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A note on testing for unit roots in the unobservable trend component of a structural model

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Resumen

Las pruebas de raíces unitarias son una práctica común en procesos estocásticos observables y se encuentra literatura abundante sobre este tema. Sin embargo, en ocasiones, aunque el problema es el mismo, los procesos de interés son latentes o no observables. En este artículo se obtienen distribuciones empíricas de las estadísticas de prueba usuales de raíces unitarias para el componente de tendencia de algunos modelos estructurales particulares, basadas en predicciones óptimas (como los datos *observados*) del proceso estocástico de tendencia. Se encuentra que estas pruebas estadísticas tienden a ser más potentes que las pruebas usuales de Dickey-Fuller.

Palabras claves: *Modelos estructurales, raíces unitarias, procesos no observables.*

Abstract

Testing for unit roots is a common practice in observable stochastic processes and there is abundant literature on this topic. However, sometimes, one is faced with the same problem but in the case where the processes of interest are latent or unobservable. In this paper, empirical distributions of the usual unit-root test statistics are obtained for the trend component of some particular structural models, which are based on optimal predictions (as the *observed* data) of the trend stochastic process. It is found that these statistical tests tend to be most powerful than the usual Dickey-Fuller tests.

Keywords: *Structural models, Unit roots, Unobservable process.*

1. Introduction

Usually, the well known unit-root tests are applied to observable stochastic processes in order to decide if, at least, a first difference of the underlying process

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is needed to get stationarity. This is the case, for example, of the works of Dickey & Fuller (1979), Phillips & Perron (1988) and Kwiatkowski, Phillips & Shin (1992).

Sometimes, the processes which we are interested in are unobservable; for example: (1) in some countries, the Gross National Product (GDP) is not observed at high-frequency periods of time, quarters say, and we might be interested in knowing if the quarterly process is integrated of order one; (2) in the Harvey's (1989) structural models approach, the so-called trend component can be assumed to be a random walk and we might be interested in testing this assumption without using tests on the variance error terms; and (3) in the unobserved components models of Schmidt & Phillips (1992) and Kaiser & Maravall (2001), it is assumed that the component processes obey ARIMA models, in particular, that they can be integrated of order one; thus, we could be interested in checking directly that assumption.

At present, there is abundant literature about the topic of testing for the presence of unit roots in observable processes, but in the case of unobservable processes only Kwiatkowski et al. (1992), Schmidt & Phillips (1992) and Harvey (2001) have addressed the problem. These authors have worked in a state-space-model context and they have tested if the variance of the so-called system-equation error terms is zero, in order to check if this decision would imply that the observable process is stationary. In this paper, and as a first step in the direction of obtaining a more general result in the future, we propose to use an optimal prediction, in the minimum mean square error (MMSE) sense, of the unobservable trend process as the *data* at hand, and then to apply the Dickey-Fuller test statistic. We remark here that this approach is a common practice in applied work. In terms of Harvey's (1989) structural models, one way of obtaining this optimal prediction is via the fixed-interval smoother. Then, we shall proceed to find the test statistic distribution via Monte Carlo simulations, in order to compare with the well-known Dickey-Fuller distribution.

The paper is organized as follows. In Section 2, we present some basic theoretical background. Section 3 includes the simulation results for two simple structural models, the so-called random walk plus noise model and the local linear trend model. Section 4 shows an application of the results to the analysis of the monthly Colombian real production index in the sample period 1983:01-1996:12. Finally, Section 5 concludes.

2. Theoretical background

Here, we present a brief summary about the concepts of state space models, structural models and unit-roots testing. The interested reader can see Harvey's (1989) and Fuller's (1996) books for more precise details.

2.1. State space models

Let $\{Y_t\}$ be a univariate stochastic process that obeys the so-called *observation* equation:

$$Y_t = z_t \gamma_t + \varepsilon_t \ , \qquad t = 0, \pm 1, \pm 2, \ldots ,$$

where z_t is a p–dimensional deterministic row vector, γ_t is a p–dimensional random vector and $\{\varepsilon_t\}$ is a univariate noise process with $\text{var}(\varepsilon_t) = h_t$. The stochastic process $\{\gamma_t\}$ can be unobservable and its dynamic is given by the so-called *system* equation:

$$\gamma_t = T_t \gamma_{t-1} + \boldsymbol{\eta}_t$$

where T_t is a deterministic matrix of dimension $p \times p$ and $\{\boldsymbol{\eta}_t\}$ is a p–dimensional noise process with $\text{var}(\boldsymbol{\eta}_t) = Q_t$. These two equations define a state space model for the process $\{Y_t\}$.

Given a set of observations $y_1, \ldots, y_n$ from the process $\{Y_t\}$, two main estimation problems arise in the context of state space models: (1) to predict γ_t for each $t = 1, \ldots, n$ with base on $y_1, \ldots, y_n$, assuming that the model matrices z_t, h_t, T_t, and Q_t are known and that we know an initial prediction of γ_0, with minimum mean square error P_0 (a matrix). (2) to estimate eventual unknown fixed parameters in the model matrices. The second problem is solved by means of the maximum likelihood approach and the first one by means of a recursive prediction algorithm that is termed fixed-interval smoother. This is given by the following expressions: let $\gamma_{t|s}$ be the optimal prediction of γ_t with base on $y_1, \ldots, y_s$, where $s = 1, 2, \ldots, n$, and let $P_{t|s}$ be its mean square error matrix. Then, for each $t = n-1, \ldots, 1$:

$$\gamma_{t|n} = \gamma_{t|t} + P_t^* (\gamma_{t+1|n} - T_{t+1} \gamma_{t|t})$$

and

$$P_{t|n} = P_{t|t} + P_t^* (P_{t+1|n} - P_{t+1|t}) P_t^*$$

where $P_t^* = P_{t|t} T'_{t+1} P_{t+1|t}^{-1}$. The quantities $\gamma_{t|t}$ and $P_{t|t}$ are computed via the Kalman filter Harvey (1989) and $P_{t+1|t}$ is the mean square error matrix of the one-step prediction Harvey (1989).

The most important point to be taken into account, here, is that the fixed-interval smoothed gives us the tools for predicting the potential unobservable process $\{\gamma_t\}$, and, in this way, we can generate “observations” for it.

2.2. Structural models

Essentially, a structural model for the process $\{Y_t\}$ is given by the equation

$$Y_t = \mu_t + s_t + c_t + \varepsilon_t$$

where μ_t, s_t, c_t, and ε_t represent unobserved components. Tipically, μ_t is the trend component, s_t is the seasonal component, c_t represents a cycle, and ε_t is the irregular or noise component. Harvey (1989) has developed an important work for the analysis of these models. As examples, we have the so-called random walk plus noise (RWPN) model and the local linear trend (LLT) model. The first one is given by the equations

$$Y_t = \alpha_t + \varepsilon_t$$

$$\alpha_t = \alpha_{t-1} + \eta_t$$

where $\{\varepsilon_t\}$ and $\{\eta_t\}$ are Gaussian zero-mean white noise processes, with variances σ^2_ε and σ^2_η, respectively, and $\{\varepsilon_t\}$ is mutually independent of $\{\eta_t\}$, in the sense that finite-dimensional vectors of variables of each process are mutually independent. In the second, the structural equation is the same as before but, now, the trend component is given by

$$\begin{aligned} \alpha_t &= \alpha_{t-1} + \beta_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \lambda_t \end{aligned}$$

where $\{\eta_t\}$ and $\{\lambda_t\}$ are Gaussian zero-mean white noise processes with respective variances σ^2_η and σ^2_λ and $\{\varepsilon_t\}$, $\{\eta_t\}$ and $\{\lambda_t\}$ are mutually independent. Here, β_t plays the role of a time-varying slope.

A structural model can be cast into the state space form. Indeed, consider for example the LLT model and define $\gamma_t = (\alpha_t, \beta_t)'$, $z_t = (1, 0)$, $\boldsymbol{\eta}_t = (\eta_t, \lambda_t)'$, and

$$T_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Using the fixed-interval smoother we can obtain optimal predictions for the unobserved components of a strucutral model. In particular, one has predictions for the trend α_t and the slope β_t in the LLT model. These kind of predictions will be the base for the anlayses in Section 3.

2.3. Unit roots testing

One way of understanding the problem of testing for unit roots in a stochastic process is through autoregressive models. In his pionering work, Fuller (1996) put the problem under the following scheme: let us assume that the process $\{Y_t\}$ follows the AR(1) model:

$$Y_t = \rho Y_{t-1} + a_t$$

where $|\rho| \leq 1$ and $\{a_t\}$ is a zero-mean white noise process with variance σ^2. If $\rho = 1$, $\{Y_t\}$ is nonstationary but its first difference *is* stationary. To test the null hypothesis of nonstationarity of $\{Y_t\}$, i.e., $H_0 : \rho = 1$, Dickey & Fuller (1979) proposed the statistic

$$\tau \;=\; \left(s^{-2}\sum_{t=2}^{n} Y_{t-1}^2\right)^{1/2}(\hat{\rho} - 1)$$

where

$$s^2 \;=\; \frac{1}{n-2}\sum_{t=2}^{n}(Y_t - \hat{\rho}Y_{t-1})^2$$

and $\hat{\rho}$ is the least squares estimator of the autoregressive coefficient ρ. Tables with the main percentiles of the test statistic distribution, for different sample sizes, are found in Fuller's (1996) book. The reader is invited to consult that book and its tables for more details about the testing strategy.

3. The simulation study

We shall consider the RWPN and LLT models defined in the preceding section. In the case of the RWPN model, the design of the simulation experiment was the following: we simulated the noise processes $\{\varepsilon_t\}$ and $\{\eta_t\}$, 10000 times each, for the sample sizes $n = 25, 50, 100, 250, 500, 2000$. For comparison purposes with the Dickey-Fuller tables, the sample size 2000 is assimilated to the ideal sample size ∞ that was written in the Dickey & Fuller's (1979) tables. In each case, we computed firstly (simulated) observations for process $\{\alpha_t\}$ and then we obtained the corresponding time series for process $\{Y_t\}$. With these time series, we obtained the predicted process $\{\hat{\alpha}_t\}$ using the fixed-interval smoother, and then we computed the corresponding Dickey-Fuller statistic for each predicted process in each sample-size case. In this way, we obtained the empirical distribution of the τ statistic under the null hypothesis that the process $\{\alpha_t\}$ has one unit root. In Table 1 we present this distribution, where the table entries correspond to the values of the test statistic. These are the figures needed for the comparison with the Dickey & Fuller's (1979) distribution.

Table 1: Empirical quantiles for the τ statistic in the RWPN model

Quantile order(/100)	Sample size					
	25	50	100	250	500	2000
1.00	−2.10	−1.89	−1.79	−1.68	−1.72	−1.68
2.50	−1.75	−1.61	−1.50	−1.44	−1.45	−1.44
5.00	−1.44	−1.37	−1.28	−1.24	−1.26	−1.23
10.0	−1.15	−1.09	−1.05	−1.00	−1.02	−1.00
50.0	0.00	−0.01	0.01	0.03	0.04	0.02
90.0	1.78	1.78	1.78	1.86	1.80	1.82
95.0	2.30	2.34	2.35	2.42	2.30	2.36
97.5	2.80	2.86	2.87	2.92	2.79	2.80
99.0	3.36	3.48	3.45	3.47	3.43	3.28

We can see in this table that (1) the shown quantiles are greater than those of Dickey & Fuller (1979) for all of the sample sizes considered and (2) the empirical distribution is centered around 0. The first fact signals that using the Dickey-Fuller quantiles for testing the null hypothesis of unit root in $\{\alpha_t\}$, based on the *predicted* processes, could led to accept more frequently this null hypothesis. The second indicates that the distribution of the test statistic tends to be translated towards the right, when compared to that of Dickey & Fuller (1979).

Returning to the first remark above, this fact could be interpreted as a loss of power of the conventional Dickey-Fuller test. In fact, we have computed the power of the test using our found empirical quantiles in the following way: we consider the system equation $\alpha_t = \rho\alpha_{t-1} + \eta_t$, for $\rho = 0.8, 0.9, 0.95, 0.99$, and repeat for each value of ρ the simulation experiment as indicated above for the same sample sizes considered. The main findings were the following (see Table 4 in the Appendix, where the entries are the power values): (1) the more the sample size and the farther ρ is from 1, the more powerful the test is, in the sense of obtaining relative frequencies of rejection close to 1, whatever the level is. (2) for values of ρ close to 1, the frequency of rejection tends to be small, as expected. For example, for the sample size $n = 250$ and level 0.05, the null hypothesis is rejected only in 16% of the cases if $\rho = 0.99$, and for $\rho = 0.95$ it is rejected in the 89% of the simulations.

Using the Dickey & Fuller's (1979) quantiles, we also computed the power of the test for the same simulated experiment and we found the following general facts for any conventional level (see Table 5 in the Appendix): (1) for small sample sizes our suggested test is most powerful than the Dickey–Fuller one, (2) for large sample sizes, the power of both tests are practically equal to 100%.

Now, the random walk plus noise model is modified in the sense of including a drift in the process $\{\alpha_t\}$, that is to say, the system equation becomes:

$$\alpha_t \; = \; a + \alpha_{t-1} + \eta_t$$

where a is a constant. The values 1, 5, 10, -1, -5, and -10 were considered for a and the simulation experiment was repeated under the same conditions than before. In Table 2 we present the empirical distribution found in this case. As can be seen there, the empirical distribution differs from that of the non-drift model and, in particular, the quantiles values are smaller than those of the non-drift case. This also happens with the conventional Dickey–Fuller quantiles. Comparing the found quantiles with those of Dickey & Fuller (1979), we found that the empirical ones are less than those ones. With respect to the power of the test in this case, we found that this is also greater than that of the Dickey–Fuller test (see Table 7 in the Appendix), especially when ρ is far from 1 and the sample size is large.

It is important to note here that, strictly speaking, little numerical differences among the quantiles values for each drift were found and that the values reported in Table 2 are the medians of those figures. For example, for $n = 25$ the 0.01 quantiles obtained for each of the drifts considered (drift value in parenthesis) were $-3.72(-10)$, $-3.86(-5)$, $-4.04(-1)$, $-4.02(1)$, $-3.79(5)$, and $-3.79(10)$. Its median value is -3.79. These little numerical differences are due to the influence of each drift in the point prediction of α_t, but as is known, the intercept term in the system equation is not determinant for the stochastic dynamic of the process $\{\alpha_t\}$. Hence, we can take a smoothed value of these different figures in order to have a unique value.

Now we consider the LLT model. The goal here is to check for the presence of a unit root in the stochastic process $\{\beta_t\}$. Notice that if this is case, the process $\{Y_t\}$ is I(2). Once more again, we proceeded via simulation to find the corresponding

Table 2: Empirical distribution of τ in the case of the RWPN model with drift

Quantile order(/100)	Sample size					
	25	50	100	250	500	2000
1.00	−3.787	−3.562	−3.564	−3.573	−3.501	−3.443
2.50	−3.104	−3.008	−2.970	−2.974	−2.964	−2.942
5.00	−2.616	−2.524	−2.503	−2.527	−2.483	−2.469
10.0	−2.034	−1.971	−1.947	−1.963	−1.974	−1.942
50.0	−0.083	−0.048	−0.053	−0.025	−0.029	−0.021
90.0	1.841	1.887	1.875	1.876	1.906	1.896
95.0	2.410	2.421	2.450	2.427	2.451	2.430
97.5	2.859	2.914	2.936	2.892	2.904	2.910
99.0	3.499	3.498	3.485	3.414	3.455	3.443

distribution for the τ statistic using the predicted process $\{\hat{\beta}_t\}$, which is obtained by means of the fixed-interval smoother, where now the state vector is given by $(\alpha_t, \beta_t)'$. As can be seen, our interest is the second component of this vector. In Table 3 we present the empirical distribution, where we can note the following facts: (1) the distribution is centered around 0.22 for any of the sample sizes considered, and this differs of the central-tendency value for the RWPN model. (2) comparing with the same model, the distribution range is greater, which shows more dispersion in this case. (3) the empirical quantiles are less than those of the Dickey-Fuller statistic, as was also found for the RWPN model with drift.

In Table 8 in the Appendix we present the results on the test power study. As before, the larger the sample size and farther from 1 the coefficient of β_{t-1} is, the most powerful the test is. Using the Dickey-Fuller quantiles for implementing the test, we found that the test loss power, as can be seen in Table 9 in the Appendix.

4. An empirical application

We consider the monthly Colombian real production index (RPI) in the time period 1983:01-1996:12, with $T = 168$ observations. The time series is plotted in Figure 1 and the data are available from the authors upon request. Initially, we take the logarithmic transformation and then we deseasonalized the time series using the X11 procedure implemented in the SAS package. As can be seen in that Figure, one is led to specify two models; namely, the LLT model and the RWPN model with drift. Using Koopman, Harvey, Doornik & Shephard (1995) STAMP package for the analysis of the models, we found that the second model is appropriate for describing the dynamical behavior of the process. The estimated parameters were $\hat{\sigma}_\varepsilon^2 = 0.03^2$, $\hat{\sigma}_\eta^2 = 0.01^2$, and $\hat{a} = 1.0$. The model residuals exhibited good behavior at the light of conventional statistical tests.

Table 3: Empirical distribution for the τ statistic in LLT model

Quantile order(/100)	Sample size					
	25	50	100	250	500	2000
1.00	−2.13	−1.76	−1.47	−1.43	−1.35	−1.35
2.50	−1.69	−1.41	−1.27	−1.21	−1.17	−1.18
5.00	−1.38	−1.18	−1.07	−1.04	−1.00	−0.98
10.0	−1.07	−0.91	−0.87	−0.82	−0.80	−0.80
50.0	0.23	0.20	0.20	0.22	0.23	0.24
90.0	2.20	2.22	2.26	2.24	2.29	2.25
95.0	2.74	2.84	2.90	2.91	2.89	2.87
97.5	3.27	3.42	3.45	3.53	3.44	3.45
99.0	3.82	4.12	4.04	4.21	4.14	4.16

For testing the null hypothesis of a unit root in the trend process $\{\alpha_t\}$, we predicted it by means of the fixed-interval smoother and then we computed the corresponding test statistic finding that $\hat{\tau} = -2.49$. Hence, at the 5% level the null is not rejected (see Table 2). It is important to note here that we need to do an interpolation between the sample sizes 100 and 250. The decision implies that the deseasonalized process $\{\ln(RPI_t)\}$ has a unit root, but the most important conclusion here is that the *unobservable* underlying trend component of this major Colombian macroeconomic variable (in logs) *is* I(1). In Figure 2 we present the predicted trend, where it is clear the influence of the drift.

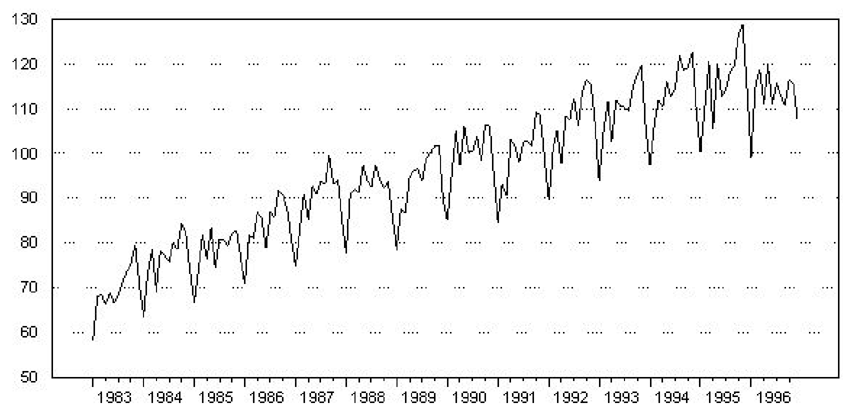

Figure 1: Colombian real production index for the sample period 1983:01-1996:12

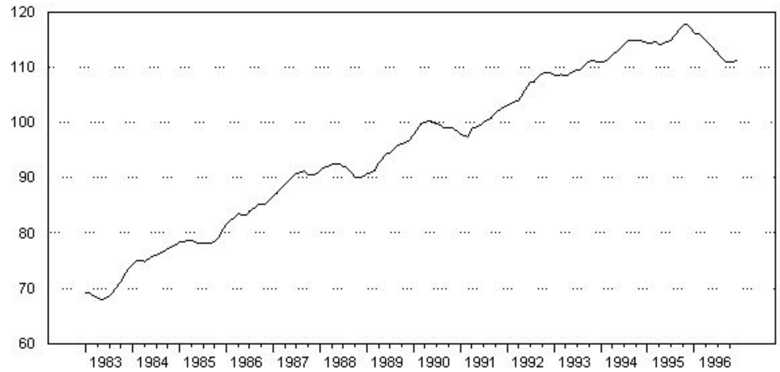

Figure 2: Predicted trend for the Colombian real production index

5. Conclusions

We have found a way for checking assumptions made directly on the unobservable trend component of an structural process, as that of having unit roots. The main finding is that the usual Dickey-Fuller test statistic distribution is not appropriate for checking the null of a unit root in the trend process, when predictions of it are used as the observed data.

The proposed statistical tests have power close to 1 for large sample sizes and from this point of view, we can say that they are very adequate. Comparing their performance with the Dickey-Fuller tests, the proposed ones tend to be most powerful, especially in small or moderate sample sizes.

The approach followed here can be used for obtaining similar statistical tests for other structural models, in Harvey's (1989) philosophy, including unobservable seasonal components and cycles. Nevertheless, some theoretical work remains to be done for checking the convergence in distribution of the proposed test statistics.

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A. Appendix

On the next pages we present Tables 4-9.

Table 4: Unit root test power using the empirical distribution for the RWPN model

Significance level (%)	Sample size					
	25	50	100	250	500	2000
$\rho = 0.80$						
1	0.07	0.34	0.93	1.00	1.00	1.00
2.5	0.18	0.57	0.98	1.00	1.00	1.00
5	0.34	0.77	0.99	1.00	1.00	1.00
10	0.56	0.91	1.00	1.00	1.00	1.00
$\rho = 0.90$						
1	0.03	0.07	0.32	0.98	1.00	1.00
2.5	0.07	0.17	0.57	1.00	1.00	1.00
5	0.16	0.31	0.76	1.00	1.00	1.00
10	0.28	0.54	0.90	1.00	1.00	1.00
$\rho = 0.95$						
1	0.02	0.03	0.08	0.52	0.97	1.00
2.5	0.04	0.07	0.19	0.75	0.99	1.00
5	0.09	0.14	0.33	0.89	1.00	1.00
10	0.17	0.27	0.54	0.96	1.00	1.00
$\rho = 0.99$						
1	0.01	0.01	0.01	0.04	0.07	1.00
2.5	0.03	0.03	0.04	0.09	0.18	1.00
5	0.06	0.06	0.08	0.16	0.31	1.00
10	0.12	0.12	0.15	0.29	0.52	1.00

Table 5: Unit root test power for the RWPN model using Dickey–Fuller quantiles

Significance level (%)	Sample size					
	25	50	100	250	500	2000
$\rho = 0.80$						
1	0.02	0.04	0.30	1.00	1.00	1.00
2.5	0.05	0.14	0.61	1.00	1.00	1.00
5	0.11	0.29	0.85	1.00	1.00	1.00
10	0.24	0.56	0.97	1.00	1.00	1.00
$\rho = 0.90$						
1	0.01	0.01	0.02	0.41	1.00	1.00
2.5	0.02	0.03	0.08	0.73	1.00	1.00
5	0.04	0.07	0.21	0.92	1.00	1.00
10	0.10	0.18	0.47	0.99	1.00	1.00
$\rho = 0.95$						
1	0.00	0.00	0.00	0.02	0.33	1.00
2.5	0.01	0.01	0.02	0.10	0.68	1.00
5	0.03	0.02	0.04	0.27	0.90	1.00
10	0.06	0.07	0.14	0.58	0.98	1.00
$\rho = 0.99$						
1	0.00	0.00	0.00	0.00	0.00	0.99
2.5	0.01	0.00	0.00	0.00	0.01	1.00
5	0.02	0.01	0.01	0.01	0.03	1.00
10	0.04	0.03	0.03	0.04	0.10	1.00

Table 6: Unit root test power for the RWPN model with drift

Significance level (%)	Sample size					
	25	50	100	250	500	2000
$\rho = 0.80$						
1	0.03	0.03	0.06	0.59	1.00	1.00
2.5	0.09	0.11	0.25	0.94	1.00	1.00
5	0.21	0.27	0.55	1.00	1.00	1.00
10	0.28	0.32	0.59	1.00	1.00	1.00
$\rho = 0.90$						
1	0.07	0.12	0.18	0.42	0.88	1.00
2.5	0.16	0.26	0.41	0.80	0.99	1.00
5	0.30	0.47	0.67	0.95	1.00	1.00
10	0.35	0.54	0.73	0.98	1.00	1.00
$\rho = 0.95$						
1	0.06	0.19	0.49	0.81	0.94	1.00
2.5	0.14	0.34	0.71	0.96	0.99	1.00
5	0.25	0.52	0.85	0.99	1.00	1.00
10	0.33	0.59	0.92	1.00	1.00	1.00
$\rho = 0.99$						
1	0.02	0.05	0.26	0.97	1.00	1.00
2.5	0.04	0.10	0.40	0.99	1.00	1.00
5	0.09	0.18	0.55	1.00	1.00	1.00
10	0.12	0.24	0.60	1.00	1.00	1.00

Table 7: Unit root test power for the RWPN model with drift using Dickey–Fuller quantiles

Significance level (%)	Sample size					
	25	50	100	250	500	2000
$\rho = 0.80$						
1	0.05	0.06	0.11	0.72	1.00	1.00
2.5	0.09	0.12	0.22	0.90	1.00	1.00
5	0.14	0.20	0.37	0.97	1.00	1.00
10	0.22	0.31	0.46	0.99	1.00	1.00
$\rho = 0.90$						
1	0.10	0.18	0.25	0.53	0.95	1.00
2.5	0.16	0.28	0.39	0.73	0.99	1.00
5	0.23	0.38	0.53	0.86	1.00	1.00
10	0.39	0.50	0.64	0.92	1.00	1.00
$\rho = 0.95$						
1	0.09	0.26	0.57	0.87	0.97	1.00
2.5	0.14	0.36	0.69	0.94	0.99	1.00
5	0.19	0.45	0.78	0.97	1.00	1.00
10	0.27	0.54	0.87	0.95	1.00	1.00
$\rho = 0.99$						
1	0.02	0.07	0.31	0.98	1.00	1.00
2.5	0.04	0.11	0.39	0.99	1.00	1.00
5	0.06	0.15	0.47	0.99	1.00	1.00
10	0.11	0.24	0.54	1.00	1.00	1.00

Table 8: Unit root test power for the LLT model

Significance level (%)	Sample size					
	25	50	100	250	500	2000
$\rho = 0.80$						
1	0.05	0.20	0.72	1.00	1.00	1.00
2.5	0.11	0.38	0.86	1.00	1.00	1.00
5	0.21	0.55	0.93	1.00	1.00	1.00
10	0.33	0.61	0.96	1.00	1.00	1.00
$\rho = 0.90$						
1	0.02	0.08	0.25	0.94	1.00	1.00
2.5	0.06	0.16	0.44	0.97	1.00	1.00
5	0.11	0.28	0.64	0.98	1.00	1.00
10	0.23	0.39	0.76	1.00	1.00	1.00
$\rho = 0.95$						
1	0.02	0.04	0.07	0.49	0.95	1.00
2.5	0.04	0.08	0.15	0.69	0.98	1.00
5	0.07	0.14	0.28	0.82	0.99	1.00
10	0.12	0.27	0.41	0.94	1.00	1.00
$\rho = 0.99$						
1	0.01	0.01	0.02	0.04	0.09	0.85
2.5	0.03	0.04	0.04	0.08	0.18	0.94
5	0.05	0.07	0.08	0.15	0.31	0.97
10	0.10	0.08	0.09	0.27	0.42	0.99

Table 9: Unit root test power in the LLT model using Dickey–Fuller quantiles

Significance level (%)	Sample size					
	25	50	100	250	500	2000
$\rho = 0.80$						
1	0.02	0.01	0.03	0.64	1.00	1.00
2.5	0.03	0.03	0.10	0.91	1.00	1.00
5	0.06	0.07	0.28	0.98	1.00	1.00
10	0.09	0.12	0.34	1.00	1.00	1.00
$\rho = 0.90$						
1	0.01	0.00	0.00	0.04	0.70	1.00
2.5	0.02	0.01	0.01	0.20	0.95	1.00
5	0.03	0.03	0.05	0.47	0.99	1.00
10	0.05	0.04	0.06	0.62	1.00	1.00
$\rho = 0.95$						
1	0.01	0.00	0.00	0.00	0.03	1.00
2.5	0.01	0.01	0.00	0.01	0.18	1.00
5	0.02	0.01	0.01	0.06	0.48	1.00
10	0.04	0.02	0.04	0.08	0.57	1.00
$\rho = 0.99$						
1	0.00	0.00	0.00	0.00	0.00	0.01
2.5	0.01	0.00	0.00	0.00	0.00	0.07
5	0.02	0.00	0.00	0.00	0.00	0.25
10	0.04	0.02	0.02	0.02	0.01	0.29