$$

$$C_m^b = \sum_a C_a^b \delta_{ma} \tag{Eq.9}$$

where,

δ_{ma} : *Binary variable indicating if tour m uses link a*

In (Eq.6) and (Eq.7), it is evident that higher flows of trucks and buses result in increased travel time on the route. The total cost is the sum of costs in all links.

(Eq.2) explains the flows of trucks and buses in link a, where t_a is the impedance function in that link. α_0 represents the minimum travel time in the link (free flow time). The second term, $\alpha_2 X_b$, accounts for the influence of bus flow on travel cost, while the third term, $\alpha_2 X_t$, does the same for truck flow. The next two terms, $\alpha_3 X_b^2, \alpha_4 X_t^2$, explain the interaction of each mode with itself, and the last term, $\alpha_5 X_b X_t$, represents the mutual interaction of both modes. Thus, (Eq.2) represents the entire cost function for the mix of traffic, and it cannot be divided into separate functions due to the interaction between bus and truck flows in the link.

4.3.2. FTTS Entropy Maximization Formulation

The Entropy Maximization (EM) approach in Freight and Transit Tour demand Synthesis (FTTS) aims to optimize flows in the network or a segment of it. This formulation analyzes the most likely arrangement for buses and trucks in shared lanes. The application of the entropy function to this multiclass formulation is a novel contribution to the field of transportation modeling.

Defining micro, meso, and macro states is crucial in formulating the EM function. Microstate represents a vehicle tour (bus or truck), meso-state encompasses tour flows for both vehicles, and macro-states involve aggregate representations of all tours, including the FTG and TTS at transportation analysis zones, total cost, and traffic counts. These states are specifically designed for the multiclass system in an urban area and are incorporated into the proposed multiclass tour synthesis (FTTS) EM formulation. The states are clearly defined below:

Microstate: Individual truck journey (starting and ending at an establishment base) following tour *m* and individual bus journey (starting and ending at a home base) following tour *m.*

Mesostate: *tm* is the number of truck journeys (tour flows) following tour *m*,

and *bm* is the number of buses journeys (tour flows) following tour *m*.

Macrostate: O_i^t is the total number of truck tours generated by node *i*

O_i^b is the total number of bus tours generated by node i

(Freight tours/ transit tours: starting and ending at a home base or establishment base),

C_{t_m} is the trucks tour impedance in the network,

C_{b_m} is the buses tour impedance in the network,

V_a^t is the truck observed traffic counts,

V_a^b is the bus observed traffic counts.

FTTS can be expressed as a maximization problem to determine the most probable distribution of freight tour flows and bus tour flows when traffic counts are available. This formulation is expressed as (Eq.10).

$$Max\ \mathrm{W}_{Mtb} = \frac{T!}{\prod_m t_m!} \times \frac{B!}{\prod_m b_m!} \qquad \text{(Eq.10)}$$

where,

W_{Mtb}: *System entropy that expresses the number of ways that truck tour flows and bus tour flows can be distributed;*

T : *Total number of truck tour flows in the network;*

t_m : *Number of freight journeys (truck tour flows) following tour m;*

B : *Total number of bus tour flows in the network;*

b_m : *Number of bus journeys (bus tour flows) following tour m;*

By following the Wilson (Wilson, 1967, 1969, 1970) methodology, the OF can be simplified by taking logarithms on both sides. Since (Eq.10) is a crescent monotonic function, maximizing the function is equivalent to maximizing the logarithm of the function. Thus, it can be rewritten as:

$$Max\ \mathrm{W}_{Mtb} = \frac{T!}{\prod_m t_m!} \times \frac{B!}{\prod_m b_m!} \qquad \text{(Eq.11)}$$

Now, taking logarithms on both sides of the (Eq.12):

$$Max\ Z' = \ln(W_{Mtb}) = \ln(T!B!) - \sum_{m=1}\sum_{t=1}\ln(t_m!b_m!) \tag{Eq.12}$$

As it is known, *Ln (T!B!)* can be removed from the OF since it is a constant term. It is possible to affirm that *Max Z' = ln* (W_{Mtb}) *= Min Z´´ = -ln* (W_{Mtb}). Therefore, the OF is rewritten as follows:

$$Min\ z'' = \sum_{m=1}\ln(t_m!b_m!) \tag{Eq.13}$$

Applying Stirling's approximation, where *ln x! = x ln x - x*, to (Eq.13), it obtains that:

$$Min\ Z = \sum_{m=1}(t_m \ln t_m - t_m + b_m \ln b_m - b_m) \tag{Eq.14}$$

The formulation to estimate Multiclass—freight and transit—tour demand synthesis is as follows:

$$Min\ Z = \sum_{m=1}(t_m \ln t_m - t_m + b_m \ln b_m - b_m) \tag{Eq.15}$$

Subject to:

$$O_i^t = \sum_{m=1}^{M} t_m \delta_{im}^t \qquad \forall i \in \{1, 2, ..., N\} \tag{Eq.16}$$

$$O_i^b = \sum_{m=1}^{M} b_m \delta_{im}^b \qquad \forall i \in \{1, 2, ..., N\} \tag{Eq.17}$$

$$C_m = C_m^t + C_m^b = \sum_m C_a^t \delta_{ma}^t + \sum_m C_a^b \delta_{ma}^b \tag{Eq.18}$$

$$\sum_m C_m \le C_{\max} \tag{Eq.19}$$

$$X_a^t = \sum_{m=1}^{M} t_m \delta_{ma}^t \qquad \forall a \tag{Eq.20}$$

$$X_a^b = \sum_{m=1}^{M} b_m \delta_{ma}^b \qquad \forall a \qquad \text{(Eq.21)}$$

$$X_a^b + X_a^t \le \mathrm{K}_a \qquad \forall a \qquad \text{(Eq.22)}$$

$$\left| X_a^t - V_a^t \right| \le \theta V_a^t \qquad \forall a \in \{1, 2, ..., Q\} \qquad \text{(Eq.23)}$$

$$\left| X_a^b - V_a^b \right| \le \theta V_a^b \qquad \forall a \in \{1, 2, ..., Q\} \qquad \text{(Eq.24)}$$

$$X_a^t \ge 0 \qquad \text{(Eq.25)}$$

$$X_a^b \ge 0 \qquad \text{(Eq.26)}$$

$$t_m \ge 0 \quad \forall m \in \{1, 2, ..., M\} \qquad \text{(Eq.27)}$$

$$b_m \ge 0 \quad \forall m \in \{1, 2, ..., M\} \qquad \text{(Eq.28)}$$

where,

M *: Total number of possible tours (freight and transit) in the system;*

N *: Total number of nodes in the system;*

Q *: Total number of links with traffic counts in the system;*

b_m *: Number of bus journeys (bus tour flows) following tour m;*

t_m *: Number of freight journeys (freight tour flows) following tour m;*

O_i^t *is the total number of truck tours generated by node i*

O_i^b *is the total number of bus tours generated by node i*

(Freight tours/ transit tours: starting and ending at a home base or establishment base),

C_m^t *: Cost of tour m corresponding to travel and handling in the truck tour;*

C_m^b *: Cost of tour m corresponding to travel and handling in the bus tour;*

C_m *: Total Cost of tour m;*

$C_{\max}$ *: Maximum Cost in the system;*

X_a^t *: Truck traffic flow in link a;*

X_a^b *: Bus traffic flow in link a;*

K_a : *Capacity in link a;*

V_a^t *: Observed truck traffic count in link a;*

V_a^b *: Observed bus traffic count in link a;*

δ_{mi}^t *: Binary parameter equal to 1 if node i is in freight tour m but equal to 0 otherwise;*

δ_{mi}^b *: Binary parameter equal to 1 if node i is in transit tour m but equal to 0 otherwise;*

δ_{ma}^t *: Binary parameter equal to 1 if freight tour m uses link a but equal to 0 otherwise;*

δ_{ma}^b *: Binary parameter equal to 1 if transit tour m uses link a but equal to 0 otherwise;*

θ_{TC} *: Parameter representing a percentage of traffic counts available.*

This formulation proposes an optimization program given by (Eq.15) using the EM function to reveal the most probable way tour flows of buses and trucks are distributed. It considers four sets of constraints: tour generation constraints for buses (O_i^b) and trucks (O_i^t) (Eq.16) and (Eq.17); impedance or cost of tour m (C_m) given by the sum of freight (C_m^t) and transit (C_m^b) (Eq.18), total cost in the system (the sum of C_m) constrained to a maximum cost ($C_{\max}$) (Eq.19); traffic flow in the network, for buses (X_a^b) and trucks (X_a^t) ((Eq.20) and (Eq.21), total flow in the links constrained to a maximum capacity (K_a) (Eq.22), and difference between flows and traffic counts (V_a^t for trucks and V_a^b for buses) seeking convergence criteria (Eq.23) and (Eq.24); finally, (Eq.25) and (Eq.26) are constraints for non-negativity of tour flows.

Existence and uniqueness of the formulation

In non-separable cost functions, they have a unique solution, if the Jacobians of the link cost functions are positive definite. In this case the cost function is a non-separable function since the travel time *t* in link *a* depends on the traffic flow of both trucks and buses. This is a continuous, monotonic, and continuously differentiable function; and precisely the Jacobian of the function is positive definite. Then, the solution is unique.

4.3.3. FTTS Entropy Maximization Formulation using fuzzy parameters

The deterministic modeling of FTTS can lead to infeasible solutions due to data variability. Flexibilization using FL is a valuable alternative to reduce uncertainty caused by variability. The model presented here, based on EM, is novel, and the use of FL in FTTS is also innovative, building on previous work by Moreno-Palacio et al. (2023; 2022). The fuzzy parameters in this formulation include bus and truck tour production, bus traffic counts, and truck volume, while the cost of tours remains fixed. The expected result is the bus and truck flow by tour and volume by link.

Then, the triangular parameters are the next:

- tour production/attraction at node *i*: $O_i^t = (O_i^{t1}, O_i^{t2}, O_i^{t3}));\ O_i^b = (O_i^{b1}, O_i^{b2}, O_i^{b3}))$ and
- truck and bus traffic counts at link *a:* $V_a^t = (V_a^{t1}, V_a^{t2}, V_a^{t3}));\ \mathrm{V}_a^b = (V_a^{b1}, V_a^{b2}, V_a^{b3}))$

Being: $O_i^{1t},\ O_i^{1b},\ V_a^{1t},\ \mathrm{V}_a^{1b}$ Vertex 1 for the minimum value (0% accomplishment), $O_i^{2t},\ O_i^{2b},\ V_a^{2t},\ \mathrm{V}_a^{2b}$ Vertex 2 for the target value (100% accomplishment), and $O_i^{3t},\ O_i^{3b},\ V_a^{3t},\ \mathrm{V}_a^{3b}$ Vertex 3 for the maximum value (0% accomplishment). The mathematical formulation optimizes freight and transit tours (t_m and b_m) using EM (Eq.29), similar to (Eq.15), and includes FL parameters.

$$Min\ Z = \sum_{m=1} (t_m \ln t_m - t_m + b_m \ln b_m - b_m) \qquad \text{(Eq.29)}$$

subject to

$$\sum_{m=1}^{M} t_m \delta_{im}^t = (O_i^{t1}, O_i^{t2}, O_i^{t3}), \qquad \forall i \in \{1, 2, ..., N\} \qquad \text{(Eq.30)}$$

$$\sum_{m=1}^{M} b_m \delta_{im}^b = (O_i^{b1}, O_i^{b2}, O_i^{b3}), \qquad \forall i \in \{1, 2, ..., N\} \qquad \text{(Eq.31)}$$

$$C_m = C_m^t + C_m^b = \sum_m C_a^t \delta_{ma}^t + \sum_m C_a^b \delta_{ma}^b \qquad \text{(Eq.32)}$$

$$\sum_{m} C_m \leq C_{\max} \tag{Eq.33}$$

$$X_a^t = \sum_{m=1}^{M} t_m \delta_{ma}^t = (V_a^{t1}, V_a^{t2}, V_a^{t3}), \qquad \forall a \tag{Eq.34}$$

$$X_a^b = \sum_{m=1}^{M} b_m \delta_{ma}^b = (V_a^{b1}, V_a^{b2}, V_a^{b3}), \qquad \forall a \tag{Eq.35}$$

$$X_a^b + X_a^t \leq \mathrm{K}_a \qquad \forall a \tag{Eq.36}$$

$$X_a^t \geq 0 \tag{Eq.37}$$

$$X_a^b \geq 0 \tag{Eq.38}$$

$$t_m \geq 0 \quad \forall m \in \{1, 2, \ldots, M\} \tag{Eq.39}$$

$$b_m \geq 0 \quad \forall m \in \{1, 2, \ldots, M\} \tag{Eq.40}$$

The formulation retains four sets of constraints. (Eq.30) and (Eq.31) involve fuzzy parameters for tour production of buses and trucks. (Eq.32) and (Eq.33) represent total costs for tour *m* and the entire system, limited by a maximum cost (Eq.34) to (Eq.35) express traffic flow for buses and trucks using fuzzy parameters, and total flow in the links constrained to a maximum capacity (K_a) in (Eq.36). Non-negativity constraints for tour flows are in (Eq.37) to (Eq.40).

The capacity constraint creates competition between buses and trucks for certain links, and evaluating different capacity levels can impact the solutions. Bays are crucial for the model, ensuring continuous flow and preventing lane capacity reduction when shared by trucks and buses.

The optimization problem with triangular fuzzy parameters (Eq.29) to (Eq.40) transforms into an equivalent bi-objective optimization problem:

$$Max = (\lambda, \sum_{m=1}^{M} (t_m - t_m \ln t_m + b_m - b_m \ln b_m)) \tag{Eq.41}$$

Subject to

$$\sum_{(m=1)}^{M} t_m \delta_{im}^{t} \leq \lambda O_i^{t2} + (1-\lambda) O_i^{t3}, \qquad i = 1, 2, ..., N \qquad \text{(Eq.42)}$$

$$\sum_{(m=1)}^{M} t_m \delta_{im}^{t} \geq (1-\lambda) O_i^{t1} + \lambda O_i^{t2}, \qquad i = 1, 2, ..., N \qquad \text{(Eq.43)}$$

$$\sum_{(m=1)}^{M} b_m \delta_{im}^{b} \leq \lambda O_i^{b2} + (1-\lambda) O_i^{b3}, \qquad i = 1, 2, ..., N \qquad \text{(Eq.44)}$$

$$\sum_{(m=1)}^{M} b_m \delta_{im}^{b} \geq (1-\lambda) O_i^{b1} + \lambda O_i^{b2}, \qquad i = 1, 2, ..., N \qquad \text{(Eq.45)}$$

$$C_m = C_m^t + C_m^b = \sum_m C_a^t \delta_{ma}^t + \sum_m C_a^b \delta_{ma}^b \qquad \text{(Eq.46)}$$

$$\sum_m C_m \leq C_{\max} \qquad \text{(Eq.47)}$$

$$\sum_{(m=1)}^{M} t_m \delta_{ma}^{t} \leq \lambda V_a^{t2} + (1-\lambda) V_a^{t3}, \qquad a = 1, 2, ..., Q \qquad \text{(Eq.48)}$$

$$\sum_{(m=1)}^{M} t_m \delta_{ma}^{t} \geq (1-\lambda) V_a^{t1} + \lambda V_a^{t2}, \qquad a = 1, 2, ..., Q \qquad \text{(Eq.49)}$$

$$\sum_{(m=1)}^{M} b_m \delta_{ma}^{b} \leq \lambda V_a^{b2} + (1-\lambda) V_a^{b3}, \qquad a = 1, 2, ..., Q \qquad \text{(Eq.50)}$$

$$\sum_{(m=1)}^{M} b_m \delta_{ma}^{b} \geq (1-\lambda) V_a^{b1} + \lambda V_a^{b2}, \qquad a = 1, 2, ..., Q \qquad \text{(Eq.51)}$$

$$X_a^b + X_a^t \leq \mathrm{K}_a \qquad \forall a \qquad \text{(Eq.52)}$$

$$X_a^t \geq 0 \qquad \text{(Eq.53)}$$

$$X_a^b \geq 0 \qquad \text{(Eq.54)}$$

$$t_m \geq 0 \;, \; \forall m \in \{1, 2, ..., M\}, \quad \lambda \in [0,1] \qquad \text{(Eq.55)}$$

$$b_m \geq 0, \ \forall m \in \{1, 2, ..., M\}, \quad \lambda \in [0,1] \tag{Eq.56}$$

The redefined model (Eq.41) to (Eq.56) has two objectives: maximizing entropy and lambda (λ). λ represents the minimum membership level of the flexible constraints. Fuzzy entropy increases the number of equations, with each constraint in the deterministic model replaced by at least two in fuzzy logic (three triangular vertices in this case, gives one constraint by every side of the triangle). A detail example about the transformation procedure to include fuzzy parameters in the formulation is available in (Moreno-Palacio et al., 2023). The problem is solved using the ε-constraint method.

$$max \sum_{(m=1)}^{M} (t_m - t_m ln(t_m) + b_m - b_m \ln(b_m)) \tag{Eq.57}$$

subject to

$$\boldsymbol{\lambda \geq \varepsilon} \tag{Eq.58}$$

and (Eq.42) *to* (Eq.56).

According to the multi-objective optimization theory, we solve this bi-objective model for different values of ε ∈ [0, 1] (López-Ospina et al., 2021).

4.4. Numerical experiments of FTTS (GAMS) using FL

This section presents the application of FTTS using FL with entropy maximization, which being a bi-objective formulation seeks to maximize both entropy and the minimum level of accomplishment, λ. Data required for modeling includes the network with nodes and links, freight and transit tour generation, and freight and transit links volume. Part of data from previous TTS and FTS research (Moreno-Palacio et al., 2023; Moreno-Palacio et al., 2022) were used, and the membership functions used were triangular.

The model used 13 bus tours and 15 truck tours as inputs, utilizing all links for at least one class. Shared preferential bus lanes with trucks and bays with sufficient size for both a truck and a bus were assumed to maintain uninterrupted flow (at least 30m in length). This numerical proof applies the formulation to data from the same network.

4.4.1. Network

The experiment was performed using the Sioux Falls (SF) network, a well-documented test network for investigating transportation problem. This was used by the first time by LeBlanc et al. (1975), in traffic assignment and, since then frequently used in transport research (Lee & Lim, 2002; Luathep et al., 2011; Meng et al., 2001). The SF network is comprised by 76 links and 24 nodes, and provides a suitable test-case, mixing demand with socio-economic and demographic features becoming it in a useful tool for transportation research (Chakirov, 2016; Chakirov & Fourie, 2014) still being a is a small scale one. It is shown in Figure 4-3.

This experiment seeks to obtain the most probable tour flows for both trucks and buses, while they used shared infrastructure.

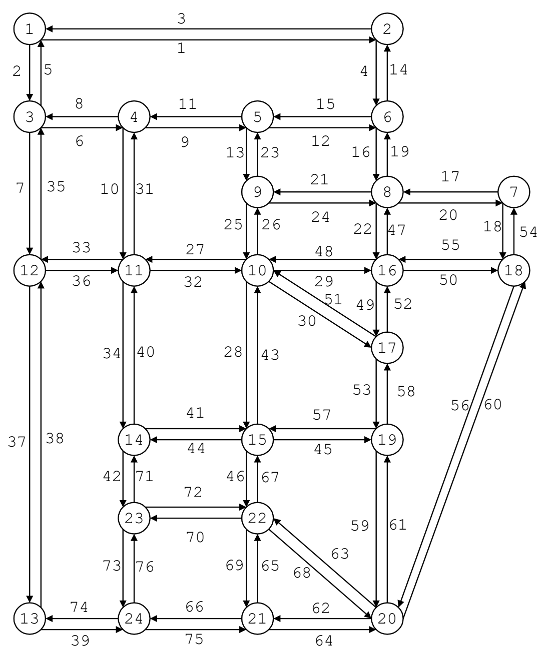


Figure 4-3. Sioux Falls Network
Source: LeBlanc et al., (1975)

To run the model and to taste the formulations—on entropy-based freight tour synthesis proposed in this paper, the code was wrote using GAMS® (General Algebraic Modeling System), a program package that includes the mathematical model obtained, the parameters, and the input data (costs, volume in links, tours production in nodes). The models were run as continuous problems.

This experiment to test the model using GAMS, assumes the use of exclusive lanes without physical separator, and with bays for stopping, for buses and trucks. This means that both buses and trucks use the same lane, with the possibility of overtaking. The model also needs to define the link performance function parameters. To do so, it was performed a simulation for a kilometer using the BPR function to obtain the travel time for each one of the records.

4.4.2. Integrated link performance function

As it was just mentioned, it was performed a simulation using the BPR function to obtain the travel time for both buses and trucks in a 1 km section. To do so, buses were assumed to constitute a maximum of 80% of the flow, with the remaining 20% being trucks. To determine the proportion of buses in the flow, a random number between 0 and 0.8 was generated and multiplied by the total flow. The rest of the flow was allocated to trucks. Both buses and trucks had a passenger car equivalent (PCE) of 1.8, as they are considered heavy vehicles. The total capacity used in this BPR simulation, was 1450 PCE/h/lane, and the speed was 60 km/h. Besides, the α value used was 0.15, and β value was 4. Without loss of generality, these assumptions did not consider factors affecting capacity such are left turns or traffic lights.

The process resulted in 1000 data points for travel time, then applying a second-order Taylor expansion regression process, the result yielded the link performance function (Eq.59). This function was used in the GAMS model for FTTS. The link performance function is as follows:

$$t_a(X_b, X_t) = 4.381\times10^{-7}\ X_b^2 + 4.814\times10^{-7}\ X_t^2 + 9.713\times10^{-7} X_b X_t \quad \text{(Eq.59)}$$

where,

X_t : *Number of truck traffic (truck flow);*

X_b : *Number of bus traffic (bus flow).*

The travel time function is quadratic and depends on both buses and trucks, with their interaction having a significant impact, approximately twice as much as their individual effects. It's crucial to highlight that the interaction between both classes is the main factor influencing travel time.

Equal time values are assumed for both buses and trucks since they are considered heavy vehicles. However, if they had different time values, the travel cost for each vehicle class per link could be computed by multiplying the time value with the link performance function.

$$c_a^b = \alpha_b(4.381\times10^{-7}\ X_b^2 + 4.814\times10^{-7}\ X_t^2 + 9.713\times10^{-7} X_b X_t)$$

(Eq.60)

$$c_a^t = \alpha_t (4.381\times10^{-7}\ X_b^2 + 4.814\times10^{-7}\ X_t^2 + 9.713\times10^{-7} X_b X_t)$$

(Eq.61)

Assuming the same time value for both, the link performance function for each one is expressed as:

$$c_a^b = c_a^t = 4.381\times10^{-7}\ X_b^2 + 4.814\times10^{-7}\ X_t^2 + 9.713\times10^{-7} X_b X_t$$

(Eq.62)

4.4.3. Analysis and Results

FTTS using FL model was tested in three scenarios. In the first scenario, high capacity and cost values were used, allowing the model to maximize lambda without constraints. The second scenario introduced capacity constraints but kept high costs unaffected. The third scenario aimed to analyze the maximum cost constraint's impact and compare the maximum flows obtained.

The non-linear optimization program with linear constraints was executed using the ε approach. Triangular membership function vertices were defined with random percentage values, and feasible solutions were found for $\varepsilon \leq 0.69$, indicating that at least one constraint must meet 69% of the target value. Higher ε values resulted in infeasible solutions due to the inherent variability in the flexible model.

4.4.3.1. Scenario 1: Analysis without capacity and cost constraints

This experiment used a capacity of 1450 PCE/hour/lane for buses in a preferential bus lane, and a maximum cost of 1000 units. The ε values ranged from 0 to 1 with a step size of 0.05, later reduced to 0.01 from $\varepsilon = 0.65$ to $\varepsilon = 0.69$.

Applying the χ2 statistical test, as suggested by Black (2018), proved useful in assessing the level of similarity between solutions (tour flows) with different ε values (López-Ospina et al., 2021; Moreno-Palacio et al., 2023; Moreno-Palacio et al., 2022), four solutions ($\varepsilon = \{0, 0.2, 0.45, 0.69\}$) were found to be statistically different in terms of tour flows. Each of these values corresponds to an entropy value as part of the solution. The experiment explored 15 thresholds for minimum accomplishment percentage.

Figure 4-4 shows achievement level versus entropy values for all solutions obtained. The x-axis represents the minimum level of accomplishment, and the y-axis shows the maximum entropy observed in each case.

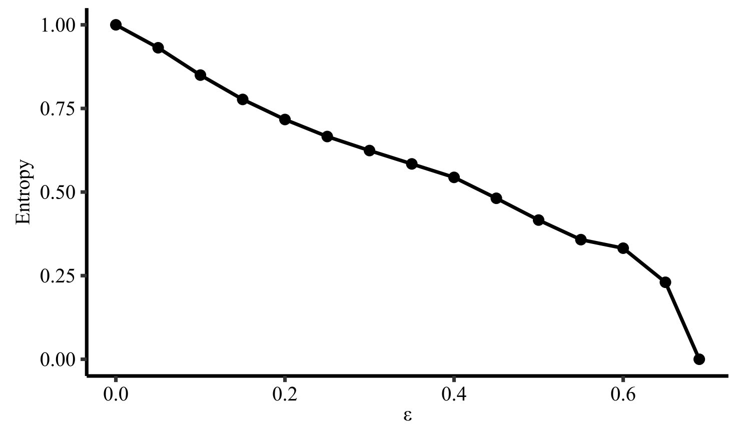


Figure 4-4. Pareto frontier Entropy vs. Epsilon.

The graph above confirms a bi-objective optimization problem, where decreasing entropy results in higher accomplishment levels, indicating a Pareto frontier. Thus, to achieve higher accomplishment levels, entropy should be reduced, and vice versa. As constraints are relaxed at different ε values, the number of trucks and buses using specific tours fluctuates significantly (Figure 4-5).

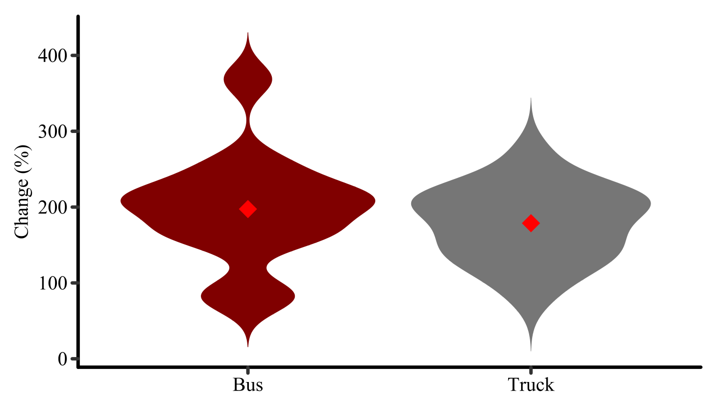


Figure 4-5. Change percentage of the flow by tour – Buses and trucks without capacity constraint

Some tours experience significant changes, with buses ranging from approximately 77% to 369%, and trucks ranging from approximately 86% to 268%. The median change is 204% for buses and 186% for trucks. On average, tour volumes changed by 197% for buses and 178% for trucks, highlighting the model's sensitivity to the epsilon value.

Figure 4-6 compares selected links to observe the volume changes as epsilon increases for both buses and trucks, focusing on statistically different solutions.

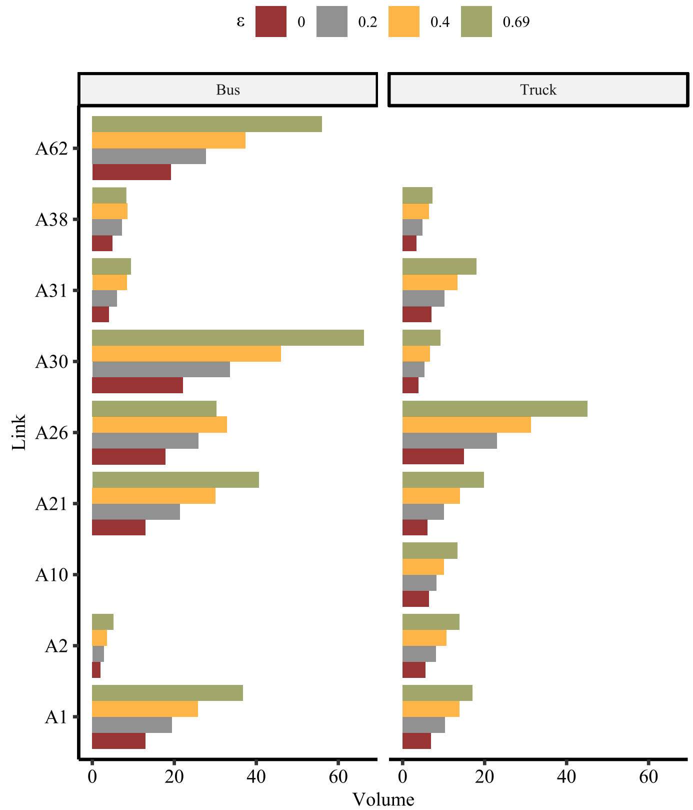


Figure 4-6. Volume by link - buses and trucks for different solutions

The overall trend in the links indicates that volume increases with higher ε-values. However, the figure reveals certain links where the trend remains consistent for initial ε-values, but for the last value, the volume decreases. Generally, as ε increases, the model facilitates an improvement in volume satisfaction for the links. However, in some cases, this value decreases, potentially due to a redistribution of tour volumes. Out of the 76 links examined, 7 experience a decrease in volume for the best solution, and all of these instances involve buses.

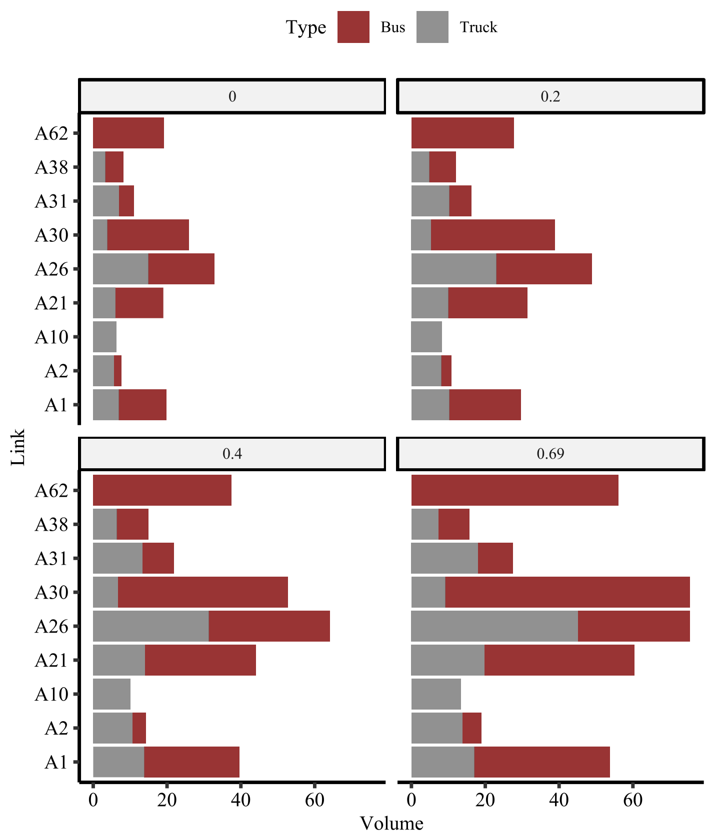


Figure 4-7. Volume by link - buses and trucks for different solutions

Figure 4-7 depicts the demand for buses and trucks on some links in each of the statistically different solutions. It is evident that, in the presented links, the volume is generally higher for buses. However, it is important to note that the solution may vary depending on the data, and the model enables us to observe demand variations in each link for both buses and trucks.

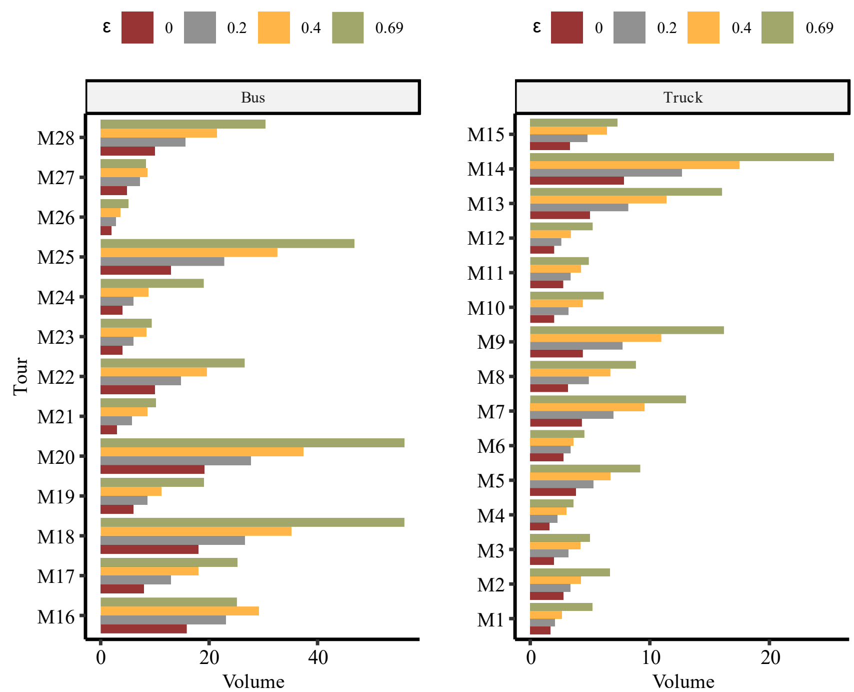


Figure 4-8. Volume by tour - buses and trucks for different solutions

Figure 4-8 showcases the 28 tours utilized in the study, with 13 designated for buses and 15 for trucks. As epsilon increases, the volume of vehicles in each tour also rises. However, there is an exception observed in tour 16, where despite the overall consistent trend, a decrease in volume is observed when transitioning from epsilon 0.4 to 0.69. This behavior may be attributed to the redistribution of links, as previously demonstrated.

During the analysis of the cost variation, it was observed that the maximum cost increased with the increment of the ε value in each solution, as depicted in Figure 4-9.

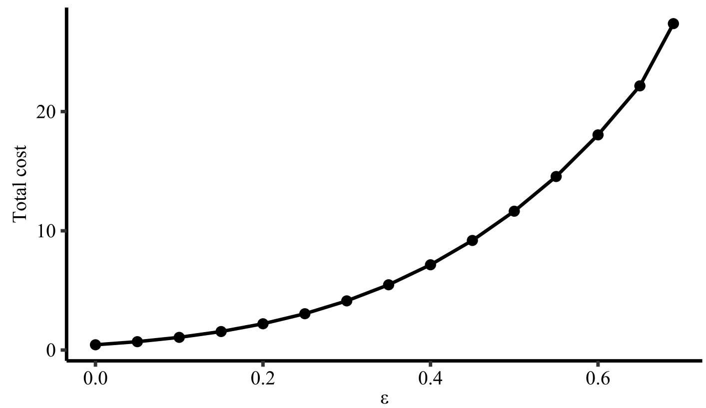


Figure 4-9. Variation of the Cost for every epsilon value

4.4.3.2. Scenario 2: Analysis with capacity constraint

This test aims to analyze the impact of capacity on the achieved solutions (tour flows). The model was run from K=75 (the first value to obtain a feasible solution with ε=0) to K=195 (ε_{max} =0.69), with steps in K of 5, resulting in 26 runs of the model. Applying the χ2 statistical test, four statistically different solutions were obtained (ε = {0, 0.19, 0.4, 0.63}).

Without loss of generality, assuming a high-capacity value of K=195 will result in a good level of service—LOS— (A, B), while a low capacity will lead to a medium-low LOS (D, E, F). This is due to the low flow of trucks and buses compared to the capacity.

Table 4-1 illustrates the impact of capacity on the ε-value. It can be observed that the best solution is obtained for a capacity of 195, and beyond this point, capacity no longer influences the solution, as the ε-value remains constant for any higher K, such as the case of K=805 (the capacity value used in section 4.4.3.1).

Table 4-1. Impact of the capacity changes on the solutions

Capacity (buses and trucks/hour/lane)	ε	Percentual tour flows change respect to the best solution					
		Buses			Trucks		
		|Min|	|Mean|	|Max|	|Min|	|Mean|	|Max|
75	0	39%	61%	79%	37%	63%	73
105	0.19	26%	45%	69%	10%	44%	61
140	0.4	3%	26%	54%	3%	27%	52
180	0.63	2%	11%	14%	1%	7	35
195	0.69	-	-	-	-	-	-
805	0.69	-	-	-	-	-	-

Additionally, the table presents the maximum, minimum, and mean variation between the tour flows obtained in each solution and those from the best solution (ε_{max} =0.69). It becomes evident that an increase in capacity leads to reduced changes across all indicators, signifying the significant role that capacity plays in determining the results.

4.4.3.3. Scenario 3: Analysis with cost constraint

The third scenario examines the impact of the maximum cost constraint on tour flows. Three comparisons are presented, using different percentages of the maximum cost: C_{min} (for ε_{max}=0), 25%, 50%, and 75% of the respective maximum cost in each case. These comparisons consider varying the subjective value of time (SVT) for buses and trucks. The scenarios include equal SVT for both, SVT_b double that of trucks, and SVT_t double that of buses. Table 4-2 shows the variation of cost and corresponding ε-values obtained. As the maximum cost increases, so does the ε-value, bringing the model closer to the best solution. The epsilon values obtained for each cost were consistent across all comparisons, with similar changes in percentage. Differences in coefficients of the link performance function for buses and trucks (see (Eq.59)) explain variations in values corresponding to C_{max} in the second and third comparisons.

Table 4-2. Impact of the cost changes on the solutions

Cost variations		ε	Percentual tour flows change respect to the best solution					
			Bus			Truck		
			|Min|	|Mean|	|Max|	|Min|	|Mean|	|Max|
Comparison Cost for STVt[1] = STVb[2]								
C_{min}	0.0178	0	37%	63%	78%	38%	61%	73%
$C_{25\%}$	0.0326	0.16	31%	50%	58%	28%	48%	64%
$C_{50\%}$	0.0652	0.38	1%	28%	45%	14%	28%	46%
$C_{75\%}$	0.0977	0.55	1%	14%	30%	1%	13%	25%
$C_{90\%}$	0.1173	0.64	2%	6%	13%	1%	6%	18%
C_{max}	0.1303	0.69	0%	0%	0%	0%	0%	0%
Comparison Cost for STVb = 2STVt								
C_{min}	0.0262	0	37%	63%	79%	39%	61%	73%
$C_{25\%}$	0.0478	0.16	32%	50%	58%	28%	48%	64%
$C_{50\%}$	0.0955	0.38	1%	28%	45%	14%	28%	46%
$C_{75\%}$	0.1433	0.55	0%	14%	30%	1%	13%	24%
$C_{90\%}$	0.1719	0.64	2%	6%	13%	1%	6%	15%
C_{max}	0.1910	0.69	0%	0%	0%	0%	0%	0%
Comparison Cost for STVt = 2STVb								
C_{min}	0.0273	0	37%	63%	78%	39%	61%	73%
$C_{25\%}$	0.0500	0.16	31%	50%	58%	28%	48%	64%
$C_{50\%}$	0.0999	0.38	1%	28%	45%	14%	29%	47%
$C_{75\%}$	0.1499	0.55	1%	14%	30%	1%	13%	26%
$C_{90\%}$	0.1798	0.64	2%	6%	13%	1%	6%	20%
C_{max}	0.1998	0.69	0%	0%	0%	0%	0%	0%

[1] Subjective value of time for trucks
[2] Subjective value of time for buses

4.5. Concluding Remarks

The experiments conducted in this paper demonstrate that both capacity and maximum cost significantly influence the solutions obtained using fuzzy parameters. When capacity is restricted, the corresponding solution's epsilon value is affected, indicating that capacity plays a crucial role in the outcome. However, once the best solution is achieved, represented by ε=0.69 in this case, further increases in capacity no longer impact the results.

It is important to acknowledge that the results may vary based on the available data. In our study, the flows did not exceed the capacity set at 1450 PCE/hour for the preferred bus lane. However, in situations where capacity is exceeded, exploring solutions for different capacity levels becomes essential. Notably, if truck stops disrupt the flow and affect capacity, the model's performance may be impacted, especially when capacity drops below 75 PCE, where feasible solutions might not be attainable. Statistical test, four statistically different solutions were obtained (ε = {0, 0.19, 0.4, 0.63}). According to results obtained and, without loss of generality, it could be assumed that a high-capacity value, such is in the case study K=195, will result in a good level of service—LOS—(A, B), while a low capacity will lead to a medium-low LOS (D, E, F). This is due to the low flow of trucks and buses compared to the capacity.

The impact of the maximum cost constraint on tour flows is significant. Increasing the maximum cost leads to higher ε-values, bringing the model closer to the best solution. This finding emphasizes the importance of considering the cost constraint in optimizing tour flows for buses and trucks. The consistent ε-values obtained across different subjective value of time (SVT) scenarios highlight the robustness of the model's performance. The observed variations in ε-values due to differences in link performance function coefficients for buses and trucks offer valuable insights into the sensitivity of tour flows to cost constraints.

The application of the FTTS model using fuzzy logic and entropy maximization offers a novel approach to estimate tour flows for shared lanes between different vehicle classes. The model's ability to incorporate traffic counts and fuzzy parameters simplifies the process and reduces study time, providing immediate and relevant results for decision-making. Additionally, the consideration of trucks and buses interacting in a multiclass formulation allows for a more accurate representation of real-world traffic conditions, accounting for congestion in traffic assignments.

In conclusion, the FTTS model using fuzzy logic and entropy maximization proves to be a promising tool for optimizing tour flows and shared infrastructure between freight and transit systems. Its ability to handle data variability and uncertainty is valuable for practical transportation planning and resource allocation, contributing to improved traffic management and more efficient infrastructure usage. This model's findings can serve as a valuable guide for decision-makers, helping them make informed choices in urban transportation planning and infrastructure development.

4.6. References

Agrawal, A. W., T. Goldman, and N. Hannaford. Shared-Use Bus Priority Lanes on City Streets: Case Studies in Design and Management. Journal of Public Transportation, Vol. 16, No. 4, 2013, pp. 25–41. https://doi.org/http://doi.org/10.5038/2375-0901.16.4.2.

Bastida, C., and J. Holguin-Veras. Freight Generation Models. Comparative Analysis of Regression Models and Multiple Classification Analysis. Transportation Research Record: Journal of the Transportation Research Board, Vol. 2097, No. 1, 2009, pp. 51–61. https://doi.org/10.3141/2097-07.

Ben-Dor, G., E. Ben-Elia, and I. Benenson. Assessing the Impacts of Dedicated Bus Lanes on Urban Traffic Congestion and Modal Split with an Agent-Based Model. Procedia Computer Science, Vol. 130, 2018, pp. 824–829. https://doi.org/10.1016/j.procs.2018.04.071.

Black, J. (2018). *Urban transport planning: Theory and practice* (Vol. 4). Routledge.

Board, T. R., of S. Engineering, and Medicine. Using Commodity Flow Survey Microdata and Other Establishment Data to Estimate the Generation of Freight, Freight Trips, and Service Trips: Guidebook. The National Academies Press, Washington, DC, USA, 2016.

Bureau of Public Roads. Traffic Assignment Manual. Washington, D.C., 1964.

Campbell, S., J. Holguin-Veras, D. G. Ramirez-Rios, C. A. Gonzalez-Calderon, L. Kalahasthi, and J. Wojtowicz. Freight and Service Parking Needs and the Role of Demand Management. European Transport Research Review, Vol. 10, No. 2, 2018, pp. 1–13. https://doi.org/http://dx.doi.org/10.1186/s12544-018-0309-5.

Chakirov, A. Sioux Falls. In The Multi-Agent Transport Simulation MATSim (A. Horni, K. Nagel, and K. W. Axhausen, eds.), Ubiquity Press, London, pp. 385–388.

Chakirov, A., and P. J. Fourie. Enriched Sioux Falls Scenario with Dynamic and Disaggregate Demand. Singapore, 2014.

Cruz-Daraviña, P. A. ;, I. Sanchez-Diaz, and J. P. Bocarejo Suescún. Bus Rapid Transit (BRT) and Urban Freight-Competition for Space in Densely Populated Cities. Sustainability, Vol. 13, No. 6611, 2021. https://doi.org/10.3390/su13126611.

Gonzalez-Calderon, C. A. Multiclass Equilibrium Demand Synthesis. PhD dissertation. Doctor of Philosophy, Rensselaer Polytechnic Institute, 2014.

Gonzalez-Calderon, C. A., and J. Holguín-Veras. Entropy-Based Freight Tour Synthesis and the Role of Traffic Count Sampling. Transportation Research Part E: Logistics and Transportation Review, Vol. 121, No. November 2017, 2019, pp. 63–83. https://doi.org/10.1016/j.tre.2017.10.010.

Gonzalez-Calderon, C. A., I. Sanchez-Diaz, I. Sarmiento-Ordosgoitia, and J. Holguin-Veras. Characterization and Analysis of Metropolitan Freight Patterns in Medellin, Colombia. European Transport Research Review, Vol. 10, No. 2, 2018, p. 23. https://doi.org/10.1186/s12544-018-0290-z.

Gonzalez-Feliu, J., and I. Sánchez-Díaz. The Influence of Aggregation Level and Category Construction on Estimation Quality for Freight Trip Generation Models. Transportation Research Part E: Logistics and Transportation Review, Vol. 121, 2019, pp. 134–148. https://doi.org/10.1016/j.tre.2018.07.007.

Gunes, S., A. Goodchild, C. Greene, and V. Nemani. Evaluating Traffic Impacts of Permitting Trucks in Transit-Only Lanes. Transportation Research Record, 2021, p. 03611981211031888.

Holguin-Veras, J. Freight Demand Modeling: State of the Art and Practice. 2013.

Holguin-Veras, J., and E. Thorson. Practical Implications of Modeling Commercial Vehicle Empty Trips. Transportation Research Record, No. 1833, 2003, pp. 87–94. https://doi.org/10.3141/1833-12.

Holguin-Veras, J., and G. R. Patil. Observed Trip Chain Behavior of Commercial Vehicles. Transportation Research Record, Vol. 1906, No. 1, 2005, pp. 74–80.

Holguin-Veras, J., and M. Cetin. Optimal Tolls for Multi-Class Traffic: Analytical Formulations and Policy Implications. Transportation Research Part A: Policy and Practice, Vol. 43, No. 4, 2009, pp. 445–467. https://doi.org/10.1016/j.tra.2008.11.012.

Holguin-Veras, J., J. Amaya-Leal, J. Wojtowicz, M. Jaller, C. A. Gonzalez-Calderon, I. Sanchez-Diaz, X. Wang, D. G. Haake, S. S. ; Rhodes, S. Darville Hodge, R. J. Frazier, M. K. Nick, J. Dack, and L. Casinelli. Improving Freight System Performance in Metropolitan Areas: A Planning Guide. Washington, D.C., 2015.

Holguin-Veras, J., M. Jaller, I. Sanchez-Diaz, S. Campbell, and C. Lawson. Freight Generation and Freight Trip Generation Models. In Modelling Freight Transport, Elsevier Inc., pp. 43–63.

Holguin-Veras, J., M. Jaller, L. Destro, X. Ban, C. Lawson, and H. Levinson. Freight Generation, Freight Trip Generation, and Perils of Using Constant Trip Rates. Transportation Research Record: Journal of the Transportation Research Board, Vol. 2224, 2011, pp. 68–81. https://doi.org/10.3141/2224-09.

Holguín-Veras, J., S. Hodge, J. Wojtowicz, C. Singh, C. Wang, M. Jaller, F. Aros-Vera, K. Ozbay, A. Weeks, M. Replogle, C. Ukegbu, J. Ban, M. Brom, S. Campbell, I. Sanchez-Díaz, C. González-Calderón, A. Kornhauser, M. Simon, S. McSherry, A. Rahman, T. Encarnación, X. Yang, D. Ramírez-Ríos, L. Kalahashti, J. Amaya, M. Silas, B. Allen, and B. Cruz. The New York City Off-Hour Delivery Program: A Business and Community-Friendly Sustainability Program. Interfaces, Vol. 48, No. 1, 2018, pp. 70–86. https://doi.org/10.1287/inte.2017.0929.

Holguin-Veras, J., T. Encarnacion, D. Ramirez-Rios, X. (Sean) He, L. Kalahasthi, S. Perez-Guzman, I. Sanchez-Diaz, and C. A. Gonzalez-Calderon. A Multiclass Tour Flow Model and Lts Role in Multiclass Freight Tour Synthesis. Transportation Science, 2020. https://doi.org/10.1287/trsc.2019.0936.

Konstantinos, K., and K. Matthew. Transit Route Network Design Problem: Review. Journal of Transportation Engineering, Vol. 135, No. 8, 2009, pp. 491–505. https://doi.org/10.1061/(ASCE)0733-947X(2009)135:8(491).

Krisztin, T. Semi-Parametric Spatial Autoregressive Models in Freight Generation Modeling. Transportation Research Part E: Logistics and Transportation Review, Vol. 114, 2018, pp. 121–143. https://doi.org/10.1016/j.tre.2018.03.003.

LeBlanc, L. J., E. K. Morlok, and W. P. Pierskalla. An Efficient Approach to Solving the Road Network Equilibrium Traffic Assignment Problem. Transportation Research, Vol. 9, No. 5, 1975, pp. 309–318. https://doi.org/10.1016/0041-1647(75)90030-1.

Lee, S., and J. Lim. Effectiveness And Macroeconomic Impact Analysis Of Policy Instruments For Sustainable Transport In Korea: A CGE Modelling Approach. WIT Transactions on the Built Environment, Vol. 60, 2002, pp. 1–12.

López-Ospina, H., C. E. Cortés, J. Pérez, R. Peña, J. C. Figueroa-García, and J. Urrutia-Mosquera. A Maximum Entropy Optimization Model for Origin-Destination Trip Matrix Estimation with Fuzzy Entropic Parameters. Transportmetrica A: Transport Science, 2021, pp. 1–38. https://doi.org/10.1080/23249935.2021.1913257.

Luathep, P., A. Sumalee, W. H. K. Lam, Z.-C. Li, and H. K. Lo. Global Optimization Method for Mixed Transportation Network Design Problem: A Mixed-Integer Linear Programming Approach. Transportation Research Part B: Methodological, Vol. 45, No. 5, 2011, pp. 808–827.

Mcleod, F., and T. Cherrett. Modelling the Impacts of Shared Freight-Public Transport Lanes in Urban Centers. 9.

Meng, Q., H. Yang, and M. G. H. Bell. An Equivalent Continuously Differentiable Model and a Locally Convergent Algorithm for the Continuous Network Design Problem. Transportation Research Part B: Methodological, Vol. 35, No. 1, 2001, pp. 83–105.

Moreno-Palacio, D. P., C. A. Gonzalez-Calderon, H. López-Ospina, J. K. Gil-Marin, and J. J. Posada-Henao. Freight Tour Synthesis Based on Entropy Maximization with Fuzzy Logic Constraints. Transportation, 2023. https://doi.org/10.1007/s11116-023-10407-y.

Moreno-Palacio, D. P., C. A. Gonzalez-Calderon, J. J. Posada-Henao, H. Lopez-Ospina, and J. K. Gil-Marin. Entropy-Based Transit Tour Synthesis Using Fuzzy Logic. Sustainability. 21. Volume 14.

Nagy, A., Z. Ercsey, J. Tick, and Z. Kovács. Bus Transport Process Network Synthesis. Acta Polytechnica Hungarica, Vol. 16, No. 7, 2019, pp. 25–43. https://doi.org/10.12700/APH.16.7.2019.7.2.

National Association of City Transportation Officials. Transit Street Design Guide. Washington, 2016.

Noriega, Y., and M. Florian. Multi-Class Demand Matrix Adjustment. CIRRELT, Canada, 2007.

Sánchez-Díaz, I., J. Holguín-Veras, and X. Wang. An Exploratory Analysis of Spatial Effects on Freight Trip Attraction. Transportation, Vol. 43, No. 1, 2016, pp. 177–196. https://doi.org/10.1007/s11116-014-9570-1.

Trubia, S., A. Severino, S. Curto, F. Arena, and G. Pau. On BRT Spread around the World: Analysis of Some Particular Cities. Infrastructures, 2020. https://doi.org/10.3390/infrastructures5100088.

Viegas, J., and B. Lu. Turn of the Century, Survival of the Compact City, Revival of Public Transport. Bottlenecks in Transportation and the Port Industry.(H. Meersman, Ed). Antwerp, Belgium, 1996, pp. 55–63.

Wang, Q., and J. Holguin-Veras. Tour-Based Entropy Maximization Formulations of Urban Commercial Vehicle Movements. 2009.

Wigan, M., M. Browne, J. Allen, and S. Anderson. Understanding the Growth in Service Trips and Developing Transport Modelling Approaches to Commercial, Service and Light Goods Movements. Association for European Transport, 2002, p. 15.

Wilson, A. G. A Statistical Theory of Spatial Distribution Models. Transportation Research, Vol. 1, No. 3, 1967, pp. 253–269. https://doi.org/10.1016/0041-1647(67)90035-4.

Wilson, A. G. Inter-regional Commodity Flows: Entropy Maximizing Approaches. Geographical Analysis, Vol. 2, No. 3, 1970, pp. 255–282. https://doi.org/10.1111/j.1538-4632.1970.tb00859.x.

Wilson, A. G. The Use of Entropy Maximising Models, in the Theory of Trip Distribution, Mode Split and Route Split. Journal of Transport Economics and Policy, Vol. 3, No. 1, 1969, pp. 108–126.

Wirasinghe, S. C., L. Kattan, M. M. Rahman, J. Hubbell, R. Thilakaratne, and & S. Anowar. Bus Rapid Transit-a Review. International Journal of Urban Sciences, Vol. 17, No. 1, 2013, pp. 1–31. https://doi.org/10.1080/12265934.2013.777514.

5. Conclusions and recommendations

5.1. Conclusions

A multi-class demand synthesis model for both transit and freight had been proposed in this research. It was obtained **applying analytical techniques**, such is entropy maximization **considering trip chain behavior using secondary data**—traffic counts— and using fuzzy parameters which incorporated flexibility and achieved include data uncertainty into the model.

The use of fuzzy logic (FL) concept into the modeling process marked a significant enhancement over classical modeling, **introducing a novel dimension to the results presented in this dissertation.**

Because of finite resources, such are capacity of the roads or parking places available, decision-makers must make the best possible decisions with the resources available. For instance, in freight tour case, they must know what routes are more prone to truck traffic to plan an infrastructure that meets their necessities without affecting other users on the road. Thus, the use of flexible models allows for considering cases where it is impossible to accomplish all constraints. Moreover, those accomplished are just part of such constraints most of the time but are still helpful in decision-making. This is a significant advantage of FL over models with fixed parameters where no solution is generated to help in the decision-making process when all constraints cannot be accomplished.

The FL is a powerful tool when a problem implies variability and uncertainty. The reason behind this is that in optimization with the fuzzy constraints technique, the obtained results are more reliable because of the more realistic shape of the models. Accordingly, FL modeling has great importance in freight transport planning (among others). On another note, the FL modeling allows for estimating relevant aspects in planning in greater detail, such as congestion behaviors, diagnostics related to emissions levels, and accidentality. This methodology can generate helpful information for decision-makers. The solutions obtained can well represent the natural behavior of transport or any other problem.

Integrating FL into the modeling process grants the model the required flexibility, which is essential as reality necessitates a certain degree of adaptability. The primary objective of this modeling approach is to meticulously represent reality to the best extent possible. FL has the potential to significantly enhance transportation planning for agencies, thereby improving urban mobility in cities and urban areas. The accuracy of results directly correlates with the quality of subsequent decisions. FL plays a pivotal role in incorporating the inherent parameter variability of daily life into input data, a reflection that should be echoed in the corresponding outputs.

In this research, triangular membership functions, which is the simplest and very common, given that it is closest to the most common distribution in data, even the choice of this function is frequently made by experts' opinion.

It is crucial to emphasize the flexibility that the fuzzy logic (FL) model offers in terms of modeling uncertainty. In light of this, a comprehensive investigation was undertaken to compare the adaptable FL model with deterministic counterparts. In FTS five deterministic scenarios were employed for this purpose. The results distinctly illustrate that the solution derived from the fuzzy model demonstrates superior performance, with its accomplishment percentage surpassing that of the deterministic cases across all dimensions. FL widens the picture for planning professionals and allows, for example, for deciding how the behavior of trucks would be if they had exclusive lanes or if the delivery time improves at a specific time in a specific zone. FTS using FL model allows the incorporation of uncertainty in the input data, such as tour production, traffic counts, and total cost in the system, which is helpful in calibrating the FTS.

In TTS the indicator "*Distance from the target value* (Δ)*"* was created to measure the distance between the model's obtained solution and the requested value or target value, which corresponds to $\lambda=1$. The problem was run several times seeking to know the resultant solutions of relaxing the parameters one by one and simultaneously. All those cases were compared with the flexible TTS model proposed solution and the FL model obtained smaller Δ values, than the ones obtained in the No FL models. This means that the use of the fuzzy parameters undoubtedly improves the performance of the model with better solutions. When considering what is real, it is not always possible to fully comply with the constraints. Planners and decision-makers must try finding an equilibrium, and these formulations can contribute to being closer to that point, decreasing the level of uncertainty. This is due, the FL model allows to obtain "satisfying solutions" instead of No solutions, when deterministic problem is unfeasible. The inclusion of fuzzy logic with the entropy maximization approach to estimate bus tour flows significantly improves the quality of the results.

The FTTS model was based on the FTS and TTS models, all of them including fuzzy parameters. Both FTS and TTS using FL models were also developed as part of this research.

In the three models, the existence of a Pareto frontier proves that the optimization problem is multi-objective, as it was presented in every correspondent chapter. For decision-maker the Pareto frontier could be a guide to choose the right model. As the Pareto frontier shows the accomplishment level versus the entropy values, it is useful to see that while the entropy decreases, the accomplishment level increases and vice versa, which means if an increase in the accomplishment level is desired, the entropy should be decreased. This face to the decision-makers to the question of which is their desired or need. For more certain in the entropy the model says that the level of accomplishment of the constraints must be low.

The problems were solved using ε approach, and when the minimum value of the membership (the lowest value for ε) is a feasible solution, it means the nonflexible problem does not have a solution with the constraints applied. This result implies that the problem has a partial solution which, for practitioners, is better than nothing. The proposed model precisely works on that option. It helps to get the best possible solution when it is not possible to satisfy each constraint fully.

Some cases exist where a feasible solution corresponds to the lowest values for ε in the minimum value of membership. This condition implies that the nonflexible problem does not have a feasible solution with the used constraints. Nevertheless, a partial solution is better than nothing for practitioners. The proposed method allows for obtaining the best solution possible when a total accomplishment of each constraint is not possible.

The experiments conducted with FTTS model demonstrate that both capacity and maximum cost significantly influence the solutions obtained using fuzzy parameters. When capacity is restricted, the corresponding solution's epsilon value is affected, indicating that capacity plays a crucial role in the outcome.

It is important to acknowledge that the results may vary based on the available data. In study presented in chapter 4, the flows did not exceed the capacity set for the preferred bus lane. However, in situations where capacity is exceeded, exploring solutions for different capacity levels becomes essential. Notably, if truck detentions disrupt the flow and affect capacity, the model's performance may be impacted, especially when capacity drops below of a value where feasible solutions might not be attainable.

Sensitivity analyses were applied for FTTS model. Those were about capacity and costs. In the case of capacity, the results obtained, and without loss of generality, shows that it could be assumed that a high-capacity value (such is in the case study K=195) will result in a good level of service—LOS— (A, B), while a low capacity will lead to a medium-low LOS (D, E, F). This is due to the low flow of trucks and buses compared to the capacity. These results shows that the model could be used **to analyze capacity and level of service of the infrastructure in the network when multi-class demand model for freight and transit is applied.**

Additionally, other **sensitivity analyses were conducted related to the maximum cost**. It verifies that the impact of the maximum cost constraint on tour flows is significant. Increasing the maximum cost leads to higher ε-values, bringing the model closer to the best solution. This finding emphasizes the importance of considering the cost constraint in optimizing tour flows for buses and trucks. The consistent ε-values obtained across different subjective value of time (SVT) scenarios highlight the robustness of the model's performance. The observed variations in ε-values due to differences in link performance function coefficients for buses and trucks offer valuable insights into the sensitivity of tour flows to cost constraints.

The application of the FTTS model using fuzzy logic and entropy maximization offers a novel approach to estimate tour flows for shared lanes between different vehicle classes. The model's ability to incorporate traffic counts and fuzzy parameters simplifies the process and reduces study time, providing immediate and relevant results for decision-making. Additionally, the consideration of trucks and buses interacting in a multiclass formulation allows for a more accurate representation of real-world traffic conditions, accounting for congestion in traffic assignments. **This served to examine the model’s feasibility considering congestion.**

The FTTS model using fuzzy logic and entropy maximization proves to be a promising tool for optimizing tour flows and shared infrastructure between freight and transit systems. Its ability to handle data variability and uncertainty is valuable for practical transportation planning and resource allocation, contributing to improved traffic management and more efficient infrastructure usage. This model's findings can serve as a valuable guide for decision-makers, helping them make informed choices in urban transportation planning and infrastructure development.

Governmental agencies responsible for overseeing mobility face the imperative of strategizing various elements, including freight and transit routes, traffic enhancement endeavors, as well as the orchestration of truck and bus routes to regulate the ebb and flow of vehicular activity in specific zones. The precision of the available data directly corresponds to the authenticity of the decisions made, influencing considerations such as the introduction of new loading/unloading bays or the expansion of parking facilities. Furthermore, this accuracy informs the identification of optimal locations for shared stops, and even the designation of corridors wherein the shared utilization of lanes could potentially outweigh the exclusive use limited to buses.

5.2. Recommendations

There are matters related to modelling process developed that should be considered in future studies. One of them is that other forms for the membership function applied to constraint flexibilization could be explored and analyze if that could have better performance.

While this research offers significant contributions to entropy-based tour demand modeling for buses and trucks, it is restricted to these two categories. Consequently, future studies should encompass a broader range of transport categories, such as the passenger car. This approach would enhance result accuracy and provide a richer dataset, enabling the identification of potential areas where shared lane and bay utilization could be implemented. This, in turn, would lead to a more efficient use of existing resources. This suggestion may provide an opportunity to be closer to reality conditions. To do so, for instance, Caliper software such are TransCad and TransModeler could be used to simulate cases study.

Moreover, an interesting analysis could have place on extended time-space networks and, as a way to prove and highlight the fuzzy logic advantages, dynamic simulations using microsimulators could be occupied.

These models had been tested using Sioux Falls network and simulated data. That work is enough and proves the robustness of the models. Even though, future studies should validate the results using the proposed models in bigger networks in order to generalize the applicability of them in urban areas.

Scientific activities and contributions during the Ph.D.

This doctoral thesis is a part of a Ph.D. process, which has consisted of the following research activities:

Publications during Ph.D. studies

- **Moreno-Palacio, D. P.,** Gonzalez-Calderon, C. A., López-Ospina, H., Gil-Marin, J. K., & Posada-Henao, J. J. (2023). Freight-Transit Tour Synthesis Entropy-Based Formulation: Sharing Infrastructure for Buses and Trucks. Accepted to presentation in *2024 TRB Annual meeting.* Track number *TRBAM-24-05924*
- **Moreno-Palacio, D. P.,** Gonzalez-Calderon, C. A., López-Ospina, H., Gil-Marin, J. K., & Posada-Henao, J. J. (2023). Freight tour synthesis based on entropy maximization with fuzzy logic constraints. *Transportation*. https://doi.org/10.1007/s11116-023-10407-y
- **Moreno-Palacio, D. P**., Gonzalez-Calderon, C. A., Posada-Henao, J. J., Lopez-Ospina, H., & Gil-Marin, J. K. (2022). Entropy-Based Transit Tour Synthesis Using Fuzzy Logic. In *Sustainability* (Vol. 14, Issue 21). https://doi.org/10.3390/su142114564
- Gonzalez-Calderon, C. A., **Moreno-Palacio, D. P**., Posada-Henao, J. J., Quintero-Giraldo, R., & Múnera, C. C. (2022). Service trip generation modeling in urban areas. *Transportation Research Part E: Logistics and Transportation Review*, *160*, 102649. https://doi.org/10.1016/J.TRE.2022.102649
- Gonzalez-Calderon, C. A., Posada-Henao, J. J., Granada-Muñoz, C. A**., Moreno-Palacio, D. P.,** & Arcila-Mena, G. (2022). Cargo bicycles as an alternative to make sustainable last-mile deliveries in Medellin, Colombia. *Case Studies on Transport Policy*, *10*(2), 1172–1187. https://doi.org/10.1016/J.CSTP.2022.04.006

Participation in conferences and academic meetings

- Entropy-Based Transit Tour Synthesis Using Fuzzy Logic. PANAM 2023. (August 2023). Guayaquil (Ecuador).

- Freight Tour Synthesis based on Entropy Maximization and Fuzzy Logic. 102nd TRB Annual meeting. (January 2023). Washington D.C. (USA).
- Freight Tour Synthesis based on Entropy - Fuzzy Logic. MIT SCALE Latin America Conference. (June 2022). USA
- Freight Tour Synthesis based on Entropy - Fuzzy Logic. 32nd POMS virtual conference 2022. (April 2022). USA
- Service trips generation modeling: an empirical investigation. 31st POMS virtual conference 2021. (December 2021). Lima (Peru).
- Integrated use of shared lanes and bays for transit and commercial vehicles. 31st POMS virtual conference 2021. (December 2021). Lima (Peru).
- Service trips generation modeling: an empirical investigation. . PANAM 2020. (August 2021). Lima (Peru).
- Integrated use of shared lanes and bays for transit and commercial vehicles. PANAM 2020. (August 2021). Lima (Peru).
- Antwerp Summer School in Urban Logistics in 2020. (September 2020). (virtual version).
- Service trips generation modeling: an empirical investigation. 2019 I-NUF Conference and 8th METRANS International Urban Freight Conference. (October 2019). Long Beach, CA (USA).
- Influence of service trips on the demand for parking space. PANAM 2018 (September 2018. Medellín (Colombia).
- Influence of service trips on the demand for parking space. VREF Urban Freight Conference (October 2018). Guthenburg (Sweden). Recognition: Best Poster Award
- Influence of service trips on the demand for parking space. VREF Advanced Studies Institute on Sustainable Urban Freight Systems —VASI-SUFS— (August, 2018). Troy, NY (USA).