A New Relativistic Wave Equation for a Massive Boson and Fermion System

Julio Cesar Jaramillo Q*
Departament of Physics National University of Colombia
Bogota D.C. - Colombia

Abstract

This article pretends to show some results from the study of The Relativistic Wave Equation in order to describe a massive system of boson and fermion. We defined the scattering matrix V_{fi} on the electromagnetic case and concurrently we extended it to the strong and weak interactions of the vector meson with the fermions. We also studied some applications to hadronic physics ,Quantum Hadrondynamics (QHD-I) and neutrino physics.

1 Introduction

In the years of 1926 and 1927 Klein and Gordon proposed a relativistic wave equation for particles with spin 0, which originally is Schrodinger Relativistic Equation [1][2]. Duffin - Kemmer - Petiau developed a relativistic theory that described the dynamics of a system of particles of spin 0 and 1 [3][4][5][6]. Proca stated a relativistic wave equation for massive particles with spin 1 [7]. Dirac in 1938 proposed a wave equation that unifies the special relativity with the spin of particles, particulary the electron [8]. Fierz - Pauli proposed a relativistic wave equation for the arbitrary spin particles [10]. Rarita - Schwinger stated a relativistic wave equation which describes the dynamics of the particles with spin 3/2 [11]. Bhabha described at the beginning of his theory the dynamics of protons through a relativistic wave equation as an exact copy of Dirac's equation [12]. Concurrently in the 40th many theories were developed in which are described the interactions of elementary particles with the gauge bosons which are the cause of the interaction, and these theories are known as Quantum Field Theories. In the outset it was developed the Quantum Electrodynamics (QED) which describes the interaction of the fermions with the electromagnetic field as the first approach to the lowest perturbative order. In the case of the strong interaction, it was stated a field theory as a copy of quantum electrodynamics which corresponds to Quantum Chromodynamics (QCD) that describes the interaction of nucleons with the gluons. Thung defined relativistic wave equation and the field theories with arbitrary spin [13]. An extension of quantum chromodynamics is the one presented by Quantum Hadrondynamics (QHD-I) which describes the dynamics of the scalar and vector mesons with the baryons [14]. Niederle defines a wave equation for the massive particles with an arbitrary rational spin number [15]. Silenko verifies the accuracy of the wave equations for particles with spin 1 [30]. Tutik and Kulikov describe the dynamics of fermions with massive bosons with integer spin through a relativistic wave equation [23]. This article contains the relativistic wave equation for massive bosons and fermions on the basis to the works referred above. For the electromagnetic case we defined the electromagnetic tensor which describes the fields produced by this system of particles. Furthermore some applications are mentioned. The relevance of this equation to hadronic physics is the deduction of the wave function of the hadrons as a product of the baryonic function with the meson function in terms of the quarks in a full manner. Another definition of the electromagnetic tensor considering the case the coupling given by the vectorial meson and the electromagnetic field on the basis of the work of Walecka [18]. The scattering matrix element V_{fi} is defined on the basis of Bjorken - Drell text about the propagator theory [19] and we propose an extension of the strong, weak and electromagnetic interaction of the vector meson with the fermions and an applications to neutrino physics.

*jcjaramilloq@unal.edu.co

2 Relativistic Wave equation for the fermion - boson with spin 1 system

The starting point in order to deduct the relativistic wave equation for this system is writing the Dirac and Proca equations:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \tag{1}$$

$$\partial_\lambda \psi^{\lambda\nu} - m_0^2 \psi^\nu = j^\nu \tag{2}$$

multiplying the eq.(1) on the right by $\psi^{\lambda\nu}$ and the eq. (2) on the left by $i\gamma^\lambda\psi$ and adding, taking into account that $i\gamma^\mu\partial_\mu = i\gamma^\lambda\partial_\lambda = i\gamma^0\frac{\partial}{\partial t} - i\boldsymbol{\gamma}\cdot\nabla$, we obtain

$$(i\gamma^\mu \partial_\mu - m)\Psi^{\lambda\nu} - im_0^2\gamma^\lambda\psi^\nu\psi = i\gamma^\lambda j^\nu\psi \tag{3}$$

and we can write $\gamma^\lambda\psi^\nu = \frac{1}{2}(\gamma^\lambda\psi^\nu - \gamma^\nu\psi^\lambda) + \frac{1}{2}(\gamma^\lambda\psi^\nu + \gamma^\nu\psi^\lambda)$ and $\gamma^\lambda j^\nu = \frac{1}{2}(\gamma^\lambda j^\nu - \gamma^\nu j^\lambda) + \frac{1}{2}(\gamma^\lambda j^\nu + \gamma^\nu j^\lambda)$ [1], finally:

$$(i\gamma^\mu \partial_\mu - m)\Psi^{\lambda\nu} - i\frac{q_0}{2}m_0^2\epsilon^{\lambda\nu\sigma}\gamma^\lambda\psi^\nu\psi = \frac{i}{2}\epsilon^{\lambda\nu\sigma}\gamma^\lambda j^\nu\psi \tag{4}$$

This is a relativistic wave equation for a the massive boson and fermion system, where q_0 is a constant [2], m is a fermion mass, m_0 is a massive boson mass and $\Psi^{\lambda\nu} = \psi^{\lambda\nu}\psi$ is the function as a product of the wave function of the fermions and the wave function for the bosons in agreement with Pitaevskii - Liftshitz - Beretetskii text which is a similar tensor to the electromagnetic tensor [38].To obtain the wave function for this system we multiply the matrix form of the bosonic field $\psi^{\lambda\nu}$ (take $\lambda, \nu = 0, 1, 2, 3$, analogue to matrix form of the electromagnetic field) with the Dirac spinor ψ we obtain

$$\Psi^{\lambda\nu} = \psi^{\lambda\nu}\psi = \begin{pmatrix} 0 & \psi^{01} & \psi^{02} & \psi^{03} \\ \psi^{10} & 0 & \psi^{12} & \psi_{13} \\ \psi^{20} & \psi^{21} & 0 & \psi_{23} \\ \psi^{30} & \psi^{31} & \psi^{32} & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \psi^{01}\psi_2 + \psi^{02}\psi_3 + \psi^{03}\psi_4 \\ \psi^{10}\psi_1 + \psi^{12}\psi_3 + \psi^{13}\psi_4 \\ \psi^{20}\psi_1 + \psi^{21}\psi_2 + \psi^{23}\psi_4 \\ \psi^{30}\psi_1 + \psi^{31}\psi_2 + \psi^{32}\psi_3 \end{pmatrix} \tag{5}$$

If Kemmer - Duffin- Petiau spin 1 matrices were considered, then we could obtain a coupled relativistic wave equation for Dirac fermions and Kemmers bosons as observed in Krolikowski article [20]. In the electromagnetic case we have considered this system in presence of an external electromagnetic field. Similary as in Klein - Gordon equation we have applied the same minimal substitution $\boldsymbol{p} \longrightarrow \boldsymbol{p} - e\boldsymbol{A}$ and $E \longrightarrow E - e\phi$ into (3) and (4) obtaining:

$$\gamma^0\left(i\frac{\partial}{\partial t} - e\phi\right)\Psi^{\lambda\nu} = (\boldsymbol{\gamma}\cdot(\boldsymbol{p} - e\boldsymbol{A}) - m)\Psi^{\lambda\nu} + i\frac{q_0}{2}m_0^2\epsilon^{\lambda\nu\sigma}\gamma^\lambda\psi^\nu\psi + \frac{i}{2}\epsilon^{\lambda\nu\sigma}\gamma^\lambda j^\nu\psi \tag{6}$$

or in the covariant form

$$(i\gamma^\mu D_\mu - m)\Psi^{\lambda\nu} - i\frac{q_0}{2}m_0^2\epsilon^{\lambda\nu\sigma}\gamma^\lambda\psi^\nu\psi - \frac{i}{2}\epsilon^{\lambda\nu\sigma}\gamma^\lambda j^\nu\psi = 0 \tag{7}$$

where $D_\mu = \partial_\mu - ieA_\mu$.

3 A scattering matrix element in the electromagnetic case

In the case of this system in presence of an external electromagnetic field we have considered the indices $\lambda = \lambda, \nu = 0$ in eq. (7) obtaining the following equations system (See Appendix B):

$$[i\gamma^\mu D_\mu - m]\psi = f(q^2)\psi \tag{8}$$

$$\gamma^\mu D_\mu\psi^{\lambda 0} - \frac{q_0 m_0^2}{2}\epsilon^{\lambda 0\sigma}\gamma^\lambda\psi^0 = 0 \tag{9}$$

[1]Since $\psi^{\lambda\nu}$ is already antisymmetric we can skip symmetrical terms demanding that symmetrical terms sum up to zero

[2]This constant has been introduce due to dimensional reasons

where $f(q^2,\psi_\lambda) = 2\gamma_\mu \epsilon_{\mu 0\sigma}\psi^{\lambda 0}F(q^2)$. If we use the same procedure such as Bjorken and Drell [19] in order to give a solution to Dirac equation in terms of propagators, then we obtain the matrix element V_{fi} at first perturbative order:

$$V_{fi} = \delta_{if} - ie\epsilon_f \int d^4x \bar{\psi}_f(x) A_\mu(x)\gamma^\mu \psi_i(x) + \epsilon_f \int d^4x \bar{\psi}_f(x) f(q^2)\psi_i(x) \quad (10)$$

being $\bar{\psi}_f$ the free wave function associated to the fermions with quantum numbers f of the Dirac equation with $\epsilon_f = +1$ for solutions with positive frequency for the future and $\epsilon_f = -1$ with solutions of negative frequency for the past, ψ_i is the free wave function for incident fermions and $\psi^{\lambda 0}$ is the associated tensor for the wave function with spin 1 [38] [19], and $f(q^2)$ are the form factor of fermion and massive boson.

4 Extension to the strong and weak interactions

Walecka develops a detailed application of scalar and vector bosons and vector mesons [18]. According to the Walecka works we obtain a generalization of the matrix element (10) for vectorial mesons that interacting electromagnetically, strongly, and weakly with fermions to first perturbative order taking the minimal coupling $D_\mu = \partial_\mu - ieA_\mu - \frac{ig\vec{\tau}_L}{2}\vec{W}_\mu - ig_s G^a_\mu T_a$ [3] in the equation (8):

$$\begin{aligned} V_{fi} = \delta_{if} + ie\epsilon_f \int d^4x \bar{\psi}_f(x)\gamma^\mu A_\mu(x)\psi_i(x) + ie\epsilon_f \int d^4x \bar{\psi}_f(x) f(q^2)\psi_i(x) \\ + i\frac{G_f\epsilon_f}{\sqrt{2}} \int d^4x \bar{\psi}_f(x)\gamma^\mu(1-\gamma^5)W_\mu(x)\psi_i(x) \\ + i\epsilon_f \frac{G_F}{\sqrt{2}} \int d^4x \bar{\psi}_f(x) f_2(q^2)\psi_i(x) + \\ ig_s\epsilon_f \int d^4x \bar{\psi}_f(x)\gamma^\mu G_\mu(x)\psi_i(x) + ig_s\epsilon_f \int d^4x \bar{\psi}_f(x) f_3(q^2)\psi_i(x) \end{aligned} \quad (11)$$

and second perturbative order:

$$\begin{aligned} V_{fi} = ie^2 \int d^4x \int d^4y [\bar{\psi}_f(y)\gamma^\mu\psi_i(y)] D_F(x-y)[\bar{\psi}_f(x)\gamma^\mu\psi_i(x)] \\ + ie^4 \int d^4x \int d^4y [\bar{\psi}_f(y)\gamma^\mu\psi_i(y)]^2 \left|f_1(q^2)\right|^2 [\bar{\psi}_f(x)\gamma^\mu\psi_i(x)]^2 \\ + ig_s^2 \int d^4x \int d^4y [\bar{\psi}_f(y)\gamma^\mu\psi_i(y)] D_F^{gluon}(x-y)[\bar{\psi}_f(x)\gamma^\mu\psi_i(x)] \\ + ig_s^4 \int d^4x \int d^4y [\bar{\psi}_f(y)\gamma^\mu\psi_i(y)]^2 \left|f_2(q^2)\right|^2 [\bar{\psi}_f(x)\gamma^\mu\psi_i(x)]^2 \\ + i\frac{G_F^2}{4} \int d^4x \int d^4y [\bar{\psi}_f(y)\gamma^\mu(1-\gamma^5)\psi_i(y)] W_F(x-y)[\bar{\psi}_f(x)\gamma^\mu(1-\gamma^5)\psi_i(x)] \\ + i\frac{G_F^4}{4} \int d^4x \int d^4y [\bar{\psi}_f(y)\gamma^\mu(1-\gamma^5)\psi_i(y)]^2 \left|f_3(q^2)\right|^2 [\bar{\psi}_f(x)\gamma^\mu(1-\gamma^5)\psi_i(x)]^2 \end{aligned} \quad (12)$$

where G_F is the Fermi constant, g_s the coupling constant of the strong interaction and f_1, f_2, f_3 are functions that depend on the form factor of fermion and vectorial meson and the density charge.

5 Applications

The first application is the definition of the baryons and mesons wave functions in terms of the quarks as defined in Nambu [21], Henley and Garcia [25], [26] and Close [27]. According to the works of the Nambu , Henley and Garcia and Close , from (4) we defined the baryonic and mesonic wave function in terms of quarks

[3]The index a to indicate the eight color gluon charge

multiplied by the color, spin and isospin wave functions given by the Quark Model. The wave functions in terms of the quarks its defined in Henley and García text [25] , and Nambu text [24]. In the hadronic case the wave function for the baryon - meson system in terms of the quarks is given by $q^{\alpha\beta\delta}_{hadron} = q^{baryon}_{\alpha} \otimes q^{meson}_{\beta\delta}$. The baryon wave function as defined at Richard [28] and similarly we have defined the wave function for mesons. In the quarks model the wave function for mesons and baryons were defined as follows:

$$\begin{aligned} q^{baryon}_{\alpha} &= N_b \sum_{ijk} q_i q_j q_k(x,y,z,t)(q^{\uparrow}_i q^{\uparrow}_j q^{\uparrow}_k)_{spin}(q^{\uparrow}_i q^{\uparrow}_j q^{\uparrow}_k)_{isospin} \sum_{abc} \varepsilon_{abc} q_a q_b q_c \\ &\simeq N_b \sum_{ijk} q_{i0} q_{j0} q_{k0}(q^{\uparrow}_i q^{\uparrow}_j q^{\uparrow}_k)_{spin}(q^{\uparrow}_i q^{\uparrow}_j q^{\uparrow}_k)_{isospin} \sum_{abc} \varepsilon_{abc} q_a q_b q_c \exp(iq\cdot x) \end{aligned} \tag{13}$$

$$q^{meson}_{\beta\delta} = N_m \sum_{ij} q_i \bar{q}_j(\boldsymbol{x},t)(q^{\uparrow}_i \bar{q}^{\uparrow}_j)_{spin} \sum_{ab} \delta^{ab} q_a \bar{q}_b \simeq N_m \sum_{ij} (q_{i0}\bar{q}_{j0})^{\beta\delta}(q^{\uparrow}_i \bar{q}^{\uparrow}_j)_{spin} \sum_{ab} \delta^{ab} q_a \bar{q}_b \exp(ip\cdot x) \tag{14}$$

Where p and q are the four momentum associated to the mesons and the baryons, with $i,j,k = 1,2,3$ and a,b,c as an color indices. We have considered as well the baryon - meson system in terms of quarks , so we could obtain the relativistic wave equations though (8) and (9), but in absence of an external electromagnetic field:

$$(\gamma^{\mu} q_{\mu} - m)(q_i q_j q_k) = f(q^2)(q_i q_j q_k) \tag{15}$$

$$\gamma^{\mu} q_{\mu}[(q_{i0}\bar{q}_{j0})^{\mu}] + \frac{q_0}{2} m_0^2 \epsilon^{\lambda 0 \sigma} \gamma^{\lambda} (q_{i0}\bar{q}_{j0})^0 = 0 \tag{16}$$

where q^{μ} is the four momentum associate to quarks [4], m_0 is the meson mass, m the baryon mass. The second application has been developed based on works about relativistic mean field theory stated by Walecka [18] and Quantum Hadrondynamics (QHD-I) also stated by Walecka and Serot [14],[18].

$$(i\gamma^{\mu} D_{\mu} - m^*_{baryon})\psi = f(q^2)\psi \tag{17}$$

$$\gamma^{\mu} D_{\mu} V^{\mu 0} + \frac{q_0}{2} m_0^2 \epsilon^{\lambda 0 \sigma} \gamma^{\lambda} V^0 = 0 \tag{18}$$

$$D_{\mu} = \partial_{\mu} - i g_V V_{\mu} \tag{19}$$

In the neutrino physics, from the eq. (4) we defined the relativistic wave equations for the massless left and right handed fermions and massive bosons system multiplying on the left by γ^5 and on the right by γ^0 to obtain the equations system in terms of $\boldsymbol{\Sigma}$ matrices as defined in Giunti and Kim [29] :

$$\left[-\gamma^5 p_0 - \boldsymbol{\Sigma}\cdot\boldsymbol{p}\right](\psi^{\mu\nu}\psi_L) - i\frac{q_0 m_0^2}{2}(\boldsymbol{\Sigma}\times\boldsymbol{\psi})\psi_L = i\boldsymbol{\Sigma}\times\boldsymbol{J}\psi_L \tag{20}$$

$$\left[\gamma^5 p_0 - \boldsymbol{\Sigma}\cdot\boldsymbol{p}\right](\psi^{\mu\nu}\psi_R) - i\frac{q_0 m_0^2}{2}(\boldsymbol{\Sigma}\times\boldsymbol{\psi})\psi_R = \frac{i}{2}\boldsymbol{\Sigma}\times\boldsymbol{J}\psi_R \tag{21}$$

From the eqs. (22) and (23) we can defined the relativistic wave equations in terms of the helicity $\hat{h}$ also stated in Giunti and Kim [29]

$$\left[-\gamma^5 p_0 - |\boldsymbol{p}|\,\hat{h}\right]\boldsymbol{p}(\psi^{\mu\nu}\psi_L) - i\frac{q_0 m_0^2}{2}|\boldsymbol{p}|^2(\hat{\boldsymbol{u}}\cdot\boldsymbol{\psi})\psi_L = \frac{i}{2}|\boldsymbol{p}|^2\,\hat{\boldsymbol{u}}\cdot\boldsymbol{J}\psi_L \tag{22}$$

$$\left[\gamma^5 p_0 - |\boldsymbol{p}|\,\hat{h}\right]\boldsymbol{p}(\psi^{\mu\nu}\psi_R) - i\frac{q_0 m_0^2}{2}|\boldsymbol{p}|^2(\hat{\boldsymbol{u}}\cdot\boldsymbol{\psi})\psi_R = \frac{i}{2}|\boldsymbol{p}|^2\,\hat{\boldsymbol{u}}\cdot\boldsymbol{J}\psi_R \tag{23}$$

According with this equations we introduced a new operator $\hat{\boldsymbol{u}}$ that releate the momentum $\boldsymbol{p}$ with the $\boldsymbol{\Sigma}$ (See Appendix C) matrices defined as:

$$\hat{\boldsymbol{u}} = \frac{\boldsymbol{\Sigma}\times\boldsymbol{p}}{|\boldsymbol{p}|} \tag{24}$$

[4] In this context q^{μ} is not a momentum transfer

6 Summary and Discusion

In section (2), the equations (4) and (5) describe a composite system of fermions with massive bosons with spin 1. Here we have not taken into account the Foldy - Wouthyusen transformation for the bosonic symmetry of Dirac equation as observed in the works of Silenko [30] and Simulik and Krivsky [31], furthermore, neither we take into account the Pauli matrices for particles with integer spin defined by Curtright, Fairlie and Zachos, C [32]. If we considered a potential V in the equation (4) then we can see that is an interacting potential between massive bosons and the fermions, analytically it might be attractive potential Coulomb or Yukawa type considering the case of nucleons. In the section (3), we consider the electromagnetic case. The equation (8) is interpreted as electromagnetic field produced by a massive boson with a fermion on their covariant form, without taking into account the charge conjugation operator that relates the Dirac spinor with the electromagnetic field as established by Simulik and Krisvky [33] [5]. In section (4), the matrix element (11) describes how is the interaction of the system [6] at the lowest perturbative order with the electromagnetic field. There are many works about coupling baryons and mesons as the case of Wu, Shang and Yao [34], they show a particular case about baryons coupling with ω-mesons through a lagrangian, and how the baryonic current preserves and the propagator formulation without the electromagnetic field coupling. In section (5), the matrix element V_{fi} (12) and (13) are interpreted as a weak, strong and electromagnetic interaction between vector meson [7] and a fermion at first and second perturbative order. From an experimental point of view we have not observed a process [8] which could show all the interactions at the same time. Regarding the experimental results on interactions of fermions with vector bosons, we just consider the relevant terms. In the section (6), the equations (16) and (17) describes the baryon - meson system in absence of an external electromagnetic field in terms of quarks. If we consider a quark - antiquark system in the rest frame with $\boldsymbol{p} \neq 0, q_0 \approx 1$, then we obtain from the eq.(17) the charged Klein - Gordon equations for the mesons in terms of quarks:

$$(q^2 + m_0^2)(q_{i0}\bar{q}_{j0})^0 = -g^{0\nu}\rho \tag{25}$$

$$(q^2 + m_0^2)(q_{i0}\bar{q}_{j0})^i = -g^{i\nu}J_i \tag{26}$$

Where q^2 is the Lorentz invariant for 4 - momentum associated to the quarks [9], $i = 1, 2, 3$, m_0 the meson mass and ρ, J_i are the charge and current density produced by the massive bosons (in this case the mesons). The estimate solution to eq. (16) is the matrix element defined in eq. (11), for the case interaction or a process among quarks as the case of creation to quarks pairs in $t = 0$, Fuda [10] describes in his works. Respect to the second application, Djukanovic, Lee and Yang and Velo developed works about the coupling of the electromagnetic field with the vector meson field [36] [37] [22]. From the eq. (21) we can consider the problem raised by Walecka as an application to the mean field theory of a hadron confined to a volume V which baryonic charge density is ρ_B, using the electromagnetic operator defined in Walecka [18]; and the third application on eqs. (22), (23), we consider the massless fermions as Weyl neutrinos and we propose a new operator stated in the eq. (26) of the which the interpretation is unknown and a constant q_0 that appear in eqs. (22), (23), (24) and (25) and the value of which, at this time is also unknown. However, take into account the eq. (27) and (28), the value of constant q_0 is less or equal than 1. We need to consider an interaction of fermions with vectorial bosons, experimentally established, in order to determine the defined matrix elements in eq. (18) and (19) and then calculate the effective cross section. As for now it is not know a decay that involves vector mesons which could interact weakly, electromagnetically and strongly simultaneously with fermions. If we have four or more particles it is important verify which interactions are presented. Moreover which therms consider relevant within eq.(19) and therefore an approach. Finally, we can deduce that functions f_1, f_2, f_3 can vary on the basis of the type of the process studied. So it is possible to determinate these functions and their interpretations. It is necessary to investigate more about new operator $\boldsymbol{u}$ and its implications to the neutrino physics. In the application to hadronic physics, Pitaevskii and Liftshitz worked on hadrons electrodynamics [38]. Based on these works is possible to determinate the electromagnetic fields in terms of form factors, electric and magnetic.

[5] there is a unitary relationship between quantum electrodynamics and relativistic quantum mechanics without taking into account the covariant formulation concerned in classical electrodynamics

[6] In this context refers to fermion boson with spin 1

[7] Bosons with odd parity $J^P = 1^-$ can see Particle Data Booklet (2012) [26]

[8] Refers a decays, particle production i.e. pairs creation electron - positron from two photons

[9] The Lorentz Invariant for the 4 - momentum its defined as $q^\mu q_\mu = q^2 = (E^2 - \boldsymbol{q}^2)$

7 A. Derivation of the electromagnetic tensor coupling with the massive boson and fermion system

From the eq. (7) we can write:

$$i\gamma^{\mu}\partial_{\mu}\Psi^{\lambda\nu} - m\Psi^{\lambda\nu} - \frac{q_0}{2}im_0^2\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}\psi^{\nu}\psi - \frac{1}{2}i\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}j^{\nu}\psi = e\gamma^{\mu}A_{\mu}\Psi^{\lambda\nu} \tag{27}$$

multiplying by $\Psi_{\lambda\nu}\gamma_{\mu}$:

$$\begin{aligned} &i(\gamma_0^2 - \boldsymbol{\gamma}^2)\Psi_{\lambda\nu}\partial_{\mu}\Psi^{\lambda\nu} \\ &\quad - m\Psi_{\lambda\nu}\gamma_{\mu}\Psi^{\lambda\nu} - \frac{q_0}{2}im_0^2\Psi_{\lambda\nu}\gamma_{\mu}\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}\psi^{\nu}\psi - i\frac{1}{2}\Psi_{\lambda\nu}\gamma_{\mu}\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}j^{\nu}\psi = e(\gamma_0^2 - \boldsymbol{\gamma}^2)A_{\mu}\left|\Psi^{\lambda\nu}\right|^2 \end{aligned} \tag{28}$$

we can rewrite the eq.(30) as:

$$\begin{aligned} &(\gamma_0^2 - \boldsymbol{\gamma}^2)\Psi_{\lambda\nu}p_{\mu}\Psi^{\lambda\nu} \\ &\quad - m\Psi_{\lambda\nu}\gamma_{\mu}\Psi^{\lambda\nu} - \frac{q_0}{2}im_0^2\Psi_{\lambda\nu}\gamma_{\mu}\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}\psi^{\nu}\psi - i\frac{1}{2}\Psi_{\lambda\nu}\gamma_{\mu}\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}j^{\nu}\psi = e(\gamma_0^2 - \boldsymbol{\gamma}^2)A_{\mu}\left|\Psi^{\lambda\nu}\right|^2 \end{aligned} \tag{29}$$

applying the four - derivative ∂_{ν} and multiplying by -1:

$$\begin{aligned} &-(\gamma_0^2 - \boldsymbol{\gamma}^2)\partial_{\nu}(\Psi_{\lambda\nu}p_{\mu}\Psi^{\lambda\nu}) \\ &\qquad + m\partial_{\nu}(\Psi_{\lambda\nu}\gamma_{\mu}\Psi^{\lambda\nu}) \\ &\quad + \frac{q_0}{2}im_0^2\partial_{\nu}(\Psi_{\lambda\nu}\gamma_{\mu}\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}\psi^{\nu}\psi) + \frac{i\partial_{\nu}}{2}(\Psi_{\lambda\nu}\gamma_{\mu}\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}j^{\nu}\psi) = -e(\gamma_0^2 - \boldsymbol{\gamma}^2)\partial_{\nu}A_{\mu}\left|\Psi^{\lambda\nu}\right|^2 \end{aligned} \tag{30}$$

And interchanging ν by μ into the eq. (32)

$$\begin{aligned} &-(\gamma_0^2 - \boldsymbol{\gamma}^2)\partial_{\mu}(\Psi_{\lambda\mu}p_{\nu}\Psi^{\lambda\mu}) \\ &\qquad + m\partial_{\mu}(\Psi_{\lambda\mu}\gamma_{\nu}\Psi^{\lambda\mu}) \\ &\quad + \frac{q_0}{2}im_0^2\partial_{\mu}(\Psi_{\lambda\mu}\gamma_{\nu}\epsilon^{\lambda\mu\sigma}\gamma^{\lambda}\psi^{\mu}\psi) + \frac{i\partial_{\nu}}{2}(\Psi_{\lambda\mu}\gamma_{\nu}\epsilon^{\lambda\mu\sigma}\gamma^{\lambda}j^{\mu}\psi) = -e(\gamma_0^2 - \boldsymbol{\gamma}^2)\partial_{\mu}A_{\nu}\left|\Psi^{\lambda\mu}\right|^2 \end{aligned} \tag{31}$$

subtracting (32) with (33) and take account that $|\Psi_{\lambda\nu}|^2 = |\Psi_{\lambda\mu}|^2 = |\Psi|^2$ then we obatin:

$$\begin{aligned} F_{\mu\nu}^{bf} & \\ &= k\left\{(\gamma_0^2 - \boldsymbol{\gamma}^2)(\partial_{\mu}(\psi_{\lambda\mu}p_{\nu}\psi^{\lambda\mu}) - \partial_{\nu}(\psi_{\lambda\nu}p_{\mu}\psi^{\lambda\nu}))\right\} \\ &\quad - k\left\{\frac{q_0 m_0^2}{2}\left[p_{\mu}(\psi_{\lambda\mu}\gamma_{\nu}\epsilon^{\lambda\mu\sigma}\gamma^{\lambda}\psi^{\mu}\psi) - p_{\nu}(\psi_{\lambda\nu}\gamma_{\mu}\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}\psi^{\nu}\psi)\right]\right\} \\ &\quad - k\left\{m\left[\partial_{\mu}(\psi_{\lambda\mu}\gamma_{\mu}\psi^{\lambda\nu}) - \partial_{\nu}(\psi_{\lambda\nu}\gamma_{\mu}\psi^{\lambda\nu})\right] + \frac{p_{\mu}}{2}(\psi_{\lambda\mu}\gamma_{\nu}\epsilon^{\lambda\mu\sigma}\gamma^{\lambda}j^{\mu}\psi) - \frac{p_{\nu}}{2}(\psi_{\lambda\nu}\gamma_{\mu}\epsilon^{\lambda\nu\sigma}\gamma^{\lambda}j^{\nu}\psi)\right\} \end{aligned} \tag{32}$$

where $k = \frac{1}{e(\gamma_0^2 - \boldsymbol{\gamma}^2)|\Psi|^2}$.

8 B. Derivation of the matrix element V_{fi}

As starting point we take the eq. (7) for $\lambda = \lambda, \nu = 0$.

$$(i\gamma^\mu D_\mu - m)\Psi^{\lambda 0} - \frac{iq_0}{2} m_0^2 \epsilon^{\lambda 0 \nu} \gamma^\lambda \psi^0 \psi - \frac{i}{2}\epsilon^{\lambda 0 \sigma} \gamma^\lambda \rho \psi = 0 \tag{33}$$

$$(i\gamma^\mu D_\mu \psi^{\lambda 0})\psi + \psi^{\lambda 0}(i\gamma^\mu D_\mu \psi) - m\psi^{\lambda 0}\psi - \frac{iq_0}{2} m_0^2 \epsilon^{\lambda 0 \nu} \gamma^\lambda \psi^0 \psi - \frac{i}{2}\epsilon^{\lambda 0 \sigma} \gamma^\lambda \rho \psi = 0 \tag{34}$$

Separating the fermionic and bosonic cases:

$$\psi^{\lambda 0}(i\gamma^\mu D_\mu \psi - m)\psi = \frac{i}{2}\epsilon^{\lambda 0 \sigma} \gamma^\lambda \rho \psi = 0 \tag{35}$$

$$\gamma^\mu D_\mu \psi^{\lambda 0} - \frac{iq_0}{2} m_0^2 \epsilon^{\lambda 0 \nu} \gamma^\lambda \psi^0 = 0 \tag{36}$$

In the fermionic case:

$$\psi^{\lambda 0}(i\gamma^\mu D_\mu \psi - m\psi) = \frac{i\epsilon^{\lambda 0 \sigma} \gamma^\lambda \rho \psi}{2} \tag{37}$$

to solve this equation we need to define a new function in terms of the momentum transfer [10] related by the form factor $F(q^2)$, and $\psi^{\lambda 0}$

$$f(q^2, \psi_\lambda) = -2ie\epsilon_{\mu 0 \sigma} \gamma_\mu \psi^{0\lambda} F(q^2) \tag{38}$$

inserting this equation into eq. (39) on the left hand:

$$\psi^{\lambda 0}(i\gamma^\mu D_\mu \psi - m\psi) = f(q^2, \psi^\lambda)\frac{i\epsilon^{\lambda 0 \sigma} \gamma^\lambda \rho \psi}{2}$$
$$\psi^{\lambda 0}(i\gamma^\mu D_\mu - m)\psi = \epsilon_{\mu 0 \sigma} \gamma_\mu \psi^{0\lambda} F(q^2) \epsilon^{\lambda 0 \sigma} \gamma^\lambda \rho \psi$$
$$\psi^{\lambda 0}(i\gamma^\mu D_\mu - m)\psi = \epsilon_{\mu 0 \sigma} \gamma_\mu \epsilon^{\lambda 0 \sigma} \gamma^\lambda F(q^2) \psi^{0\lambda} \psi \rho$$
$$\psi^{\lambda 0}(i\gamma^\mu D_\mu - m)\psi = \epsilon_{\mu 0 \sigma} \epsilon^{\lambda 0 \sigma} \gamma_\mu \gamma^\lambda F(q^2) \psi^{0\lambda} \psi \rho$$

using the Levi Civita property $\epsilon_{\mu 0 \sigma}\epsilon^{\lambda 0 \sigma} = \delta^\mu_\lambda$ we have:

$$\psi^{\lambda 0}(i\gamma^\mu D_\mu - m)\psi = \delta^\lambda_\mu \gamma_\mu \gamma^\lambda F(q^2) \psi^{\lambda 0} \psi \rho$$
$$\psi^{\lambda 0}(i\gamma^\mu D_\mu - m)\psi = (\gamma_0^2 - \boldsymbol{\gamma}^2) F(q^2) \psi^{0\lambda} \psi \rho$$
$$\psi^{\lambda 0}\left[(i\gamma^\mu D_\mu - m)\psi - (\gamma_0^2 - \boldsymbol{\gamma}^2) F(q^2) \psi \rho\right] = 0$$

we can define $f(q^2) = (\gamma_0^2 - \boldsymbol{\gamma}^2) F(q^2)\rho$ and we obtain

$$\psi^{\lambda 0}\left[(i\gamma^\mu D_\mu - m)\psi - f(q^2)\psi\right] = 0 \tag{39}$$

Now we will take the term:

$$(i\gamma^\mu D_\mu - m)\psi - f(q^2)\psi = 0 \tag{40}$$

and we applying the same procedure as shown in the reference [19] pp. 83 and 96 to order to obtain:

$$\psi(x) = \psi_0(x) + e\int d^4y D_F(x-y)\gamma^\mu A_\mu(y)\psi(x) + \int d^4y D_F(x-y) f(q^2) \tag{41}$$

According to the Feynman conditions this solution contain only positive frequency in the future and negative frequency in the past:

[10] In this case fermion boson interaction

a In the future

$$\psi(x) - \psi_0(x) \simeq \int d^3p \sum_{r=1}^{2} \psi_p^r(x) \left[-ie \int d^4y \bar{\psi}_p^r(y) \gamma^\mu A_\mu(y) \psi(x) - ie \int d^4y \bar{\psi}_p^r(y) f(q^2) \psi(x) \right] \tag{42}$$

b In the past

$$\psi(x) - \psi_0(x) \simeq \int d^3p \sum_{r=3}^{4} \psi_p^r(x) \left[ie \int d^4y \bar{\psi}_p^r(y) \gamma^\mu A_\mu(y) \psi(x) + ie \int d^4y \bar{\psi}_p^r(y) f(q^2) \psi(x) \right] \tag{43}$$

For the eq.s (54) and (55) we can identify the matrix element V_{fi} as coefficients of the free-wave solutions $\psi^r(x)$:

$$V_{fi} \simeq \delta_{fi} - ie\epsilon_f \int d^4y \bar{\psi}_f(y) \gamma^\mu A_\mu(y) \psi_i(y) - ie\epsilon_f \int d^4y \bar{\psi}_f(y) f(q^2) \psi_i(y) \tag{44}$$

Similary we obtain the matrix element to perturbative second order taking account the same procedure as shown in the eq (6.57) of the reference [19] and we can stated the matrix element to perturbative first and second order inserting the minimal coupling for the weak, strong and electromagnetic interaction $D_\mu = \partial_\mu - ieA_\mu - \frac{ig\vec{\tau}_L}{2}\vec{W}_\mu - ig_s G_\mu^a T_a$ into eq.(42).

9 C. Derivation of the Relativistic Wave Equations in terms of the helicity and the vector $\hat{\boldsymbol{u}}$

Consider the massless fermion *left- handed, right - handed* and massive boson system. In this case the eq.(4) can be rewrite as

$$(\gamma^0 p_0 - \boldsymbol{\gamma} \cdot \boldsymbol{p})\psi_L^{\lambda\nu} - \frac{iq_0 m_0^2}{2} \epsilon^{\lambda\nu\sigma} \gamma^\lambda \psi^\nu \psi_L = \frac{i}{2} \epsilon^{\lambda\nu\sigma} \gamma^\lambda j^\nu \psi_L \tag{45}$$

multiplying on the right by γ^0 and on the right by γ^5 and according to the definition of the $\boldsymbol{\Sigma}$ matrix and the chirality on the reference [29] (pag.16, eq. (2.79), pag. 18, eqs. (291), (292)) we get:

$$[-\gamma^5 p_0 - \boldsymbol{\Sigma} \cdot \boldsymbol{p}]\psi_L^{\lambda\nu} - \frac{iq_0 m_0^2}{2} \epsilon^{\lambda\nu\sigma} \Sigma^\lambda \psi^\nu \psi_L = \frac{i}{2} \epsilon^{\lambda\nu\sigma} \Sigma^\lambda \psi^\nu \psi_L \tag{46}$$

and multiplying by $1/|\boldsymbol{p}|$ we obtain the relativistic wave equation in term of the helicity operator:

$$[-\gamma^5 p_0 - \hat{h}|\boldsymbol{p}|]\psi_L^{\lambda\nu} - \frac{iq_0 m_0^2}{2} |\boldsymbol{p}| \epsilon^{\lambda\nu\sigma} \Sigma^\lambda \psi^\nu \psi_L = \frac{i}{2} |\boldsymbol{p}| \epsilon^{\lambda\nu\sigma} \Sigma^\lambda j^\nu \psi_L \tag{47}$$

Considering the spatial coordinates of the bosonic current $\boldsymbol{J}$:

$$[-\gamma^5 p_0 - \hat{h}|\boldsymbol{p}|]\psi_L^{\lambda\nu} - \frac{iq_0 m_0^2}{2} |\boldsymbol{p}| (\boldsymbol{\Sigma} \times \boldsymbol{\psi}) \psi_L = \frac{i}{2} |\boldsymbol{p}| (\boldsymbol{\Sigma} \times \boldsymbol{J}) \psi_L \tag{48}$$

Applying the momentum operator $\boldsymbol{p}$ over $\psi_L^{\lambda\nu}$ and the property of the scalar triple product:

$$[-\gamma^5 p_0 - \hat{h}|\boldsymbol{p}|]\boldsymbol{p}\psi_L^{\lambda\nu} - \frac{iq_0 m_0^2}{2} |\boldsymbol{p}| [(\boldsymbol{\Sigma} \times \boldsymbol{p} \cdot \boldsymbol{\psi})] \psi_L = \frac{i}{2} |\boldsymbol{p}| \boldsymbol{p} \cdot (\boldsymbol{\Sigma} \times \boldsymbol{J}) \psi_L \tag{49}$$

$$[-\gamma^5 p_0 - \hat{h}|\boldsymbol{p}|]\boldsymbol{p}\psi_L^{\lambda\nu} - \frac{iq_0 m_0^2}{2} |\boldsymbol{p}| [(\boldsymbol{\Sigma} \times \boldsymbol{p}) \cdot \boldsymbol{\psi}] \psi_L = \frac{i}{2} |\boldsymbol{p}| (\boldsymbol{\Sigma} \times \boldsymbol{p}) \cdot \boldsymbol{J} \psi_L \tag{50}$$

we substituting $\boldsymbol{\Sigma} \times \boldsymbol{p}$ by $|\boldsymbol{p}|\hat{\boldsymbol{u}}$ finally we get

$$\left[-\gamma^5 p_0 - |\boldsymbol{p}|\,\hat{h}\right] \boldsymbol{p}(\psi^{\mu\nu}\psi_L) - i\frac{q_0 m_0^2}{2} |\boldsymbol{p}|^2 (\hat{\boldsymbol{u}} \cdot \boldsymbol{\psi}) \psi_L = \frac{i}{2} |\boldsymbol{p}|^2 \,\hat{\boldsymbol{u}} \cdot \boldsymbol{J} \psi_L \tag{51}$$

This result can be extend similary with the *right-handed fermion* obtaining:

$$\left[\gamma^5 p_0 - |\boldsymbol{p}|\,\hat{h}\right] \boldsymbol{p}(\psi^{\mu\nu}\psi_R) - i\frac{q_0 m_0^2}{2} |\boldsymbol{p}|^2 (\hat{\boldsymbol{u}} \cdot \boldsymbol{\psi}) \psi_R = \frac{i}{2} |\hat{\boldsymbol{p}}|^2 \,\hat{\boldsymbol{u}} \cdot \boldsymbol{J} \psi_R \tag{52}$$

References

[1] Gordon W, Z, *Phys* 40, pp. 117–133, (1926 -1927)

[2] Klein O, Z, "Quantum Theory and Five-Dimensional Theory of Relativity", *Phys* 37, (1926) pp. 895–906, dx.doi.org/10.1007/BF01397481.

[3] Petiau G,*Acad. R. Belg. Cl. Sci. Mem. Collect.* (8) 16, No. 2. (1936)

[4] Duffin R Y (1938), "On The Characteristic Matrices of Covariant Systems" *Phys. Rev. C* 54, 1114. dx.doi.org/10.1103/PhysRev.54.1114

[5] Kemmer N, "The Particle Aspect of Meson Theory" *Proc. R. Soc. Lond. A.*, 173, (1939), 91–116. doi: 10.1098/rspa.1939.0131.

[6] Okninski A, "Duffin–Kemmer–Petiau and Dirac Equations—A Supersymmetric Connection", *Sym*, 4, (2012) pp.427-440; doi:10.3390/sym4030427.

[7] Proca A J, Proca A J, *Phys. Radium* (7) 8, (1936), pp.347-353

[8] Dirac P. A. M. "Quantum Theory of Electron" ,*Proc. R. Soc. Lond. A.* 117, (1928), doi: 10.1098/rspa.1928.0023.

[9] Dirac P. A. M. "Quantised Singularities in the Electromagnetic Field" *Proc. R. Soc. Lond. A.* (1931) ,133, doi: 10.1098/rspa.1931.0130 .

[10] Fierz M and Pauli W, "On relativistic wave equations for particles of arbitrary spin in an electromagnetic field", *Proc. R. Soc. Lond. A* 173, (1939), doi: 10.1098/rspa.1939.0140.

[11] Rarita W and Schwinger J, " On a Theory of Particles with Half-Integral Spin " *Phys. Rev.* (60) 61 (1941). dx.doi.org/10.1103/PhysRev.60.61.

[12] Bhabha. H. J "Relativistic Wave Equations for the Elementary Particles" *Proc. Ind. Acad. Sci. A ,India*.21. 1945. pp 241 - 264. dx.doi.org/10.1103/RevModPhys.17.200

[13] Wu-Ki Tung, "Relativistic Wave Equations and Field Theory for Arbitrary Spin" *Phys. Rev.* (5) 156. (1967), dx.doi.org/10.1103/PhysRev.156.1385.

[14] Serot B D and Walecka J D (1981) *Adv. Nucl. Phys* (16), (1981).

[15] Niederle J and Nikitin A G, "Relativistic wave equations for interacting massive particles with arbitrary half-intreger spins" (2001) arXiv:hep-th/0412213v, doi: 10.1103/PhysRevD.64.125013.

[16] Silenko A, "Verification of Relativistic Wave Equations for Spin-1 Particles" arXiv:hep-th/0401183v1, (2004).

[17] Kulikov D A and Tutik R S, " Relativistic wave equation for one spin-1/2 and one spin-0 particle", (2007) arXiv:0706.1010v1 [hep-th].

[18] Walecka J D ,"Theoretical Nuclear and Subnuclear Physics" , World Scientific publishing and Imperial College Press, London, pp. 136 - 139 (2004).

[19] Bjorken J D, Drell S D. (1964). Relativistic Quantum Mechanics . McGraw - Hill Companies (1964) p.96.

[20] Krolikowski W, "Relativistic Two-Body Equation for One Dirac and One Duffin - Kemmer Particle", (1982) *Act. Phys. Pol.* (1982) Vol.**B** 14.

[21] Lifshitz E M, Pitaevskii L P and Berestetskii V B, "Relativistic Quantum Theory part 1",Pergamon Press pp. 41, 43 (1971).

[22] Lee T D, Yang C N, "Theory of Charged Vector Mesons Interacting with the Electromagnetic Field" *Phys. Rev* 128 (885),(1961). dx.doi.org/10.1103/PhysRev.128.885.

[23] Kulikov D A and Tutik R S, " Relativistic wave equation for one spin-1/2 and one spin-0 particle", (2007) arXiv:0706.1010v1 [hep-th].

[24] Nambu Y, "Quarks: Frontiers in Elementary Particle Physics", United States: Word Scientific, pp. 106 - 107, (1981).

[25] Henley E M and Garcia A, Subatomic Physics , Word Scientific, United States, (2007), p. 479.

[26] "Particle Data Booklet" (2013). p.10.

[27] Close .F. E. "An Introduction to Quarks and Partons", Academic Press, London , (1979) p.5

[28] Richard J D (2012) "An Intoduction to Quark Model" ,arXiv:1205.4326 [hep-php], (2012), p. 44.

[29] Giunti C and Kim W C, "Fundamental of Neutrino and Astrophysics" , Oxford University Press, New York, (2007), pp. 16 - 17.

[30] Silenko A, "Verification of Relativistic Wave Equations for Spin-1 Particles" arXiv:hep-th/0401183v1, (2004).

[31] Simulik V and Krivsky I, "Bosonic symmetries of the Dirac equation", *Phys. Lett. A* 375, (2011) pp.2479–2483, doi:10.1016/j.physleta.2011.03.058.

[32] Curtright T, Fairlie D, and Zachos (2004), "A Compact Formula for Rotations as Spin Matrix Polynomials" (2004), arXiv:1402.3541. doi:10.3842/SIGMA.2014.084.

[33] Simulik V and Krivsky I *Ann. Foundation. Louis De Broglie* **27**. 2 (2002).

[34] Wu S S, Shang H X and Yao Y J, "Vector Meson Propagator and Baryon Current Conservation" (2001) arXiv:nucl-th/0101044v2 29 Apr. doi:10.1016/S0370-2693(01)00772-9.

[35] Fuda M (2012),"Relativistic quantum mechanics and the quark-pair creation model", *Phys. Rev. C.* 86, 055205. dx.doi.org/10.1103/PhysRevC.86.055205.

[36] Djukanovic D, Schindler M, Gegelia J, Scherer, "Quantum Electrodynamics for Vector Mesons" (2005) arxiv:hep-ph/0505180v. doi: 10.1103/PhysRevLett.95.012001.

[37] Velo G ,*Commun.math.Phys.43*, (1974).

[38] Lifshitz E M and Pitaevskii V B, "Realtivistic Quantum Theory part 2" ,Pergamon Press, pp. 624 - 648 (1971).