UNIVERSIDAD NACIONAL DE COLOMBIA

EL MODELO DE GAUGE $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X \otimes U(1)_{B-L}$

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The $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)'_X \otimes U(1)_{B-L}$ gauge model:
Extending the standard model in search of explanations for Dark matter and to the mass hierarchy problem

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RESUMEN

El modelo estándar ha sido probado de manera experimental en repetidas ocasiones. Sin embargo, falla al intentar explicar ciertos fenómenos ya observados como la masa de los neutrinos, la abundancia de materia oscura, y la marcada diferencia entre las masas de las distintas generaciones de partículas, conocido como el problema de jerarquía de masas. En este trabajo se propone una extensión del modelo estándar, libre de anomalías, que en forma exitosa, logra dar explicación a estos problemas.

ABSTRACT

The standard model has been proven experimentally in many ocassions. However, it fails to explain certain phenomena as observed neutrino masses, dark matter abundance, and the observed mass hierarchy between generations of particles. In this work we propose an anomally free extension to the standard model which succesfully gives explanations to this problems.

Failure is not an option. Everyone has to succeed.

— Arnold Schwarznegger

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1 INTRODUCTION

The standard model of particle physics is a theory which models the behavior of fundamental particles. It is a gauge theory, which means that it is invariant under transformations which belong to a certain gauge group. This is a very important feature for any theory to be useful; the physics must be the same under gauge transformations. [1]

The standard model was constructed because there were multiple experiments which showed that nucleons were composed by more fundamental particles. There was a need to understand the behavior of those particles and the forces by which they interact. Many theories were tried, but finally, thanks to the works made by Glashow, Weinberg and Salam [2, 3, 4] a consistent formulation of the standard model was made.

The standard model has been proved multiple times, such as when the Higgs boson was discovered [5], also when the W and Z bosons were discovered [6, 7]; also precision tests, such as the one-loop contribution to the anomalous magnetic moment of the electron match with theory up to 10 significant digits, which is an astonishing accuracy of one part in a billion [8].

But as with any other theory there are still some phenomena it lacks to explain. An example of such phenomena is the observed mass difference between particle generations; this is shown on Tabs. 1 and 2. Their masses have been observed to differ by several orders of magnitude, but according to the standard model such difference is unexpected. Another well known problem of the standard model is that it predicts that neutrinos are massless. Studies made on particles arriving earth from the sun have gathered data that can only be explained if neutrinos

Quark	Mass (MeV/c^2)
u	$2.3\pm^{0.7}_{0.5}$
d	$4.8\pm^{0.5}_{0.3}$
c	$(1.275 \pm 0.025) \times 10^3$
s	95 ± 5
t	$(173.21\pm^{0.71}_{0.51}) \times 10^3$
b	$(4.18 \pm 0.03) \times 10^3$

Table 1: Quarks classified by generations. Each quark can have any of the three color charges (R, G, B), so there is a total of 18 different quarks (without counting antiparticles). Taken from [9]

Lepton	Mass (MeV/c^2)
e^-	0.510998928(22)
ν_e	$<$ 2eV (Theoretically 0 eV)
μ^-	105.6583715(22)
ν_μ	$<$ 2eV (Theoretically 0 eV)
τ^-	1776.82(16)
ν_τ	$<$ 2eV (Theoretically 0 eV)

Table 2: Masses of different lepton types classified by generation. Leptons do not have color, so there are just six of them (without counting antiparticles). Taken from [9]

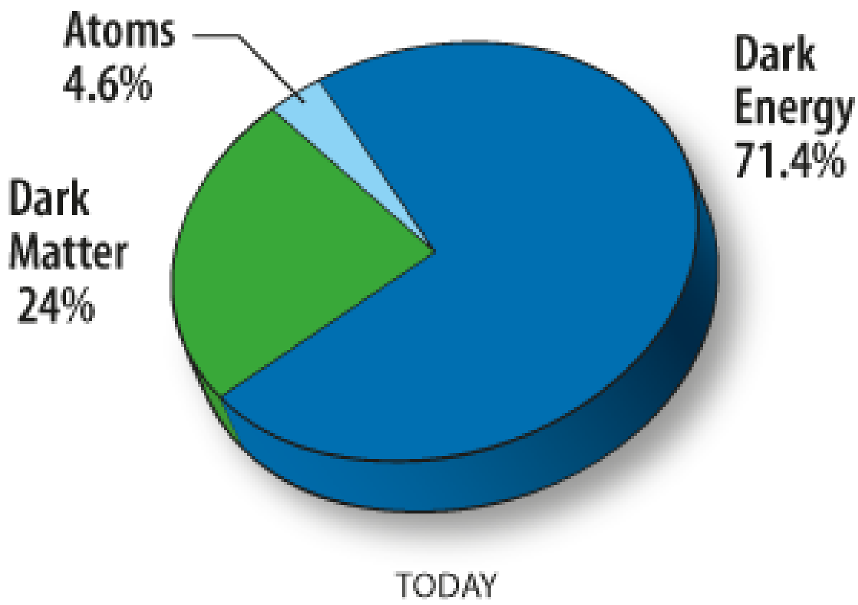

Figure 1: Universe composition according to WMAP data. Taken from [12]

are massive [10, 11]. As shown on Tab. 2 neutrino masses are still undetermined; experiments show that their masses must be very light but nonzero.

Finally, results obtained by Hubble space telescope and WMAP have shown that universe is composed approximately as shown in fig. 1. The standard model is a theory about ordinary matter and gives no hints about dark matter. This clearly contradicts observations. Because of this problems, but thanks to its unprecedent success when contrasted with experiments, researchers have opted to propose new theories based on the standard model, usually extending it. In our work we will extend the standard model adding a $U(1)'_X$ symmetry analogus to [13]; and will also add a $U(1)_{B-L}$ symmetry.

The first $U(1)'_X$ symmetry, which corresponds to an exotic hypercharge, is expected to generate an horizontal symmetry. That means that it will provide a mechanism to distinguish between different generations of particles when coupling to the Higgs field; this will provide different masses for each one, thus giving and answer to the mass hierarchy problem.

On the other hand, the new $U(1)_{B-L}$ symmetry, which corresponds to the barionic minus leptonic number, is expected to provide a natural mechanism to account for dark matter stability. This is normally done by imposing a discrete symmetry group, this makes impossible for dark matter particles to decay to ordinary matter ones; but this is a rather mathematical approach. Our method uses a continuous symmetry group that, when broken, explains the decouple between ordinary and dark matter. This is analogus to unification of weak and electromagnetic interactions in the Glashow Weinberg Salam model before symmetry was broken.

In other words, by proposing this symmetry group we are saying that on early stages of the universe there were no mass hierarchy between generations of particles, nor a distinction between dark and ordinary matter.

It is also expected that when breaking symmetry in this model neutrinos will acquire mass.

An interesting future work will be to study the phenomenology of the model. It could have great consequences both on the cosmological background, but also in observed phenomena that still lacks explanation from the standard model.

Now, let us discuss briefly the ideas that will be treated in the following chapters. In Chapter 2 we consider both some mathematical aspects of group theory, and the physical significance of groups. Every mathematical group is known to have a representation. The first step towards building a useful and consistent gauge model is to choose the correct representation of the group so that it cancels anomalies.

An anomaly is a symmetry of the system that exists in the classical level, but that disappears in the quantization process. In quantum field theory it is mandatory that anomalies are cancelled. The reason for this request is that behind any symmetry on the system the Noether theorem states that there are both a conserved current and a conserved charge associated with it. If anomalies do not vanish the conserved current diverges, therefore the theory is unrenormalizable [14]. Renormalisability is a key feature to have when constructing new theories, because it is what makes possible to compare predictions and measurements.

In Chapter 3 it is shown that anomalies are proportional to the completely symmetric constant factor i. e.

$$D_{\alpha\beta\gamma} := \frac{1}{2}\mathrm{tr}[\{T_\alpha, T_\beta\}T_\gamma], \tag{1}$$

where the T's are the group generators for some representation. This is the reason why not any representation of the group can be chosen. The generators of the "correct" representation will satisfy $D_{\alpha\beta\gamma} = 0$.

Once the adequate representation has been chosen, the next step is to define the adecuate Higgs sector in order to give particles mass when symmetry is broken.

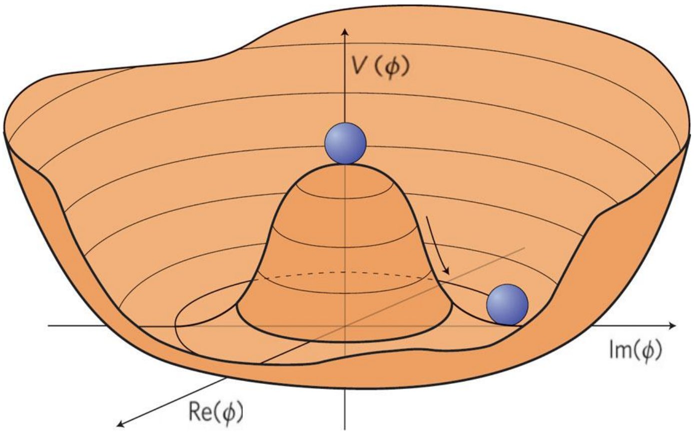

Figure 2: Representation of the Standard model Higgs potential. The vacuum state decays to a randomly chosen point at the bottom of the "hat". This fixes universe chirality and breaks vacuum symmetry while leaving lagrangian symmetry untouched. Taken from [15]

In order to understand this mechanism, a review from the standard model is made in Chapter 4. After that, the calculations for this model are carried out in Chapter 5.

When the Higgs sector is defined, a single or multiple Higgs fields can be introduced; they can also be vectorial or scalar, and even have a real or complex nature. Such characteristics depend on the model, and are needed in order to guarantee consistence, invariance and renormalisability.

Finally, in order to break symmetry the vacuum expectation value for the Higgs field must be chosen. This will define the masses of different particles. For the standard model on early stages of the universe the Higgs potential behaves as shown on fig. 2; the blue sphere is a pictoric representation of the vacuum state, so in order to minimize energy it decays to a random point in the bottom of the potential. Note that this breaks vacuum symmetry (it does not have rotation symmetry anymore), but lagrangian symmetry remains untouched.

When some generators act on the vacuum state the spontaneous symmetry break process makes their expectation values to be non-zero. This is known in quantum field theory jargon as a broken generator. This causes the coupling constant between the associated field and the Higgs field to be non-zero, thus giving it mass.

The standard model breaks symmetry from $SU(2)_L \otimes U(1)_Y$ to $U(1)_Q$. In the process the bosons W^+, W^- and Z^0 acquire mass while the photon remains massless.

In our model we will break symmetry following this scheme (VEV means that the mentioned field(s) acquires a vacuum expectation value different from zero)

$$
\begin{array}{cl}
SU(2)_L \otimes U(1)_Y \otimes U(1)'_X \otimes U(1)_{B-L} & \\
\downarrow & \phi \text{ acquires VEV} \\
SU(2)_L \otimes U(1)_Y \otimes U(1)'_X & \\
\downarrow & \chi_3^0 \text{ acquires VEV} \\
SU(2)_L \otimes U(1)_Y & \\
\downarrow & \eta \text{ and } \rho \text{ acquire VEV} \\
U(1)_Q. &
\end{array}
$$

We expect to recover the standard model results and also to find neutrino masses and an explanation to the mass hierarchy problem.

2 GROUP THEORY IN PARTICLE PHYSICS

In physics it is usual to find some transformations that leave the system (lagrangian) untouched (invariant). This symmetry transformations form a *group*, hence the study of group theory is important for physicists. As with any method used for studying a complex problem it has some advantages and disadvantages; physics cannot be very clear from the pure mathematical point of view, but the consequences of existing symmetries can be deduced formally without going through cumbersome dynamical calculations [18].

Symmetries can be classified in two categories. Global symmetries and local symmetries. The later are much more restrictive than the former ones, but have great importance in physics since they are necessary for a consistent formulation of physical theories involving the behavior of fundamental particles.

For example in quantum electrodynamics one must have charge conservation. If only global symmetry was demanded, then it would be ok for a charge to vanish from Earth and suddenly appear in Andromeda, but that kind of phenomena has never been observed. Hence local symmetry must be imposed; with this new choice, charge is conserved locally and the continuity equation is respected, in agreement with nature [17].

In this chapter the basic knowledge of group theory needed to understand notation will be treated. Then the reasons that motivate the standard model symmetry group and the presented extension will be discussed.

BASIC ELEMENTS OF GROUP THEORY

A *group* $G(A, \cdot)$ is nothing more than a set of elements $A = \{a_i\}$ with an operation (which for our case will be multiplication, noted by $\cdot$), which satisfies the following [18]:

CLOSURE If $a, b \in A$, then $c = a \cdot b$ is also in A.

ASSOCIATIVE $\forall a, b, c \in A$ we have that $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

IDENTITY $\exists e \in A \mid e \cdot a = a \cdot e = a \; \forall a \in A$

INVERSE $\forall a \in A \; \exists a^{-1} \mid a \cdot a^{-1} = a^{-1} \cdot a = e$

If $\cdot$ also conmutes, then the group is said to be Abelian, in other case it is called non-Abelian.

All transformations that leave the lagrangian invariant form a group. To show this, suppose that A and B are transformations that leave the lagrangian invariant, hence $A\mathcal{L} = \mathcal{L}$ and $B\mathcal{L} = \mathcal{L}$. Using this, is easy to see that a combination of such transformations satisfies the closure condition: $AB\mathcal{L} = A\mathcal{L} = \mathcal{L}$. In an analogus way it is easy to see how associativeness of multiplication is satisfied. There exists an identity, and is simply obtained by doing nothing to lagrangian, and also an applied transformation to lagrangian can be undone, rendering the original lagrangian, hence showing that there are inverses for each element of the group. This group is called the symmetry group of the theory [19]. Given any two groups $G(\{g_i\}, \cdot_G)$, $H(\{h_i\}, \cdot_H)$ the *direct product group* is defined as $(G \otimes H)(\{d_i\} = \{g_i h_i\}, \cdot)$, where multiplication is (explicitly):

$$\begin{aligned} d_a \cdot d_b &= g_a h_a \cdot g_b h_b \\ &= (g_a \cdot_G g_b)(h_a \cdot_H h_b). \end{aligned} \tag{2}$$

If a group can be written as a direct product of smaller groups the study of its properties is simplified [18]. For example $U(n) = SU(n) \otimes U(1)$.

A *representation* is a set of matrices $D(a)$ associated with the elements of the group, a, so that multiplication between matrices preserves group multiplication i. e.

$$D(a)D(b) = D(a \cdot b). \tag{3}$$

A representation can be *reducible*, this happens if there exists a non singular matrix M that is independent from the group elements that satisfies

$$MD(a)M^{-1} = diag(d_1(a), d_2(a), ...) \qquad \forall a; \tag{4}$$

if M does not exist then that particular representation is called *irreducible*. A reducible representation implies that a subset of states (on which transformations act) is never connected to other states. On the other hand an irreducible representation means that all states can be conected together between group transformations [18]. That is the reason because in physics we work only with irreducible representations of symmetry groups; otherwise (un)physical decouplings will be introduced by hand, which is unacceptable.

Groups can also be discrete or continuous. Discrete groups can have a finite or infinite set of elements, but the important thing is that its elements do not vary in a smooth way. Examples of such kind of groups are $G(\mathbb{Z}, +)$ or $G(\{-1, 0, 1\}, +)$. In physics, examples of thee kind of symmetries are time reversal, parity or charge conjugation; thee groups are also usually used to force decouplings. An example of this is the use of Z_2 to explain the decoupling between ordinary matter and dark matter [20, 21].

On the other hand continuous groups have infinite elements which are labelled by continuous parameters. An example of this groups is the group of rotations. A general element of any continuous group can be written (in matrix representation) as $U(\beta_a, \beta_b, ..., \beta_r)$, r being the total number of parameters needed to uniquely determine a group element (matrix). Parameters can always be chosen in such a way that they are zero for the identity. Then, any group element can be written in terms of group generators T_a (just like a vector space can be generated from base vectors), which are defined as:

$$T_a := i \left.\frac{\partial U}{\partial \beta_a}\right|_{\beta=0}. \tag{5}$$

Within continuous groups there are Lie groups. Thee type of groups have some special properties, but for the purpose of this work we will just limit to say that its elements can be written as

$$U = \exp(-i\beta_a T_a), \tag{6}$$

where summation convention over repeated indices is assumed [19].

In physics we are specially interested in Lie groups that have unitary representation i. e. $UU^\dagger = U^\dagger U = 1$. This is because they allow us to use the usual analytic methods, and also because transformations in quantum mechanics are demanded to be unitary operators in Hilbert space [18]. From thee, symplectic and unitary groups such as $U(n)$ and $SU(n)$ are specially appealing; the reason for this interest is that thee groups were proven to lead to renormalizable theories when anomalies vanish [22, 23]. Any useful theory must predict measurable results, which is ensured by renormalizability.

Now we go back to classification of symmetries. More precisely, global symmetries correspond to space-time symmetries and its parameters are independent of space time. They relate the fields at different space time points; an example of this are Lorentz transformations [19].

On the other hand, gauge symmetries correspond to internal symmetries i. e. they do not alter the coordinate system. In contrast with global symmetries, parameters from a local (gauge) symmetry depend on space-time coordinates. They connect different fields or different components of the same field at the same space-time point. An example of this is a gauge transformation in quantum electrodynamics (QED) i. e.

$$\psi \to \psi' = \exp(-ieQ\theta(x))\psi, \tag{7}$$

$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \theta, \tag{8}$$

which leaves

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - m)\psi - eQ\overline{\psi}\gamma^\mu \psi A_\mu \tag{9}$$

invariant. Gauge symmetries require the presence of new particles called in general gauge bosons [19]. In order to guarantee invariance of the derivate under the gauge symmetry a gauge boson is introduced for each generator of the group e. g. a $U(1)$ group has one generator, so a single gauge boson will be introduced. This is the case of QED, which has a single gauge boson: the photon.

For this work we extend the standard model symmetry using two $U(1)$ groups. This group is the group of phase shifts i. e. transformations of the form $e^{i\delta}$ [18].

SYMMETRIES OF THE STANDARD MODEL

There are different motivations to elevate a certain property to a symmetry. For example there were some discoveries that led to the postulation of a new quantum "charge" called colour. First of all, by Pauli exclusion principle, it is known that two identical fermions cannot occupy the same state while having the same spin. The discovery of the ´$^{++}$ particle (which is composed by three up quarks) posed a problem. It has isospin 3/2, and since it is a ground state for the system of three up quarks its wave function has zero total angular momentum, hence is totally symmetric. Also, in order to have spin $s_3 = 3/2$ the spin function must be also symmetric (which means that the spins of the quarks are aligned in the same direction), so overall, the wavefunction of ´$^{++}$ must be symmetric; again, this is forbidden for fermions by Pauli exclusion principle.

The solution to this problem emerged thanks to the works made by Greenberg, Han and Nambu [24, 25, 26]. As mentioned above they proposed the existence of a new hidden degree of freedom called colour. Each flavour of particles *u, d, c, s, b, t* corresponded to a colour triplet. The colour group operators could change the quark colour but not its flavour. They also postulated that only colour singlets were observable states [18].

There are SU(3) singlets in the product of $3 \times 3^* = 1 + 8$ and $3 \times 3 \times 3 = 1 + 8 + 8 + 10$ (for a detailed explanation of this notation see [18, Ch. 4]), hence only $q\overline{q}$ and qqq configurations can be observed; states like q, qq or $qqqq$ (which were though to be possible but were not observed) are ruled out by this new formalism, hence having agreement with nature.

Also with this proposal the ´$^{++}$ wave function becomes antisimmetric (satisfying the Pauli exclusion principle) i. e.

$$´^{++} = u_\alpha(x_1)u_\beta(x_2)u_\gamma(x_3)\epsilon^{\alpha\beta\gamma}, \tag{10}$$

were $\alpha,\ \beta,\ \gamma = 1, 2, 3$ are colour indices, and $\epsilon^{\alpha\beta\gamma}$ is the Levi-Civita symbol.

Later a theory of interactions between quarks inside hadrons was developed, it is known as quantum chromodynamics. That theory is the one of the strong (nuclear) force. Quarks interact with each other exchanging gluons, which are colour carriers; this is analogus to QED in which charged particles interact with each by photons exchange.

This new quantum number (colour) explained the particle spectrum in a better way than earlier theories (as was mentioned above). It also has the SU(3) symmetry.

Now, let us introduce some important concepts for the following discusion.

- Parity: It is a transformation which changes the signs of the coordinate system i. e. $(x, y, z) \to (-x, -y, -z)$.

- Helicity: Is the sign of the projection of the particle spin over its motion direction. Note that this quantity is dependent on the reference frame.

- Chirality: It tells us if the particle transforms in a right (or left)-handed representation of the Poincaré group. Dirac spinors have both kind of components, so it is useful to define projectors which extract those components and work with them independently i. e. $P_{L(R)}\psi = \psi_{L(R)}$, where

$$P_{L(R)} = \frac{1 \mp \gamma_5}{2} \tag{11}$$

It has been observed that nature can distinguish between left and right handed systems [27]. Moreover, it has been observed that only particles with left chirality (also called left-handed particles) interact through weak force [17, Ch. 19]. Left-handed particles also form multiplets under $SU(2)$, while right(-helicity) particles do not interact and also form singlets under $SU(2)$.

In massless fermion theories the lagrangian can be split like

$$\mathcal{L} = \psi^{\dagger}_{Li} i\overline{\sigma} \cdot \partial\psi_{Li} + \psi^{\dagger}_{Ri} i\sigma \cdot \partial\psi_{Ri}. \tag{12}$$

Since right and left contributions are independent the system can be then coupled to a gauge field by assigning left particles to a representation of the group and the right ones to the singlet representation i. e. they are invariant under $SU(2)$ transformations. Hence

$$\mathcal{L} = \psi^{\dagger}_{Li} i\overline{\sigma} \cdot (\partial_\mu - igA^a_\mu t^a_r)\psi_{Li} + \psi^{\dagger}_{Ri} i\sigma \cdot \partial\psi_{Ri} \tag{13}$$

Particles can only interact through exchange of gauge bosons (force carriers), so this clearly shows that right particles do not interact in a massless fermion theory [17].

Finally, the last symmetry present in the standard model group corresponds to the weak hypercharge Y. It is a new quantity which is motivated by The Gell-Mann–Nishijima relation, which was proposed by Nishijima and Nakano in 1953 [28] and later (independently) by Gell-Mann in 1953 [29]. It related charge Q and isospin T_3 like

$$Q = T_3 + \frac{Y}{2}. \tag{14}$$

Let us recall how generators were chosen (following the ideas from [18]). To see this we limit to the simplest case i. e. the gauge theory between an electron and its neutrino. For this case the contribution of the weak interaction to the lagrangian is given by

$$\mathcal{L}_W = g(J_\lambda W^\lambda + h.c.), \tag{15}$$

with

$$J_\lambda = \overline{\nu_e}\gamma_\lambda(1 - \gamma_5)e \tag{16}$$

Also, the electromagnetic part of the lagrangian is

$$\mathcal{L}_e m = eJ^{em}_\lambda A^\lambda, \tag{17}$$

where the electromagnetic current is

$$J^{em}_{\lambda} = -\bar{e}\gamma_{\lambda}e \tag{18}$$

In a theory which unifies weak and electromagnetic interaction at least three vector gauge bosons are needed. This is because, as stated above weak interaction has SU(2) symmetry, and the electromagnetic interaction has $U(1) \supset SU(2)$ symmetry, hence the simplest group that unifies them both is SU(2). As mentioned before, each gauge boson is associated with a group generator; hence theory must have three gauge bosons.

But there is a problem, and is that Noether currents do not form a closed algebra; and they must because they are proportional to the generators. Remember that closure is one of the conditions to have a group. In order to see that this claim is true the weak and electric charges are defined as

$$T_{+}(t) = \frac{1}{2}\int d^3x J_0(x) = \frac{1}{2}\int d^3x \nu_e^{\dagger}(1-\gamma_5)e \tag{19a}$$

$$T_{-}(t) = T_{+}^{\dagger}(t) \tag{19b}$$

$$Q(t) = \int d^3x J_0^{em}(x) = -\int d^3x e^{\dagger}e. \tag{19c}$$

Taking into account the anticonmutation relations for fermions i.e.

$$\left\{\psi_i^{\dagger}(\vec{x},t), \psi_j(\vec{x},t)\right\} = \delta_{ij}\delta^3(\vec{x}-\vec{x}') \tag{20}$$

it can be shown that

$$\begin{aligned}[T_{+}(t), T_{-}(t)] &= \frac{1}{2}\int d^3x(\nu_e^{\dagger}(1-\gamma_5)\nu_e - e^{\dagger}(1-\gamma_5)e) \\ &= 2T_3(t) \neq Q \end{aligned} \tag{21}$$

Now a new gauge boson is introduced associated with T_3. This new generators now form the group $SU(2) \otimes U(1)$. The contribution to lagrangian from the field tensors now reads

$$\mathcal{L}_1 = -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}, \tag{22}$$

where

$$F^i_{\mu\nu} = \partial_{\mu}A^i_{\nu} - \partial_{\nu}A^i_{\mu} + g\epsilon^{ijk}A^j_{\mu}A^k_{\nu} \qquad i = 1,2,3 \tag{23}$$

$$G_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}. \tag{24}$$

Now in order to see how quantum numbers are assigned consider the first generation of the standard model; this approximation is self contained, and the purpose here is just to show how this leads to Y observed values. We have 15 particles, namely ν_{eL}, e_L, e_R, u_L, u_R, d_L, d_R (remember that u and d come in three different colours) and that the left or right components are obtained by using the helicity projectors P_L and P_R (eq. 11). With this on mind, the charges now read as

$$T_+ = \int (\nu_{eL}^\dagger e_L + u_L^\dagger d_L) d^3x, \quad (25a)$$

$$T_- = T_+^\dagger, \quad (25b)$$

$$T_3 = \frac{1}{2}\int (\nu_{eL}^\dagger \nu_{eL} - e_L^\dagger e_L + u_L^\dagger u_L - d_L^\dagger d_L) d^3x, \quad (25c)$$

$$Q = \int (-e^\dagger e + \frac{2}{3}u^\dagger u - \frac{1}{3}d^\dagger d) d^3x \quad (25d)$$

$$= \int (-e_L^\dagger e_L - e_R^\dagger e_R + \frac{2}{3}u_L^\dagger u_L + \frac{2}{3}u_R^\dagger u_R - \frac{1}{3}d_L^\dagger d_L - \frac{1}{3}d_R^\dagger d_R) d^3x \quad (25e)$$

The U(1) representation must be chosen so that the Gell-Mann–Nishijima relation (eq. 14) is satisfied, so

$$Q - T_3 = \int [-\frac{1}{2}(\nu_{eL}^\dagger \nu_{eL} + e_L^\dagger e_L) + \frac{1}{6}(u_L^\dagger u_L + d_L^\dagger d_L) - e_R^\dagger e_R + \frac{2}{3}(u_R^\dagger u_R) - \frac{1}{3}d_R^\dagger d_R] d^3x \quad (26)$$

It is trivial to see now that the group generators form a closed algebra i. e.

$$[Q - T_3, T_i] = 0 \qquad i = 1, 2, 3. \quad (27)$$

So Y is chosen to be

$$Y = 2(Q - T_3) \quad (28)$$

So as was shown, the weak hypercharge (Y) obbeys a U(1) symmetry. In a similar manner X, which can be though of an other generalized "charge" (also called exotic hypercharge), was introduced in some works following a $U'(1)$ structure [13]. The difference made by introducing $U'(1)_X$ is that it acts as a horizontal symmetry thanks to its non-universality (implied by the prime) i. e. it assigns different X charges to different generations.

The idea of implementing a "horizontal" symmetry comes from the mass hierarchy problem. There is no satisfactory answer to date to that problem. There are many questions related with that topic, for instance, one may ask if there are just three generations of particles, or if there are more. But as happened with colour, those questions can only be answered if a theory which explains that is formulated.

Then an idea (inspired by colour) to solve that problem is to formulate a theory which connects different generations [30]. For example, suppose a theory that groups quarks under SU(3) triplets, so that (u, c, t) are a triplet, and the same happens for the down-like quarks and leptons. Remember that interacting particles come in doublets like

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$
$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu \\ \nu_\tau \end{pmatrix} \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

so the name of horizontal symmetry is a reminder of the fact that transformations of this part of the group act on quarks "horizontally", i. e. it acts on triplets (u, c, t), (d, s, b), (e, μ, τ), $(\nu_e, \nu_\mu, \nu_\tau)$.

Now, it is interesting to introduce $B - L$ as a gauge symmetry. This number is conserved in many processes. An example of this claim can be seen in the β decay e. g.

	$n^0 \rightarrow$	$p^+ +$	$e^- +$	$\overline{\nu_e}$
B	1	1	0	0
L	0	0	1	-1
B-L	1	1		

so this shows that the physical origin of the $B - L$ quantity.

In next chapter the subject of anomalies is presented. The reason for talking about this seemingly unrelated topic is because it will determine the form in which the representation of the symmetry group is chosen in order to build a consistent model.

3 ANOMALIES

There are three types of symmetry breaking in physics. Two of them are widely known, whilst the third one, despite having a crucial importance on construction of new gauge models is still seen as something arcane. The first two kinds of symmetry break are: explicit, this is when the Lagrangian contains a term which violates the invariance; spontaneous, this is when the Lagrangian is invariant but the vacuum state does not share its symmetry, this will be treated on Chapter 5.

The third kind of symmetry breaking is usually referred as *anomaly*. This is the subject of this chapter.

Anomalies can be though of as a quantum symmetry breaking. This is, a symmetry exists when the system is studied in the classical level, but dissapears when the system is quantized. There are plenty of examples of anomalies in many fields of physics [31, 32, 33, 34]; Jackiw evidenced on Ref. [35] that this can be evidenced in a rather simple system on quantum mechanics.

EXAMPLE OF AN ANOMALY IN QUANTUM MECHANICS

Here the same example from Ref. [36] is shown. First of all consider a freely moving particle; then, it obeys the time independent Schrödinger equation i. e.

$$-\frac{1}{2m}\nabla^{2-}(\mathbf{x}) = E^{-}(\mathbf{x}), \tag{29}$$

where $E = k^2/2m$. That equation has the solution

$$^{-}(\mathbf{x}) = \frac{1}{r}\chi_k(\mathbf{r})P_l(\cos\theta), \tag{30}$$

where the radial function satisfies

$$\left(-\frac{d^2}{dr^2} + \frac{l(l+1)}{r^2} + k^2\right)\chi_k(r) = 0. \tag{31}$$

Note that under transformations of the form

$$r \to \lambda r \tag{32a}$$

$$k \to \frac{1}{\lambda}k \tag{32b}$$

the eq. 31 remains invariant (it is also said that the equation is symmetric under those kind of transformations); so its solution must share that symmetry, hence a solution is

$$\chi_k(r) = krj_l(kr). \tag{33}$$

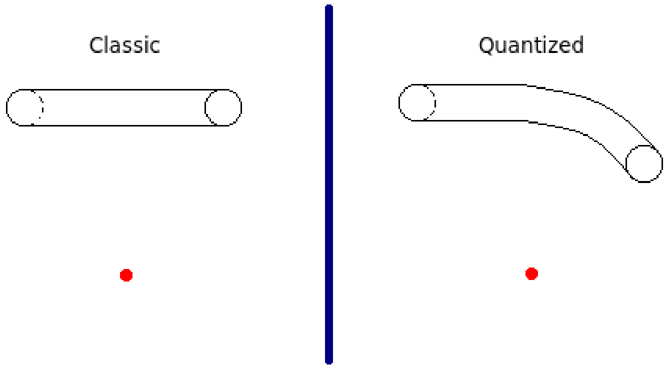

Figure 3: Example of an anomaly in quantum mechanics. The red dot is the δ-like potential. Note that the axial symmetry of the beam is lost by the quantization process.

With this, the plane-wave solution of eq. 29, using a partial wave expansion in the limit $r \to \infty$ takes the form

$$^{-(+)}(r) \sim e^{ikz} + \frac{1}{\sqrt{r}} e^{i(kr+\pi/4)} f(\theta), \tag{34}$$

where the scattering amplitude $f(\theta)$ is

$$f(\theta) = -i \sum_{-\infty}^{\infty} \frac{e^{2i\delta_m(k)} - 1}{\sqrt{2\pi k}} e^{im\theta}. \tag{35}$$

Now consider the case

$$V(r) = \lambda \delta^2(r) \tag{36}$$

Note that this is a non $1/r^2$ potential, so that (in two dimensions) scale invariance is preserved. At classical level it is evident that particles with non-vanishing impact parameter will not be deflected. But in the quantum approach that is no longer the case; because the wave-like nature particles acquire even if their impact parameter is non-zero they will feel its presence (see Fig. 3), hence the nonforward scattering cross section will have a value different from zero (this only occours for $m = 0$; remember that partial waves with $m \neq 0$ vanish at the origin).

Also note that invariance under eq. 32 would require $\delta_0(k) = \mathtt{const}$, but this is no longer the case!

The classical symmetry has been broken by the quantization process. This is an anomaly.

The energy dependence of the phase shift arises because the potential is divergent, but in order to predict, calculations need to be finite. In order to do that, in the regularization process a quantity with energy dimensions is introduced and by doing that, the scale invariance (eq 32) is violated.

Another way to see this is shown on [37]. The regularization can be made by defining

$$V(r) = \lim_{\epsilon \to 0} \lambda \delta^2(r - \epsilon), \tag{37}$$

which will later lead to the cross-section (for large k)

$$\frac{d\sigma}{d^{\cdot}} \sim \frac{1}{4k}\frac{2\pi}{(\ln k/\kappa)^2} \tag{38}$$

The parameter κ must be determined empirically, but the important result is that eq. 38 violates invariance under scale transformations i. e. eq. 32.

ANOMALIES IN QUANTUM FIELD THEORY

In quantum field theory the appearance of divergences is very common, even in renormalizable theories. In order to make calculations finite it is usual to introduce a regulator or a cut-off. This new quantities may violate the symmetries, and at the end of calculations -when they are removed- they may leave traces of the aforementioned symmetry violation. That is the reason why anomalies appear in quantum field theory.

Two kinds of anomalies can be identified. Those which violate a global symmetry; in this case there is no problem with theory. And those which violate gauge symmetries; this ones indeed pose a problem to the theory because this kind of violations cause it to be inconsistent. Hence anomaly cancellation becomes a constraint when building physical gauge theories.

In the 1960s it was believed that the pion was a boson associated with an spontaneously broken $SU(2) \otimes SU(2)$ symmetry of strong interactions. This view had repeated successes but also some serious flaws. For example, the dominant decay mode of the neutral pion $\pi^0 \to 2\gamma$ had a predicted decay rate of $(1.9 \times 10^{13})\ s^{-1}$, whilst the observed decay rate was of $(1.19 \pm 0.08) \times 10^{16}\ s^{-1}$! Similar problems appeared when calculating the rates of other processes.

The solution of this problem was found by Bell and Jackiw [38]; they found that the regulator needed to deduce the consequences of the conservation of neutral axial vector currents violated the chiral symmetry, thus inducing the anomaly. This was later confirmed and generalized by Adler [39].

Now let us deduce that anomalies are affected by the choice of the representation of the symmetry group.

The method used to study anomalies is the one of direct calculation (the same as on Ref. [14]). The problem was originally treated this way, and also it helps to discuss anomalies in more general theories in a straightforward way.

Consider the one-loop three point function

$$`^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z) \equiv \left\langle T\left\{ j^{\mu}_{\alpha}(x), j^{\nu}_{\beta}(y), j^{\rho}_{\gamma}(z)\right\}\right\rangle_{VAC}, \tag{39}$$

where j^{μ}_{α} is the fermionic current; it can be calculated in terms of the free fields as

$$j^{\mu}_{\alpha}(x) = -i\overline{\chi}T_{\alpha}\gamma^{\mu}\chi. \tag{40}$$

Expanding eq. 39 yields

$$`^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z) = -i\mathrm{Tr}\left[S(x-y)T_\beta\gamma^\nu P_L S(y-z)T_\gamma\gamma^\rho P_L S(z-x)T_\alpha\gamma^\mu P_L\right] \\ - i\mathrm{Tr}\left[S(x-z)T_\gamma\gamma^\rho P_L S(z-y)T_\beta\gamma^\nu P_L S(y-x)T_\alpha\gamma^\mu P_L\right], \quad (41)$$

where P_L is the projector of left helicity i. e.

$$P_L = \left(\frac{1+\gamma_5}{2}\right), \quad (42)$$

and $S(x)$ is the propagator of a massless fermion i. e.

$$S(x) = \frac{-i}{(2\pi)^4}\int d^4p \left(\frac{-i\not{p}}{p^2 - i\epsilon}\right) e^{ipx}. \quad (43)$$

Then, eq. 41 can be rearranged as

$$`^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z) = \frac{i}{(2\pi)^{12}}\int d^4k_1 d^4k_2 e^{-i(k_1+k_2)x} e^{ik_1y} e^{ik_2z}\int d^4p \\ \times\left\{\mathrm{tr}\left[\frac{\not{p}-\not{k}_1+\not{a}}{(p-k_1+a)^2-i\epsilon}\gamma^\nu\frac{\not{p}+\not{a}}{(p+a)^2-i\epsilon}\gamma^\rho\frac{\not{p}+\not{k}_2+\not{a}}{(p+k_2+a)^2-i\epsilon}\gamma^\mu\frac{1+\gamma_5}{2}\right]\times\mathrm{tr}[T_\beta T_\gamma T_\alpha]\right. \\ \left.+\,\mathrm{tr}\left[\frac{\not{p}-\not{k}_2+\not{b}}{(p-k_2+b)^2-i\epsilon}\gamma^\rho\frac{\not{p}+\not{b}}{(p+b)^2-i\epsilon}\gamma^\nu\frac{\not{p}+\not{k}_1+\not{b}}{(p+k_1+b)^2-i\epsilon}\gamma^\mu\frac{1+\gamma_5}{2}\right]\times\mathrm{tr}[T_\gamma T_\beta T_\alpha]\right\}, (44)$$

where tr is a trace over the corresponding Dirac or group indices; a and b are arbitrary constant 4-vectors which are introduced for later convenience. Using the definitions of $D_{\alpha\beta\gamma}$ the symmetric group constant and $C_{\alpha\beta\gamma}$ the structure constant i. e.

$$D_{\alpha\beta\gamma} = \frac{1}{2}\mathrm{tr}[\{T_\alpha, T_\beta\}T_\gamma], \quad (45)$$

$$\left[T_\alpha, T_\beta\right] = C_{\alpha\beta\gamma}T^\gamma, \quad (46)$$

the three-point function can be separated into terms that are symmetric and anti-symmetric in the group indices, this is done by putting

$$\mathrm{tr}[T_\beta T_\gamma T_\alpha] = D_{\alpha\beta\gamma} + \frac{1}{2}iNC_{\alpha\beta\gamma}, \quad (47)$$

$$\mathrm{tr}[T_\gamma T_\beta T_\alpha] = D_{\alpha\beta\gamma} - \frac{1}{2}iNC_{\alpha\beta\gamma}. \quad (48)$$

Where N satisfies

$$\mathrm{tr}[T_\alpha T_\beta] = N\delta_{\alpha\beta}. \quad (49)$$

Finally, taking the divergence of eq. 44 and using the identity

$$\not{k}_1 + \not{k}_2 = (\not{p}+\not{k}_2+\not{a}) - (\not{p}-\not{k}_1+\not{a}) = (\not{p}+\not{k}_1+\not{b}) - (\not{p}-\not{k}_2+\not{b}), \quad (50)$$

renders

$$\frac{\partial}{\partial x^\mu}\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z) = \left[\frac{\partial}{\partial x^\mu}\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)\right]_{formal} + \left[\frac{\partial}{\partial x^\mu}\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)\right]_{anom}, \tag{51}$$

where

$$\begin{aligned}\left[\frac{\partial}{\partial x^\mu}\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)\right]_{formal} &= -iC_{\alpha\beta\delta}\delta^4(x-y)\left\langle j^\nu_\delta(y) j^\rho_\gamma(z)\right\rangle_{VAC} \\ &\quad - iC_{\alpha\gamma\delta}\delta^4(x-z)\left\langle j^\nu_\beta(y) j^\rho_\delta(z)\right\rangle_{VAC},\end{aligned} \tag{52}$$

and

$$\begin{aligned}\left[\frac{\partial}{\partial x^\mu}\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)\right]_{anom} &= \frac{i}{(2\pi)^{12}}D_{\alpha\beta\gamma}\int d^4k_1 d^4k_2 e^{-i(k_1+k_2)x}e^{ik_1y}e^{ik_2z} \\ \times \int d^4p \Bigg\{ &\mathrm{tr}\left[\frac{\not p - \not k_1 + \not a}{(p-k_1+a)^2 - i\epsilon}\gamma^\nu \frac{\not p + \not a}{(p+a)^2 - i\epsilon}\gamma^\rho \frac{1+\gamma_5}{2}\right] \\ &-\mathrm{tr}\left[\frac{\not p + \not a}{(p+a)^2 - i\epsilon}\gamma^\rho \frac{\not p + \not k_2 + \not a}{(p+k_2+a)^2 - i\epsilon}\gamma^\nu \frac{1+\gamma_5}{2}\right] \\ &+\mathrm{tr}\left[\frac{\not p - \not k_2 + \not b}{(p-k_2+b)^2 - i\epsilon}\gamma^\rho + \frac{\not p + \not b}{(p+b)^2 - i\epsilon}\gamma^\nu \frac{1+\gamma_5}{2}\right] \\ &-\mathrm{tr}\left[\frac{\not p + \not b}{(p+b)^2 - i\epsilon}\gamma^\nu \frac{\not p + \not k_1 + \not b}{(p+k_1+b)^2 - i\epsilon}\gamma^\rho \frac{1+\gamma_5}{2}\right]\Bigg\}.\end{aligned} \tag{53}$$

Integrals within eq. 52 can be renormalized and pose no problem for theory. Now focusing on the anomalous part, and grouping the first and last traces, and the rest, we have

$$\begin{aligned}\left[\frac{\partial}{\partial x^\mu}\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)\right]_{anom} &= \frac{i}{(2\pi)^{12}}D_{\alpha\beta\gamma}\int d^4k_1 d^4k_2 e^{-i(k_1+k_2)x}e^{ik_1y}e^{ik_2z} \\ \times \Bigg\{\mathrm{tr}\left[\gamma^\kappa\gamma^\nu\gamma^\lambda\gamma^\rho\frac{1+\gamma_5}{2}\right] & I_{\kappa\lambda}(a-b-k_1, b, b+k_1) \\ &+ \mathrm{tr}\left[\gamma^\kappa\gamma^\rho\gamma^\lambda\gamma^\nu\frac{1+\gamma_5}{2}\right] I_{\kappa\lambda}(b-a-k_2, a, a+k_2)\Bigg\}\end{aligned} \tag{54}$$

where

$$I_{\kappa\lambda}(k,c,d) \equiv \int dp^4[f_{\kappa\lambda}(p+k,c,d) - f_{\kappa\lambda}(p,c,d)] \tag{55}$$

$$f_{\kappa\lambda}(p,c,d) \equiv \frac{(p+c)_\kappa (p+d)_\lambda}{[(p+c)^2 - i\epsilon][(p+d)^2 - i\epsilon]}. \tag{56}$$

With some work it can be shown that choosing $a = -b$ the only terms which survive are the ones involving the totally antisymmetric tensor $\epsilon^{\kappa\nu\lambda\rho}$ (with $\epsilon^{0123} = 1$) i. e.

$$\mathrm{tr}[\gamma^\kappa\gamma^\nu\gamma^\lambda\gamma^\rho] = -4i\epsilon^{\kappa\nu\lambda\rho}. \tag{57}$$

Hence

$$\left[\frac{\partial}{\partial x^{\mu}}\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)\right]_{anom} = \frac{2}{(2\pi)^{12}} D_{\alpha\beta\gamma}\int d^4k_1\, d^4k_2\, e^{-i(k_1+k_2)x} \times e^{ik_1 y} e^{ik_2 z}\pi^2 \epsilon^{\kappa\nu\lambda\rho} a_{\kappa}(k_1+k_2)_{\lambda} \quad (58)$$

Then, the anomaly (eq. 58) in the current $J^{\mu}_{\alpha}(x)$ can be eliminated if a is chosen to be proportional to $k_1 + k_2$. Sadly by making that choice the anomaly reappears in $(\partial/\partial y^{\nu})\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)$ or in $(\partial/\partial z^{\rho})\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)$. The anomaly in $(\partial/\partial y^{\nu})\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)$ only dissapears if $a + k_2 \propto k_1$; and will vanish in $(\partial/\partial z^{\rho})\grave{}^{\mu\nu\rho}_{\alpha\beta\gamma}(x,y,z)$ if $a - k_1 \propto k_2$. It is impossible to choose a so that all the anomalies vanish simultaneously.

Then, the only way in which anomalies can be cancelled is if and only if

$$\boxed{D_{\alpha\beta\gamma} = 0.} \quad (59)$$

This treatment is completely general, so eq. 59 imposes a set of constraints for the representation of the group. In other words the theory will only be consistent if the chosen representation of the group satisfies eq. 59.

For our case we will have the following anomalies,

$$\begin{aligned}
[SU(3)_c]^2\, U(1)_X &\rightarrow A_1\\
[SU(2)_L]^2\, U(1)_X &\rightarrow A_2\\
[U(1)_Y]^2\, U(1)_X &\rightarrow A_3\\
U(1)_Y\, [U(1)_X]^2 &\rightarrow A_4\\
[U(1)_X]^3 &\rightarrow A_5\\
[Grav]^2 \otimes U(1)_X &\rightarrow A_6\\
[SU(3)_c]^2\, U(1)_{(B-L)} &\rightarrow A_7\\
[SU(2)_L]^2\, U(1)_{(B-L)} &\rightarrow A_8\\
[U(1)_Y]^2\, U(1)_{(B-L)} &\rightarrow A_9\\
U(1)_Y \left[U(1)_{(B-L)}\right]^2 &\rightarrow A_{10}\\
[U(1)_X]^2\, U(1)_{(B-L)} &\rightarrow A_{11}\\
U(1)_X \left[U(1)_{(B-L)}\right]^2 &\rightarrow A_{12}\\
\left[U(1)_{(B-L)}\right]^3 &\rightarrow A_{13}\\
[Grav]^2 \otimes U(1)_{(B-L)} &\rightarrow A_{14}.
\end{aligned}$$

Which, written explicitly, are

$$\mathcal{A}_1 = \sum_Q X_{Q_L} - \sum_Q X_{Q_R}, \tag{60a}$$

$$\mathcal{A}_2 = \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L}, \tag{60b}$$

$$\mathcal{A}_3 = \sum_{\ell,Q}\left[Y_{\ell_L}^2 X_{\ell_L} + 3Y_{Q_L}^2 X_{Q_L}\right] - \sum_{\ell,Q}\left[Y_{\ell_R}^2 X_{\ell_R} + 3Y_{Q_R}^2 X_{Q_R}\right], \tag{60c}$$

$$\mathcal{A}_4 = \sum_{\ell,Q}\left[Y_{\ell_L} X_{\ell_L}^2 + 3Y_{Q_L} X_{Q_L}^2\right] - \sum_{\ell,Q}\left[Y_{\ell_R} X_{\ell_R}^2 + 3Y_{Q_R} X_{Q_R}^2\right], \tag{60d}$$

$$\mathcal{A}_5 = \sum_{\ell,Q}\left[X_{\ell_L}^3 + 3X_{Q_L}^3\right] - \sum_{\ell,Q}\left[X_{\ell_R}^3 + 3X_{Q_R}^3\right], \tag{60e}$$

$$\mathcal{A}_6 = \sum_{\ell,Q}\left[X_{\ell_L} + 3X_{Q_L}\right] - \sum_{\ell,Q}\left[X_{\ell_R} + 3X_{Q_R}\right], \tag{60f}$$

$$\mathcal{A}_7 = \sum_Q (B-L)_{Q_L} - \sum_Q (B-L)_{Q_R} \tag{60g}$$

$$\mathcal{A}_8 = \sum_\ell (B-L)_{\ell_L} + 3\sum_Q (B-L)_{Q_L}, \tag{60h}$$

$$\mathcal{A}_9 = \sum_{\ell,Q}\left[Y_{\ell_L}^2 (B-L)_{\ell_L} + 3Y_{Q_L}^2 (B-L)_{Q_L}\right] - \sum_{\ell,Q}\left[Y_{\ell_R}^2 (B-L)_{\ell_R} + 3Y_{Q_R}^2 (B-L)_{Q_R}\right] \tag{60i}$$

$$\mathcal{A}_{10} = \sum_{\ell,Q}\left[Y_{\ell_L} (B-L)_{\ell_L}^2 + 3Y_{Q_L} (B-L)_{Q_L}^2\right] - \sum_{\ell,Q}\left[Y_{\ell_R} (B-L)_{\ell_R}^2 + 3Y_{Q_R} (B-L)_{Q_R}^2\right] \tag{60j}$$

$$\mathcal{A}_{11} = \sum_{\ell,Q}\left[X_{\ell_L}^2 (B-L)_{\ell_L} + 3X_{Q_L}^2 (B-L)_{Q_L}\right] - \sum_{\ell,Q}\left[X_{\ell_R}^2 (B-L)_{\ell_R} + 3X_{Q_R}^2 (B-L)_{Q_R}\right] \tag{60k}$$

$$\mathcal{A}_{12} = \sum_{\ell,Q}\left[X_{\ell_L} (B-L)_{\ell_L}^2 + 3X_{Q_L} (B-L)_{Q_L}^2\right] - \sum_{\ell,Q}\left[X_{\ell_R} (B-L)_{\ell_R}^2 + 3X_{Q_R} (B-L)_{Q_R}^2\right] \tag{60l}$$

$$\mathcal{A}_{13} = \sum_{\ell,Q}\left[(B-L)_{\ell_L}^3 + 3(B-L)_{Q_L}^3\right] - \sum_{\ell,Q}\left[(B-L)_{\ell_R}^3 + 3(B-L)_{Q_R}^3\right] \tag{60m}$$

$$\mathcal{A}_{14} = \sum_{\ell,Q}\left[(B-L)_{\ell_L} + 3(B-L)_{Q_L}\right] - \sum_{\ell,Q}\left[(B-L)_{\ell_R} + 3(B-L)_{Q_R}\right] \tag{60n}$$

Here, sum over Q means over all quarks of the model; sum over ℓ means over all leptons of the model. X, Y, $B-L$ means sum of thee quantum numbers (exotic hypercharge, hypercharge, barionic minus leptonic number) corresponding to the summed particle. Subscripts L and R imply summation over left-handed or right-handed particles.

Anomaly equations for the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ part of the group are ommited because that is the standard model sector and have been proven to cancel (for details see [17, Ch. 20], [14, Sec. 22.4]). For our case, the representation which vanishes eq. 60 gives the particle content shown in Tabs. 3 and 4. An alternative aproach to evaluate anomalies is presented on Appendix A.

The quantity P presented in Tabs. 3 and 4 is defined as

$$P = (-1)^{3(B-L)+2s}, \tag{61}$$

where s is the spin. The dark matter candidates for this model are the fields which have $P = -1$.

Table 3: Left sector particle content. $a = 1,2,3$, $\alpha = 1,2$. G_{SM} means charges under the standard model like (# of colors, doublet or singlet under $SU(2)_L$, Y).

Spectrum	G_{SM}	X	$B-L$	P
Leptons				
$\psi_{aL} = \begin{pmatrix} \nu_a \\ e_a \end{pmatrix}_L$	(1,2,-1)	-1/3	-1	1
$(N_{aR})^c$	(1,1,0)	-1/3	0	-1
Quarks				
$Q_{\alpha L} = \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}_L$	(3,2,1/3)	0	1/3	1
$Q_{3L} = \begin{pmatrix} u_3 \\ d_3 \end{pmatrix}_L$	(3,2,1/3)	1/3	1/3	1
$J_{\alpha L}$	(3,1,-2/3)	0	-2/3	-1
T_L	(3,1,4/3)	1/3	4/3	-1

Table 4: Right sector particle content. $a = 1, 2, 3$, $\alpha = 1, 2$. G_{SM} means charges under the standard model like (# of colors, doublet or singlet under $SU(2)_L$, Y).

Spectrum	G_{SM}	X	$B-L$	P
	Leptons			
ν_{aR}	(1,1,0)	0	-1	1
e_{aR}	(1,1,-2)	-1	-1	1
	Quarks			
u_{aR}	$(3^*, 1, 4/3)$	2/3	1/3	1
d_{aR}	$(3^*, 1, -2/3)$	-1/3	1/3	1
$J_{\alpha R}$	$(3^*, 1, -2/3)$	-1/3	-2/3	-1
T_R	$(3^*, 1, 4/3)$	2/3	4/3	-1

4 QUICK REVIEW OF THE STANDARD MODEL

We have discussed the reasons why groups are so important for particle physics. We have also mentioned which characteristics must satisfy the representation of the said group in order for the model to be anomaly free.

Now the time to define interactions and masses of particles has come.

This chapter is structured as follows. First of all the particle content that makes the standard model anomaly free is presented, then we construct the lagrangian, which is roughly composed by interaction terms and mass terms. Then, we show how the spontaneus symmetry breaking mechanism acts in the standard model and gives particles mass and we also discuss some of the consequences of this mechanism.

As we mentioned in Chapter 1, one of the flaws of the standard model is that it does not account for neutrino masses. So in the final part of this chapter we will illustrate how neutrinos acquire mass in theories that go beyond the standard model by using the seesaw mechanism.

This chapter is dedicated to illustrate how mechanisms and procedures work in a well studied theory like the standard model. The calculations for our model are left for the other chapters.

PARTICLE CONTENT

The standard model is composed of three generations of particles. And is made anomaly free by choosing the representation presented in Tabs. 5 and 6 [40]. Notice how in the right sector there is a lack of neutrinos. This is because in the standard model neutrinos are massless.

Table 5: Left sector particle content. $a = 1, 2, 3$. For doublets the $\pm$ notation means that the upper particle has + charge, and the other the negative.

Spectrum	Colours	T_{3L}	Y
Leptons			
$\phi_{aL} = \begin{pmatrix} \nu_a \\ e_a \end{pmatrix}_L$	1	$\pm 1/2$	-1
Quarks			
$Q_{aL} = \begin{pmatrix} u_a \\ d_a \end{pmatrix}_L$	3	$\pm 1/2$	$1/3$

Table 6: Right sector particle content. $a = 1, 2, 3$.

Spectrum	Colours	T_{3L}	Y
	Leptons		
e_{aR}	1	0	-2
	Quarks		
u_{aR}	3^*	0	4/3
d_{aR}	3^*	0	-2/3

The Higgs sector for the standard model consists of a single complex scalar field with a doublet representation and quantum numbers shown on Tab. 7.

Table 7: Higgs sector for the standard model

	Colours	T_{3L}	Y
$\phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix}$	1	$\pm 1/2$	1

BUILDING LAGRANGIAN TERMS

Terms in the lagrangian must be

- General: It must contain all possible consistent terms.
- Renormalizable: This means that for a theory in a four dimensional universe the terms of the lagrangian must be of 4th-order at max when counting powers. [17]
- Invariant under symmetries: This is guaranteed if the sum of the different quantum charges is zero. Otherwise there will be a phase left in the term which will change when a symmetry transformation is applied, thus violating invariance.

It is easy to see that a term like $\phi^\dagger\phi = \phi^-\phi^+ + \phi_0^*\phi_0$ is suitable to be part of the lagrangian, as it does not exceed 4th order, and is clearly invariant i. e.

$$\begin{array}{lcccc} & \phi^\dagger & \phi & & \\ T_{3L} \text{ for } \phi^-\phi^+ & -1/2 & +1/2 & = & 0 \\ T_{3L} \text{ for } \phi_0^*\phi_0 & 1/2 & -1/2 & = & 0 \\ \text{Y for both} & -1 & +1 & = & 0. \end{array}$$

With this considerations in mind now we build the different parts of the lagrangian. As with any lagrangian it has a kinetic part and a potential.

Kinetic Terms

In order for them to be invariant under the gauge group, the derivate becomes a covariant derivate (in the sense of gauge fields, not in the sense of general relativity) i. e.

$$D_\mu = \partial_\mu + igA^i_\mu T_i + i\frac{g'}{2}B_\mu Y, \tag{62}$$

where g and g' are coupling constants (the prime just distinguishes them); the A^i_μ and B_μ are vectorial gauge fields and the i superscript just acts as an etiquette. The T_i are the generators from $SU(2)_L$ and Y is the generator from $U(1)_Y$. Repeated indices summation is implied.

Gauge fields transform under an infinitesimal transformation $U = \exp(i\theta \cdot T) \approx I + i\theta \cdot T$ like

$$A'^c_\mu = A^c_\mu + f^{abc}A^a_\mu\theta^b + \frac{1}{g}\partial_\mu\theta^c, \tag{63}$$

where f^{abc} are the structure constants of the group. Now we are able to build the kinetic terms.

For Gauge Fields

For gauge fields, which are the carriers of the interaction, the lagrangian is defined

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{64}$$

where F and B are the field tensors, which are defined to be

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{65a}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\mu. \tag{65b}$$

Notice that when expanding this terms we will have a sub term of the form $(\partial A)^2$, which is the pure kinetic contribution to the lagrangian for the gauge fields. Also we will have terms of the form ∂AAA and $AAAA$, which are interaction terms between gauge fields.

For Fermions

For fermions the kinetic lagrangian has two contributions, one for leptons

$$\mathcal{L}_{lep} = \bar{l}_L i\not{D} l_L + \bar{e}_R i\not{D} e_R, \tag{66}$$

and one for quarks

$$\mathcal{L}_{quarks} = \overline{Q}_L i\not{D} Q_L + \bar{u}_{aR} i\not{D} u_{aR} + \bar{d}_{aR} i\not{D} d_{aR}. \tag{67}$$

The slash notation implies that the slashed term is being contracted with the gamma matrixes.

Also notice that when expanding this terms using the covariant derivative defined in eq. 62 there will be terms of the form fermion-gauge boson-fermion. This means that the gauge interactions between the particles of the model will be given by the kinetic terms of the lagrangian. Also notice that as right particles behave as singlets under $SU(2)_L$ there is no $igA^i_\mu T_i$ term for them, so their interactions are electromagnetic in nature.

For Scalar Fields

For scalar fields the kinetic lagrangian is

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) \tag{68}$$

Expanding this will give both pure kinetic terms for scalar fields and mass terms for gauge bosons i. e. terms of the form $\partial_\mu\phi\partial^\mu\phi$ and $\text{const}\cdot A_\mu A^\mu$ respectively. In order to find the physical masses of the gauge bosons the mass matrix that appears when expanding eq. 68 must be diagonalized. To achieve this the Weinberg rotation is performed i. e.

$$W^\pm_\mu := \frac{1}{\sqrt{2}}(A_{1\mu} \mp A_{2\mu}) \tag{69a}$$

$$Z_\mu := \cos\theta_W A_{3\mu} - \sin\theta_W B_\mu \tag{69b}$$

$$A_\mu := \sin\theta_W A_{3\mu} + \cos\theta_W B_\mu. \tag{69c}$$

We will elaborate more on this later.

Mass Terms

In order to find the masses of the particles the Higgs mechanism is used. This mechanism is well studied and has been confirmed experimentally with the finding of a Higgs boson at LHC [5].

Yukawa Lagrangian

The way for fermions to acquire mass is through the Yukawa lagrangian i. e. by interacting with the Higgs fields.

In the standard model we have the particle content shown in Tabs. 5, 6 and 7 for the first generation. Clearly what would be usual mass terms are not invariant e. g. a term like

$$\mathcal{L} = -\lambda e_L e_R \tag{70}$$

is not invariant because when summing its hypercharges

$$-(-1/2) + (-1) \neq 0,$$

hence it is not invariant under hypercharge transformations.

Next, we introduce the standard model Higgs, given in Tab. 7. Now it is possible to construct invariant terms, like

$$\mathcal{L} = -\lambda \bar{L} \cdot \phi e_R.$$

With this considerations, we have that the Yukawa lagrangian for the standard model reads

$$-\mathcal{L} = \overline{Q}_L h_d \phi d_R + \overline{Q}_L h_u \tilde{\phi} u_R + \bar{l}_L h_e \phi e_R + h.c. \tag{71}$$

Higgs Potential

Finally, the Higgs potential, which consists of possible interactions between higgses is given by

$$V = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2. \tag{72}$$

SPONTANEUS SYMMETRY BREAK

The basic idea behind the mechanism is introducing a scalar field that does not anihilate in vacuum; this is done because ordinary mass terms like

$$\mathcal{L} = \lambda e_L e_R$$

are not gauge invariant. By introducing the Higgs field with the adequate gauge charges one can construct terms like

$$\mathcal{L} = \lambda \begin{pmatrix} \nu_L & e_L \end{pmatrix} \cdot \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} e_R = \lambda l_L \cdot \phi e_R \tag{73}$$

which are indeed gauge invariant (l is a leptonic spinor which contracts with ϕ). Now, the lagrangian preserves its symmetry but the Higgs field does not when we evaluate the base state; ϕ breaks the symmetry by acquiring the vacuum expectation value (VEV)

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{74}$$

In order to visualize how symmetries are broken, look at the shape of the potential in Fig. 2. The potential has a revolution symmetry, let us suppose it is the same symmetry of the lagrangian. When the system decays into a particular base state (indicated in the figure by the drop of the ball) it ceases to share the symmetry of the lagrangian; that is why it is said that the symmetry has been spontaneously broken. When this happens we plug eq. 74 into eq. 73 obtaining a mass term i. e.

$$\mathcal{L} = \lambda v e_L e_R = m e_L e_R. \tag{75}$$

However, there is no evidence that a unique Higgs boson is responsible for giving mass to all particles. Many extensions of the Higgs sector have been proposed because of this fact [41]. We propose one for this model which seeks to address both the mass hierarchy problem and dark matter stability.

Regardless of the form of the Higgs sector, first one shall construct the Higgs potential and define its minumum conditions, otherwise the particles will not acquire a stable mass, or will not be able to get mass at all.

After determining the minimum the next step is to compute the Higgs sector mass matrix, which is found by expanding the potential around its minima i. e.

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi - \phi_0)^a(\phi - \phi_0)^b \left(\frac{\partial^2}{\partial \phi^a \partial \phi^b} V \right)_{\phi_0} + \mathcal{O}(\phi^3), \tag{76}$$

where ϕ_0 is a constant field which minimizes the potential. Notice how the last term before truncation has the form of a mass term; hence the masses of the Higgs sector are given by

$$\mathcal{M}_{ab} = \left(\frac{\partial^2}{\partial \phi^a \partial \phi^b} V \right)_{\phi_0} . \tag{77}$$

In order to find the physical masses this matrix must be diagonalized. This will render the physical mass spectrum of the Higgs fields.

Particle Masses and Generation Mixing

With this considerations, for the first generation of the standard model, the Yukawa lagrangian for leptons reads

$$\mathcal{L}_e = -\lambda_e \overline{L} \cdot \phi e_R + \text{h.c.}, \tag{78}$$

so when symmetry is broken, i. e. $\langle \phi \rangle_0 = (0, v/\sqrt{2})^\dagger$, eq. 78 becomes

$$\mathcal{L}_e = -\frac{1}{\sqrt{2}} \lambda_e v \overline{e_L} e_R + \text{h.c.}, \tag{79}$$

so the electron mass is given by

$$m_e = \frac{1}{\sqrt{2}} \lambda_e v. \tag{80}$$

The same happens for quarks.

Now, when the other two generations are considered, new terms appear in the lagrangian. There are two kinds of terms that appear, ones that mix generations e. g.

$$\frac{1}{\sqrt{2}} \lambda_{ds} v \overline{d_L} s_R,$$

and mass terms for the other generations, i. e.

$$\frac{1}{\sqrt{2}}\lambda_{ss}\nu\overline{s_L}s_R,$$
$$\frac{1}{\sqrt{2}}\lambda_{bb}\nu\overline{b_L}b_R,$$

etc.

In general, there are no symmetry restrictions for Yukawa couplings, so they must be treated as general complex matrixes. This appears to pose a huge problem to discrete and flavor conservation symmetries; however, by making an apropiate chiral transformation Yukawa couplings can be simplified.

In order to find this transformation, diagonalize the hermitian matrixes obtained by squaring the different Yukawa couplings. Define unitary matrixes $U_{u,(d)}$ and $W_{u,(d)}$ (the work is analogus for down-like quarks, so it is only illustrated here for up-like quarks) so that

$$\lambda_u\lambda_u^\dagger = U_u D_u^2 U_u^\dagger \tag{81}$$
$$\lambda_u^\dagger\lambda_u = W_u D_u^2 W_u^\dagger \tag{82}$$

where D_u^2 is a diagonal matrix with positive eigenvalues. The relationship between λ_u and D^2 can be easily found. Let

$$\lambda_u = U_u D_u W_u^\dagger, \tag{83}$$

so

$$\lambda_u\lambda_u^\dagger = (U_u D_u W_u^\dagger)(U_u D_u W_u^\dagger)^\dagger \tag{84}$$
$$= U_u D_u W_u^\dagger W_u D_u^\dagger U_u^\dagger \tag{85}$$
$$= U_u D_u^2 U_u^\dagger; \tag{86}$$

an analogus procedure can be followed to arrive to eq. 82. Hence D_u is the diagonal matrix whose elements are the positive square roots of eigenvalues of D_u^2.

It has been shown that it is always possible to find such simplification. Now let us check that it does not affect theory consistence.

First we make the change of variables

$$u_R^i = W_u^{ij}u_R^{jo} \qquad d_R^i = W_d^{ij}d_R^{jo} \tag{87}$$
$$u_L^i = U_u^{ij}u_L^{jo} \qquad d_L^i = U_d^{ij}d_L^{jo} \tag{88}$$

and then we check that the Yukawa lagrangian is invariant e. g.

$$-\mathcal{L}_{yuk} = \overline{\psi}_L^o\lambda\psi_R^o + h.c. \tag{89}$$
$$= \overline{\psi}_L^o U^\dagger U D W^\dagger W \psi_R^o + h.c. \tag{90}$$
$$= \overline{\psi}_L D\psi_R + h.c., \tag{91}$$

where ψ denotes a fermionic field and the 0 superscript indicates it is not in the mass eigenstate basis. So the masses of the standard model fermions are given by

$$m_u^i = \frac{1}{\sqrt{2}} D_u^{ij} v \qquad m_d^i = \frac{1}{\sqrt{2}} D_d^{ij} v \tag{92}$$

hence eq. 88 converts the fermionic fields to the mass eigenstate basis.

Notice that there is no reason to say that coupling constants will greatly differ between generations, i. e. there is no reason to say $D_{33}^u \gg D_{22}^u \gg D_{11}^u$ (same holds for D^d). But experiments show that masses between generations differ by more than three orders of magnitude. To date there is no satisfactory explanation of this fact. This is known as the mass hierarchy problem.

Now we analyze how the different currents are affected by this transformation. For the electromagnetic current we have

$$\overline{u}_L^i \gamma^\mu u_L^i \to \overline{u}_L^i U_u^{\dagger ij} \gamma^\mu U_u^{jk} u_L^k = \overline{u}_L^i \gamma^\mu u_L^i \tag{93}$$

and same happens with the Z^0-boson current. But there is something new in the current which couples to the W boson i. e.

$$J^{\mu +} = \frac{1}{\sqrt{2}} \overline{u}_L^i \gamma^\mu d_L^i \to \frac{1}{\sqrt{2}} \overline{u}_L^i \gamma^\mu (U_u^\dagger U_d)^{ij} d_L^i \tag{94}$$

this means that weak interactions which change the charge mix the u_L^i and the d_L^i quarks. The amplitude of this mixing is given by matrix $\mathcal{K} := U_u^\dagger U_d$ which is known as the Cabibbo-Kobayasi-Maskawa (CKM) matrix, or just quark mixing matrix. This has been observed experimentally, so the simplification of the Yukawa couplings made by eq. 88 is consistent with nature.

THE WHOLE PICTURE: MASSES AND INTERACTIONS

In past two sections we reviewed the characteristics of the lagrangian i. e. it is general, renormalizable and invariant under gauge transformations. We also saw the different contributions to the lagrangian. Finally, we illustrated the Higgs mechanism, which gives particles mass; we also saw that when three generations of particles are considered we have generation mixing.

Now let us contemplate the whole picture: how the spontaneous symmetry break leads to particle masses and interactions observed in the real world.

Scalar Fields

First, let us see the effects of the covariant derivate (eq. 62) when acting on the Higgs VEV (eq. 74)

$$\begin{aligned} D_\mu \langle \phi \rangle_0 &= -\frac{iv}{2\sqrt{2}} \begin{pmatrix} gA_{3\mu} + g'B_\mu & g(A_{1\mu} - iA_{2\mu}) \\ g(A_{1\mu} + iA_{2\mu}) & -gA_{3\mu} + g'B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= -\frac{iv}{2\sqrt{2}} \begin{pmatrix} g\sqrt{2}W^+_\mu \\ -gA_{3\mu} + g'B_\mu \end{pmatrix}, \end{aligned} \tag{95}$$

when in the last step eq. 69 was used. Hence, expanding eq. 68 one finds two contributions: mass terms and kinetic terms. For mass terms we have

$$\begin{aligned} \mathcal{L}_{mass} &= \frac{v^2}{8}(2g^2 W^+_\mu W^{-\mu} + (gA_{3\mu} - g'B_\mu)^2) \\ &= \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} + \frac{1}{2}\frac{(g^2 + g'^2)v^2}{4} Z_\mu Z^\mu + 0 A_\mu A^\mu \\ &= M_W^2 W^+_\mu W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu + 0 A_\mu A^\mu, \end{aligned} \tag{96}$$

with A_μ and Z_μ defined by the Wienberg rotation (eq. 69). From last equation the masses of the gauge bosons have been determined

$$M_W = \frac{gv}{2} \qquad M_Z = \frac{gv}{2\cos\theta_W} \qquad M_A = 0. \tag{97}$$

In order to study interactions, we define $x = \sin^2\theta_W$ and perform the Weinberg rotation on the covariant derivate,

$$D_\mu = \partial_\mu - ieQA_\mu - \frac{ig}{\sqrt{2}} T^+ W^+_\mu - \frac{ig}{\sqrt{2}} T^- W^-_\mu - \frac{ig}{c}(T_{3L} - xQ)Z_\mu. \tag{98}$$

If we also define $e = g\sin\theta_W = \cos\theta_W$ and apply the covariant derivate on the perturbed Higgs field $\phi = \begin{pmatrix} \varphi^+ & \frac{h + v + i\eta}{\sqrt{2}} \end{pmatrix}^T$ we have

$$\begin{aligned} D_\mu \phi = \begin{pmatrix} \partial_\mu \varphi^+ \\ \frac{\partial_\mu h + i\partial_\mu \eta}{\sqrt{2}} \end{pmatrix} - ieA_\mu \begin{pmatrix} \varphi^+ \\ 0 \end{pmatrix} - i\frac{g}{c} Z_\mu \begin{pmatrix} \left(\frac{1}{2} - x\right)\varphi^+ \\ -\frac{h + v - i\eta}{\sqrt{2}} \end{pmatrix} \\ - i\frac{g}{\sqrt{2}} W^+_\mu \begin{pmatrix} \frac{h + v + i\eta}{\sqrt{2}} \\ 0 \end{pmatrix} - i\frac{g}{\sqrt{2}} W^-_\mu \begin{pmatrix} 0 \\ \varphi^+ \end{pmatrix} \end{aligned} \tag{99}$$

so plugging this on eq. 68 we have

$$
\begin{aligned}
\mathcal{L} &= (D_\mu\phi)^\dagger(D^\mu\phi) \\
&= ieA^\mu(\varphi^-\partial_\mu\varphi^+ - \varphi^+\partial_\mu\varphi^-) + i\frac{g}{2}\left(\frac{1}{2} - x\right) Z^\mu(\varphi^-\partial_\mu\varphi^+ - \varphi^+\partial_\mu\varphi^-) \\
&+ i\frac{g}{2}W^{\mu-}(h\partial_\mu\varphi^+ - \varphi^+\partial_\mu h) + \frac{g}{2}W^{\mu-}(\eta\partial_\mu\varphi^+ - \varphi^+\partial_\mu\eta) \\
&+ \frac{g}{2c}Z^\mu(h\partial_\mu\eta - \eta\partial_\mu h) + \frac{g^2 v}{4c^2}Z_\mu Z^\mu h + \frac{egv}{2}A_\mu W^{\mu+}\varphi^- \\
&- \frac{g^2 v x}{2c}Z_\mu W^{\mu+}\varphi^- + \frac{g^2 v}{2}W^+_\mu W^{\mu-}h + e^2 A_\mu A^\mu \varphi^+\varphi^- \\
&+ \frac{eg}{c}\left(\frac{1}{2} - x\right) A_\mu Z^\mu \varphi^+\varphi^- + \frac{g^2}{c^2}\left(\frac{1}{2} - x\right)^2 Z_\mu Z^\mu \varphi^+\varphi^- + \frac{g^2}{8c^2}Z_\mu Z^\mu(h^2 + \eta^2) \\
&+ \frac{ge}{2}A_\mu W^{\mu+}\varphi^- h + i\frac{ge}{2}A_\mu W^{\mu+}\varphi^-\eta - \frac{g^2 x}{2c}Z_\mu W^{\mu-}\varphi^+ h + i\frac{g^2 x}{2c}Z_\mu W^{\mu-}\varphi^+\eta \\
&+ \frac{g^2}{4}W^+_\mu W^{\mu-}(h^2 + \eta^2) + \frac{g^2}{2}W^+_\mu W^{\mu-}\varphi^+\varphi^- + \text{h.c.} \\
&+ iM_W\partial_\mu\varphi^+ W^{\mu-} - iM_W\partial_\mu\varphi^- W^{\mu+} + M_Z\partial_\mu\eta Z^\mu
\end{aligned}
\tag{100}
$$

For Fermions

In this section we only review the treatment for the quark sector. This is because the leptonic sector has an analogus yet simpler treatment. By breaking symmetry and expanding the Yukawa lagrangian we have (primed fields mean written in gauge eigenstate basis)

$$
\begin{aligned}
-\mathcal{L}_{\text{Yuk}} &= \overline{Q}'_L h_d \phi \mathcal{D}'_R + \overline{Q}'_L h_u \tilde{\phi} \mathcal{U}'_R + \text{h.c.} \\
&= \begin{pmatrix} \overline{\mathcal{U}}'_L & \overline{\mathcal{D}}'_L \end{pmatrix}^a h_d^{ab} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \mathcal{D}'^b_R + \begin{pmatrix} \overline{\mathcal{U}}'_L & \overline{\mathcal{D}}'_L \end{pmatrix}^a h_u^{ab} \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} \mathcal{U}'^b_R + \text{h.c.} \\
&= \overline{\mathcal{U}'}^a_L h_u^{ab} \frac{v}{\sqrt{2}} \mathcal{U}'^b_R + \overline{\mathcal{D}'}^a_L h_d^{ab} \frac{v}{\sqrt{2}} \mathcal{D}'^b_R + \text{h.c.} \\
&= \overline{\mathcal{U}'}_L M_u \mathcal{U}'_R + \overline{\mathcal{D}'}_L M_d \mathcal{D}'_R + \text{h.c.}
\end{aligned}
\tag{101}
$$

where $a, b = 1, 2, 3$ indicate different generations, $\mathcal{U} = (u, c, t)^T$ and $\mathcal{D} = (d, s, b)^T$. As it was explained on Section 4.3.1 the Yukawa couplings (noted there by λ^{ab} and here by h^{ab}) must be diagonalized using a unitary transformation (here we change notation from $W_{u(d)}$ and $U_{u(d)}$ to $U_{L(R)}$ which rotates up quarks and $V_{L(R)}$ which rotates down quarks), leaving

$$
-\mathcal{L}_{\text{Yuk}} = \overline{\mathcal{U}}_L U_L^\dagger M_u U_R \mathcal{U}_R + \overline{\mathcal{D}}_L V_L^\dagger M_d V_R \mathcal{D}_R + \text{h.c.} \tag{102}
$$

$$
= \overline{\mathcal{U}}_L M_u^{\text{diag}} \mathcal{U}_R + \overline{\mathcal{D}}_L M_d^{\text{diag}} \mathcal{D}_R + \text{h.c.} \tag{103}
$$

Notice the lack of primes; this indicates that everything is written in mass eigenstate basis. Now we have explicitly the mass terms and the generation mixing terms for the standard model quark sector, the later ones given by $\mathcal{K}$, the CKM matrix. Finally, rewriting the Yukawa lagrangian in eigenstate basis yields

$$
\begin{aligned}
-\mathcal{L}_{Yuk} &= \overline{Q}'_L h_d \phi \mathcal{D}'_R + \overline{Q}'_L h_u \tilde{\phi} \mathcal{U}'_R + h.c. \\
&= \begin{pmatrix} \overline{\mathcal{U}}'_L & \overline{\mathcal{D}}'_L \end{pmatrix}^a h_d^{ab} \begin{pmatrix} \varphi^+ \\ \dfrac{h+v+i\eta}{\sqrt{2}} \end{pmatrix} \mathcal{D}'^b_R \\
&+ \begin{pmatrix} \overline{\mathcal{U}}'_L & \overline{\mathcal{D}}'_L \end{pmatrix}^a h_u^{ab} \begin{pmatrix} \dfrac{h+v-i\eta}{\sqrt{2}} \\ -\varphi^- \end{pmatrix} \mathcal{U}'^b_R + h.c. \\
&= \frac{\sqrt{2}}{v} \Big(\overline{\mathcal{U}}_L U_L^\dagger V_L M_d^{diag} V_R^\dagger V_R \mathcal{D}_R \varphi^+ - \overline{\mathcal{U}}_R U_R^\dagger U_R M_u^{diag} U_L^\dagger V_L \mathcal{D}_L \varphi^+ + h.c. \\
&+ \overline{\mathcal{D}}_L V_L^\dagger V_L M_d^{diag} V_R^\dagger V_R \mathcal{D}_R \frac{h+i\eta}{\sqrt{2}} + \overline{\mathcal{D}}_R V_R^\dagger V_R M_d^{diag} V_L^\dagger V_L \mathcal{D}_L \frac{h-i\eta}{\sqrt{2}} \\
&+ \overline{\mathcal{U}}_L U_L^\dagger U_L M_u^{diag} U_R^\dagger U_R \mathcal{U}_R \frac{h-i\eta}{\sqrt{2}} + \overline{\mathcal{U}}_R U_R^\dagger U_R M_u^{diag} U_L^\dagger U_L \mathcal{U}_L \frac{h+i\eta}{\sqrt{2}} \Big) \\
&= \frac{g}{\sqrt{2} M_W} \overline{\mathcal{U}} \left(M_d^{diag} P_R - M_u^{diag} P_L \right) \mathcal{K} \mathcal{D} \varphi^+ + h.c. \\
&+ \frac{g}{2M_W} \left(\overline{\mathcal{D}} M_d^{diag} \mathcal{D} + \overline{\mathcal{U}} M_u^{diag} \mathcal{U} \right) h \\
&+ \frac{ig}{2M_W} \left(\overline{\mathcal{D}} M_d^{diag} \gamma_5 \mathcal{D} - \overline{\mathcal{U}} M_u^{diag} \gamma_5 \mathcal{U} \right) \eta \qquad (104)
\end{aligned}
$$

Expanding the covariant derivate on the kinetic quark lagrangian we have

$$
\begin{aligned}
\mathcal{L}_{quarks} &= \overline{u}_L i\partial\!\!\!/ u_L + \overline{u}_R i\partial\!\!\!/ u_R + \overline{d}_L i\partial\!\!\!/ d_L + \overline{d}_R i\partial\!\!\!/ d_R \\
&+ \begin{pmatrix} \overline{u} & \overline{d} \end{pmatrix}_L \left[eA\!\!\!/ \begin{pmatrix} \frac{2}{3} u \\ -\frac{1}{3} d \end{pmatrix}_L + \frac{g}{c} Z\!\!\!/ \begin{pmatrix} \left(\frac{1}{2} - \frac{2}{3}x\right) u \\ \left(-\frac{1}{2} + \frac{1}{3}x\right) d \end{pmatrix}_L + \frac{g}{\sqrt{2}} W\!\!\!/^+ \begin{pmatrix} d \\ 0 \end{pmatrix}_L + \frac{g}{\sqrt{2}} W\!\!\!/^- \begin{pmatrix} 0 \\ u \end{pmatrix}_L \right] \\
&\quad + e\left(\frac{2}{3} \overline{u}_R A\!\!\!/ u_R - \frac{1}{3} \overline{d}_R A\!\!\!/ d_R \right) + \frac{g}{c} \left(-\frac{2}{3} x \overline{u}_R Z\!\!\!/ u_R + \frac{1}{3} x \overline{d}_R Z\!\!\!/ d_R \right). \qquad (105)
\end{aligned}
$$

And writing it in the mass eigenstate basis

$$
\begin{aligned}
\mathcal{L}_{quarks} &= \overline{\mathcal{U}} i\partial\!\!\!/ \mathcal{U} + \overline{\mathcal{D}} i\partial\!\!\!/ \mathcal{D} + \frac{2}{3} e \overline{\mathcal{U}} A\!\!\!/ \mathcal{U} - \frac{1}{3} e \overline{\mathcal{D}} A\!\!\!/ \mathcal{D} \\
&+ \frac{g}{c} \overline{\mathcal{U}} Z\!\!\!/ \left(\frac{1}{2} P_L - \frac{2}{3} x \right) \mathcal{U} + \frac{g}{c} \overline{\mathcal{D}} Z\!\!\!/ \left(-\frac{1}{2} P_L + \frac{1}{3} x \right) \mathcal{D} \\
&+ \frac{g}{\sqrt{2}} \overline{\mathcal{U}}_L W\!\!\!/^+ \mathcal{K} \mathcal{D}_L + \frac{g}{\sqrt{2}} \overline{\mathcal{D}}_L W\!\!\!/^- \mathcal{K}^\dagger \mathcal{U}_L \qquad (106)
\end{aligned}
$$

The first three terms of eq. 105 are pure kinetic. The others account for interactions. Notice that the right particles just interact through photons and Z bosons. Photons and Z bosons are their own antiparticles, hence their charges and flavor numbers are zero; this interactions just exchange momentum, contrary to their left counterparts, that interact weakly, transforming up-like quarks in down-like and vice versa; this is the reason because weak interaction is said to be chiral.

Also notice how the unitarity condition of the matrixes that rotate quarks forbids the flavor change in the A_μ and Z_μ (also known as neutral) currents; this is known as Glashow-Iliopoulos-Maiani (GIM) mechanism. This agrees with experimental data at three level [42, 43, 44].

Gauge Fields

For the gauge fields we must take into account eq. 64, and the definitions of the field tensors eq. 65. By expanding and applying the Weinberg rotation we will find pure kinetic terms i. e. of the form $(\partial A)^2$, and interaction terms

$$\begin{aligned}
\mathcal{L}_{int} = &\, igs\left[\partial_\mu A_\nu (W^{\mu+}W^{\nu-} - W^{\mu-}W^{\nu+}) + A_\mu(W^+_\nu \partial^\mu W^{\nu-} - W^-_\nu \partial^\mu W^{\nu+})\right.\\
&\left. + A_\nu(W^{\nu-}\partial_\mu W^{\nu+} - W^{\mu+}\partial_\mu W^{\nu-})\right] + igc\left[\partial_\mu Z_\nu (W^{\mu+}W^{\nu-} - W^{\mu-}W^{\nu+})\right.\\
&\left. + Z_\mu(W^+_\nu \partial^\mu W^{\nu-} - W^-_\nu \partial^\mu W^{\nu+}) + Z_\nu(W^{\nu-}\partial_\mu W^{\nu+} - W^{\mu+}\partial_\mu W^{\nu-})\right]\\
&- \frac{g^2}{2}\left(W^+_\mu W^{\mu-} W^+_\nu W^{\nu-} - W^+_\mu W^{\mu+} W^-_\nu W^{\nu-}\right)\\
&- g^2 W^+_\mu W^-_\nu (c^2 Z^\mu Z^\nu + csA^\mu Z^\nu + csA^\nu Z^\mu + s^2 A^\mu A^\nu)\\
&- g^2 W^+_\mu W^{\nu-}(c^2 Z_\nu Z^\nu + 2csA_\nu Z^\nu + s^2 A_\nu A^\nu)
\end{aligned} \tag{107}$$

Notice how the photon i. e. the gauge field from $U(1)_Q$ does not present self-interaction terms. This is because $U(1)_Q$ is an abelian group.

BEYOND THE STANDARD MODEL

Neutrino Masses

The standard model was build without considering right neutrinos because experiments showed that weak interactions violated parity i. e. only the left part interacted weakly, and also neutrinos had the unique feature of allowing the formulation of a theory where particles had left chirality and antiparticles right. So the community assumed that the standard model did not need right neutrinos. [45, 46, 47]

Without right neutrinos mass terms cannot be constructed, hence in the standard model neutrinos are massless.

Now, neutrino oscillation has been evidenced by some experiments [10, 11]. The mechanism which explains oscillations between flavors of particles needs to be analogus to quark mixing. Remember that quark mixing is a consequence of quarks having mass (mixing amplitudes appear as a consequence of writing currents in mass-eigenstate basis). Hence the fact that neutrinos oscillate can only be explained if they also have mass.

Then, it is necessary to build mass terms.

But if one just adds right neutrinos and builds simple mass terms one would lack an explanation for the smallness of neutrino masses when compared to other fermions. Also, as neutrinos do not have charge, there is also a chance for them to be Majorana particles i. e. that they are their own antiparticles. Also there is the general case, when right neutrinos are added and neutrinos are also considered to be Majorana particles. This will lead to the "seesaw" mechanism. We focus our attention in reviewing it because our model also makes this considerations for neutrinos.

Seesaw Mechanism

When one considers the most general case, one has that the mass terms of the lagrangian for neutrinos have three contributions: one from ordinary left-right terms, also known as Dirac mass terms, which we note $\mathcal{L}^D_{mass}$; and two Majorana contributions i. e. of the form $m_\nu \overline{\nu^c} \nu$, one for left neutrinos, and other for right neutrinos, noted by $\mathcal{L}^L_{mass}$ and $\mathcal{L}^R_{mass}$ respectively. This leads to a mass matrix of the form

$$\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}. \tag{108}$$

Based on experiments one can impose certain conditions on the terms of this mass matrix. Right neutrinos have not been observed, hence one can say that their Majorana masses are huge; also, according to observations for left neutrinos one can postulate that their Majorana masses are so tiny that they can be neglected, and that their Dirac masses are tiny i. e. $m_D \ll m_R$. Under this considerations the mass matrix takes the form

$$\begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix}; \tag{109}$$

when it is diagonalized, one obtains

$$m_1 \approx \frac{m_D^2}{m_R}, \tag{110}$$

$$m_2 \approx m_R. \tag{111}$$

So m_1 is small if m_2 is big, that is the reason why this is called the seesaw mechanism. This ideas can be straightforwardly extended for the case of three generations.

This was a quick review of the canonic seesaw mechanism. There are other kinds of seesaw mechanisms, but are not reviewed here because the Higgs sector for the new particles added to the standard model differ on its kind from the ones in our model. Also, there is a whole lot of subtleties that should be considered when adding the Dirac–Majorana terms to the lagrangian. For more information on types of seesaw mechanisms and a complete study on adding consistently the Dirac–Majorana terms see [48, Chapter 6].

Also 1-loop corrections to neutrino masses can be found, and depending on the model parameters they can even be larger than three level contribution. For a detailed study of this cases see [49].

5 THE $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)'_X \otimes U(1)_{B-L}$ GAUGE MODEL

As was explained before, many of the standard model predictions have been confirmed by experiments, but there are other predictions that have been debunked by them. Also, there are some phenomena that it fails to explain. This motivates the scientific comunity to think that the standard model is not the ultimate theory.

Maybe is it part of a more general theory? There is no way to know. But certainly its astonishing agreements with nature make us think that it might be. Because of this we propose a new theory: an extension of the standard model with the extra symmetries $U(1)'_X$ and $U(1)_{B-L}$. The inclusion of $U(1)'_X$, a non universal symmetry, is motivated by the mass hierarchy problem. Thanks to the fact that it is not universal, one can think of particles as presenting an horizontal symmetry; this fact will allow us to build mass terms in a way that gives a natural explanation to the mass hierarchy problem.

Also, the inclusion of the $U(1)_{B-L}$ symmetry is motivated by the lack of an explanation of dark matter particles in the standard model. It is usual to find on literature that there is a decoupling between dark and barionic matter caused by a discrete symmetry (usually Z_2). However, the inclusion of the continuous $U(1)_{B-L}$ allows us to define parity as

$$P = (-1)^{3(B-L)+2s}, \tag{112}$$

which gives a natural explanation of this decoupling when $U(1)_{B-L}$ breaks. We have already seen the particle content of this model (presented in Chapter 3), and we have reviewed the structure of the standard model.

In this chapter we present the calculations for our model based on ideas treated in Chapter 4. Present chapter is structured as follows. First, we present the proposed Higgs sector for this model and its vacuum expectation values. It will, with the particle content and the gauge fields dictate the masses and interactions of particles according to this model. It is structured in a way that helps us concrete the objetives of this work; it will be explained later how they achieve that. Then we construct the Higgs potential, determine its minimum and the masses of the Higgs sector. We find some interesting predictions regarding dark matter. After that, we construct the Yukawa lagrangian and the mass matrixes.

HIGGS SECTOR AND HIGGS POTENTIAL

Remember that within the goals of this work are to give an explanation to dark matter, to neutrino masses and also to the mass hierarchy problem. This will be done by choosing a particular -extended- Higgs sector.

Our Higgs sector is composed by the fields presented in Tab. 8. Each field serves a purpose, and its quantum numbers are chosen to fullfill that purpose. Basically, the idea is that when a quantum number is zero that field does not affect that symmetry when it aquires a vacuum expectation value (VEV), otherwise, that symmetry is broken by that Higgs field. Our symmetry breaking scheme is

$$\begin{array}{cl} SU(2)_L \otimes U(1)_Y \otimes U(1)'_X \otimes U(1)_{B-L} & \\ \downarrow & \phi \text{ acquires VEV} \\ SU(2)_L \otimes U(1)_Y \otimes U(1)'_X & \\ \downarrow & \chi_3^0 \text{ acquires VEV} \\ SU(2)_L \otimes U(1)_Y & \\ \downarrow & \eta \text{ and } \rho \text{ acquire VEV} \\ U(1)_Q. & \end{array}$$

Notice how according to the ideas presented above ϕ has $Y = 0$, $X = 0$ but $B - L \neq 0$, hence breaking $B - L$ symmetry and leaving the others untouched. Same happens with the rest of the fields. Also notice that we use two-higgs doublets in order to break symmetry from the standard model group to $U(1)_Q$. This is done because it adds the fewest arbitrary parameters yet has the capability of including interesting phenomena such as charged Higgs bosons, it has also been studied for more than 30 years and has been shown to recover the results of the standard model [41].

Table 8: Higgs sector particle content. G_{SM} means charges under the standard model like (# of colors, doublet or singlet under $SU(2)_L$, Y).

Spectrum	G_{SM}	X	$B-L$	P
$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix}$	(1,2,1)	1/3	0	1
$\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \end{pmatrix}$	(1,2,1)	2/3	0	1
η_3^0	(1,1,0)	-1/3	1	-1
χ_3^0	(1,1,0)	-1/3	0	1
ϕ	(1,1,0)	0	2	1

We also define in tab. 9 the VEVs for the various Higgs fields. Notice that we are making a strong assumption: the dark matter associated Higgs field does not break symmetry. This fact will allow us to understand dark matter stability. This will be explained later.

Table 9: Vacuum expectation values (VEV) of the Higgs fields.

Spectrum	VEV
$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix} = \begin{pmatrix} \eta_1 + i\eta_2 \\ \eta_3 + i\eta_4 \end{pmatrix}$	$\langle\eta\rangle = \begin{pmatrix} 0 \\ N_\eta \end{pmatrix}$
$\rho = \begin{pmatrix} \rho^+ \\ \rho^0 \end{pmatrix} = \begin{pmatrix} \rho_1 + i\rho_2 \\ \rho_3 + i\rho_4 \end{pmatrix}$	$\langle\rho\rangle = \begin{pmatrix} 0 \\ N_\rho \end{pmatrix}$
$\eta_3^0 = \eta_{3_1}^0 + i\eta_{3_2}^0$	$\langle\eta_3^0\rangle = 0$
$\chi_3^0 = \chi_{3_1}^0 + i\chi_{3_2}^0$	$\langle\chi_3^0\rangle = N_{\chi_3^0}$
$\phi = \phi_1 + i\phi_2$	$\langle\phi\rangle = V$

Building the Higgs Potential

With the Higgs sector defined in the previous section (see tabs. 8 and 9), we are now able to construct the Higgs potential. Rememberthat it must be

- General
- Renormalizable
- Invariant under symmetries

So with this considerations we have that terms like

	$\rho^\dagger$	ρ	$\eta^\dagger$	η		
X	-2/3	+2/3	-1/3	+1/3	=	0
N	-1/3	+1/3	+1/3	-1/3	=	0
Y	-1	+1	-1	+1	=	0
B-L	0	+0	+0	+0	=	0

are suitable to be part of the potential. We have that for this model the potential which satisfies the above conditions and is also CP invariant is

$$\begin{aligned}
\mathcal{V} = {} & \mu_1^2\rho^\dagger\rho + \mu_2^2\eta^\dagger\eta + \mu_3^2\phi^*\phi + \mu_4^2\chi_3^{0*}\chi_3^0 + \mu_5^2\eta_3^{0*}\eta_3^0 \\
& + \lambda_1(\rho^\dagger\rho)^2 + \lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\phi^*\phi)^2 + \lambda_4(\chi_3^{0*}\chi_3^0)^2 \\
& + \lambda_5(\eta_3^{0*}\eta_3^0)^2 + \lambda_6\rho^\dagger\rho\eta^\dagger\eta + \lambda_7\rho^\dagger\rho\phi^*\phi + \lambda_8\rho^\dagger\rho\chi_3^{0*}\chi_3^0 \\
& + \lambda_9\rho^\dagger\rho\eta_3^{0*}\eta_3^0 + \lambda_{10}\eta^\dagger\eta\phi^*\phi + \lambda_{11}\eta^\dagger\eta\chi_3^{0*}\chi_3^0 + \lambda_{12}\eta^\dagger\eta\eta_3^{0*}\eta_3^0 \\
& + \lambda_{13}\phi^*\phi\chi_3^{0*}\chi_3^0 + \lambda_{14}\phi^*\phi\eta_3^{0*}\eta_3^0 + \lambda_{15}\chi_3^{0*}\chi_3^0\eta_3^{0*}\eta_3^0 + \lambda_{16}\rho^\dagger\eta\eta^\dagger\rho \qquad (113)
\end{aligned}$$

We stated above that in order to explain dark matter stability the field η_3^0 does not break symmetry. In order to guarantee this, the parameter $\mu_5^2 > 0$.

Also notice that in 113 there are interaction terms between different Higgses. It is particularly interesting to fix our attention in the study on interactions between η_3^0 and the other fields. This is because η_3^0 is our dark matter candidate.

For example, study the interaction term

$$\lambda_{14}\phi^*\phi\eta_3^{0*}\eta_3^0. \tag{114}$$

When first order corrections are made, then it becomes

$$\lambda_{14}\eta_3^{02}(\phi+v)^2, \tag{115}$$

by expanding this, one obtains that a three point interaction is possible, i. e.

$$\lambda_{14}v\eta_3^{02}\phi. \tag{116}$$

So this gives a link between dark matter and standard model particles.

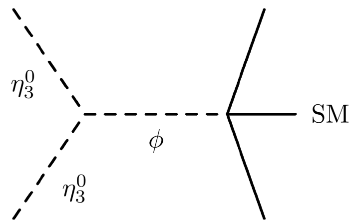

Figure 4: Possible link between dark matter and standard model particles, as predicted by eq. 116

However, notice that decay processes of dark matter are forbidden thanks to its zero VEV (guaranteed by the choice of $\mu_5^2 > 0$). Following the same procedure as above, and saying that b was η_3^0 VEV, eq. 115 will become

$$\lambda_{14}(\eta_3^0+b)^2(\phi+v)^2, \tag{117}$$

when expanding, a decay term will appear

$$\lambda_{14}b\eta_3^0\phi^2. \tag{118}$$

But as $b = 0$ this term does not exist. Now it is clear how by choosing $\mu_5^2 > 0$ dark matter stability is guaranteed.

Now, knowing that there is a link between dark matter and barionic particles (but through collisions as shown on Fig. 4) it is important to study how this can happen while respecting the experimentally observed relic density. In order to do that we review the ideas from [50].

The treatment of [50] is important because it allows to rewrite the evolution equation of a species considering coannihilations i. e. processes like the one shown in Fig. 4, in an easy comparable form with the case when they are not considered. They also show how coannihilations can increase or decrease the relic density, so its crucial to consider them.

First of all, following ideas from [51, 50, 52] we arrive to an equation for the evolution of a species.

The evolution of the phase space density $f(p, x, t)$ of a particle species is described by the Boltzmann equation. Using the Liouville operator L that gives the net rate of change in time of the phase space density f and C, the collision operator, which represents the number of particles per phase space unit that are gained or lost because of collision processes, the aforementioned Boltzmann equation can be simply written as

$$L[f] = C[f]. \tag{119}$$

By using the Friedmann–Robertson–Walker cosmological model we are assuming that f just depends on the particle energy E and the time, this is because in this model the phase space density is assumed to be isotropic and spatially homogeneus. Under this assumptions,

$$L[f] = \partial_t f - \frac{\dot{R}}{R}\frac{|p|^2}{E}\partial_E f = \partial_t f - H\frac{|p|^2}{E}\partial_E f, \tag{120}$$

where H is the Hubble parameter and R is the scale factor of the universe; the dot indicates a time derivative.

Another important quantity is the particle number density n. For g spin degrees of freedom with the same distribution it takes the form

$$n = \frac{g}{(2\pi)^3}\int d^3p f(E, t). \tag{121}$$

Now, in order to deduce an expresion for the evolution of quantity n, the Boltzmann equation is integrated over all posible momenta and summed over all the spin degrees of freedom. This calculations are presented on [52]. Their results are, for the integrated Liouville term

$$\frac{g_1}{(2\pi)^3}\int d^3p_1 L[f_1] = \dot{n}_1 + 3Hn_1, \tag{122}$$

and for the integrated collision term,

$$\frac{g_1}{(2\pi)^3}\int d^3p_1 C[f_1] = -\int \sigma v_{Ml}(dn_1 dn_2 - dn_1^{eq} dn_2^{eq}), \tag{123}$$

where the subindices refer to two particles 1 and 2, $\sigma_{all\ f} = \sum \sigma_{12\to f}$ is the total annihilation cross section, the eq super index refers to the equilibrium state, and v_{Ml} is the Møller velocity. This velocity is defined in a way that makes $v_{Ml}n_1n_2$ be lorentz invariant; it also equals the product vn_1n_2 in the rest frame of one of the particles. This allows to write the *invariant* interaction rate per time per volume be written in any reference frame as

$$\frac{dN}{dVdt} = \sigma v_{Ml} n_1 n_2. \tag{124}$$

From symmetry considerations [53, 54] it can be said that distributions in kinetic and chemical equilibrium are proportional independent of the momentum. Even if species 1 and 2 decouple, but continue in kinetic equilibrium by colliding with other particles in the thermal bath, this proportionality will hold. Hence, the integrated collision factor can be written both before and after decoupling as

$$\frac{g_1}{(2\pi)^3}\int d^3p_1 C[f_1] = -\langle\sigma v_{Ml}\rangle(n_1 n_2 - n_1^{eq} n_2^{eq}). \tag{125}$$

where the thermal average is defined as

$$\langle\sigma v_{Ml}\rangle = \frac{\int \sigma v_{Ml} dn_1^{eq} dn_2^{eq}}{\int dn_1^{eq} dn_2^{eq}}. \tag{126}$$

Plugging eqs. 122 and 125 into 119, we obtain

$$\dot{n}_1 + 3Hn_1 = -\langle\sigma v_{Ml}\rangle(n_1 n_2 - n_1^{eq} n_2^{eq}), \tag{127}$$

and an analogus equation for particle 2. So if particles 1 and 2 are identical, $n = n_1 = n_2$ and

$$\dot{n} + 3Hn = -\langle\sigma v_{Ml}\rangle(n^2 - n_{eq}^2). \tag{128}$$

Now, density decreases because universe is expanding. It is helpful to treat this decrease implicitly by instead considering variable $Y = n/s$, where s is the total entropy density of the universe, it is also convenient to change from time to universe temperature T (which is usual in cosmology). By making this change of variables, and dividing eq. 128 by $S = sR^3$ we have

$$\dot{Y} = -s\langle\sigma v_{Ml}\rangle(Y^2 - Y_{eq}^2). \tag{129}$$

It is also convenient to have $x = m/T$ as evolution parameter, with T being the photon temperature; under this change eq. 129 becomes

$$\frac{dY}{dx} = \frac{1}{3H}\frac{ds}{dx}\langle\sigma v_{Ml}\rangle(Y^2 - Y_{eq}^2). \tag{130}$$

In the standard Friedmann–Robertson–Walker cosmology the Hubble parameter is given by

$$H = \sqrt{\frac{8}{3}\pi G\rho}, \tag{131}$$

where ρ is the total energy density if the universe and G is the gravitational constant. Also, by defining the effective degrees of freedom $g_{eff}(T)$ and $h_{eff}(T)$ in such a way that $g_{eff}(T) = h_{eff}(T) = 1$ for a relativistic species with one internal degree of freedom and

$$\rho = g_{eff}(T)\frac{\pi^2}{30}T^4, \tag{132}$$

$$s = h_{eff}(T)\frac{2\pi^2}{45}T^3, \tag{133}$$

one can rewrite eq 130 as

$$\frac{dY}{dx} = -\left(\frac{45}{\pi}G\right)^{-1/2}\frac{m\sqrt{g_*}}{x^2}\langle\sigma v_{Ml}\rangle(Y^2 - Y_{eq}^2), \tag{134}$$

with

$$\sqrt{g_*} = \frac{h_{eff}}{\sqrt{g_{eff}}}\left(1 + \frac{1}{3}\frac{T}{h_{eff}}\frac{dh_{eff}}{dT}\right). \tag{135}$$

Now that we have arrived to an expresion for the evolution of a species we must find an expresion for $\langle\sigma v_{Ml}\rangle$. This can be expanded into series, however, its expansion usually models it poorly, or there are times when even that expansion is divergent [52]; hence an integral form is preferred. In order to find this integral form we follow the ideas from [50]. Again, we aim to find an integral form for $\langle\sigma v_{Ml}\rangle$ that is general, relativistic, and that only comprises one integration.

According to [50]

$$\begin{aligned}\langle\sigma_{eff}v\rangle &= \sum_{ij}\langle\sigma_{ij}v_{ij}\rangle\frac{n_i^{eq}}{n^{eq}}\frac{n_j^{eq}}{n^{eq}}\\ &= \frac{\sum_{ij}\langle\sigma_{ij}v_{ij}\rangle n_i^{eq}n_j^{eq}}{n_{eq}^2}\\ &= \frac{A}{n_{eq}^2}.\end{aligned} \tag{136}$$

The physical meaning of quantity A is the total annihilation rate at temperature T per volume unit. By using the unpolarized annihilation rate per unit volume W_{ij}, defined as

$$W_{ij} = 4p_{ij}\sqrt{s}\sigma_{ij} = 4\sigma_{ij}\sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2} = 4E_iE_j\sigma_{ij}v_{ij}, \tag{137}$$

where

$$p_{ij} = \frac{\left[s - (m_i + m_j)^2\right]^{1/2}\left[s - (m_i - m_j)^2\right]^{1/2}}{2\sqrt{s}} \tag{138}$$

is the momentum of particle $i(j)$ in the center of mass frame of the pair ij, A can be rewritten in a covariant form (notice how by the above definition W_{ij} is lorentz invariant) i. e.

$$A = \sum_{ij}\int W_{ij}\frac{g_if_id^3\mathbf{p}_i}{(2\pi)^3 2E_i}\frac{g_jf_jd^3\mathbf{p}_j}{(2\pi)^3 2E_j}. \tag{139}$$

Now the integral is reduced from 6 dimensions to 1 dimension. First rewrite the volume element in the momentum space as [52]

$$d^3\mathbf{p}_i d^3\mathbf{p}_j = 4\pi|\mathbf{p}_i|E_i dE_i\, 4\pi|\mathbf{p}_j|E_j dE_j\, \frac{1}{2}d\cos\theta \tag{140}$$

where θ is the angle between momenta i and j. Then perform the change of variables

$$E_+ = E_i + E_j \tag{141a}$$
$$E_- = E_i - E_j \tag{141b}$$
$$s = m_i^2 + m_j^2 + 2E_iE_j - 2|p_i||p_j|\cos\theta, \tag{141c}$$

so that

$$\frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}\frac{d^3\mathbf{p}_j}{(2\pi)^3 2E_j} = \frac{1}{(2\pi)^4}\frac{dE_+dE_-ds}{8}, \tag{142}$$

and the integration limits transform from $E_i \geqslant m_i, E_j \geqslant m_j, |\cos\theta| \leqslant 1$ to

$$s \geqslant (m_i + m_j)^2, \tag{143a}$$
$$E_+ \geqslant \sqrt{s}, \tag{143b}$$
$$\left|E_- - E_+\frac{m_j^2 - m_i^2}{s}\right| \leqslant 2p_{ij}\sqrt{\frac{E_+^2 - s}{s}}. \tag{143c}$$

Thanks to the Maxwell Boltzmann aproximation the product of equilibrium distributions just depends of E_+; also, when final state statistical factors are neglected W_{ij} becomes only dependent of s; so integration over E_- is just

$$\int dE_- = 4p_{ij}\sqrt{\frac{E_+^2 - s}{s}}. \tag{144}$$

Hence the volume element becomes

$$\frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}\frac{d^3\mathbf{p}_j}{(2\pi)^3 2E_j} = \frac{1}{(2\pi)^4}\frac{p_{ij}}{2}\sqrt{\frac{E_+^2 - s}{s}}dE_+ds, \tag{145}$$

now, by replacing this on 139 and integrating over E_+ renders

$$A = \frac{T}{32\pi^4}\sum_{ij}\int_{(m_i+m_j)^2}^{\infty} ds\, g_i g_j p_{ij} W_{ij} K_1\left(\frac{\sqrt{s}}{T}\right), \tag{146}$$

with K_1 being the 1st-order modified Bessel function of the second kind. This can be written in a more compact way by exchanging the sum and the integral and defining

$$p_{\text{eff}} = p_{11} = \frac{1}{2}\sqrt{s - 4m_1^2}, \tag{147a}$$
$$W_{\text{eff}} = \sum_{ij}\frac{p_{ij}}{p_{11}}\frac{g_i g_j}{g_1^2}W_{ij}. \tag{147b}$$

This renders

$$A = \frac{g_1^2 T}{32\pi^4}\int_{4m_1^2}^{\infty} ds\, p_{\text{eff}} W_{\text{eff}} K_1\left(\frac{\sqrt{s}}{T}\right), \tag{148}$$

or by changing variables $s = 4p_{\text{eff}}^2 + 4m_1^2$, $ds = 8p_{\text{eff}}dp_{\text{eff}}$, so

$$A = \frac{g_1^2 T}{4\pi^4} \int_0^\infty dp_{\text{eff}} p_{\text{eff}}^2 W_{\text{eff}} K_1 \left(\frac{\sqrt{4p_{\text{eff}}^2 + 4m_1^2}}{T} \right), \tag{149}$$

which is a one dimensional integral.

Also, using Boltzmann statistics, i. e. $f_i = \exp(-E_i/T)$, we have that

$$n^{eq} := \sum_i n_i^{eq} \tag{150}$$

$$= \sum_i \frac{g_i}{(2\pi)^3} \int d^3 p_i e^{-E_i/T} \tag{151}$$

$$= \frac{T}{2\pi^2} \sum_i g_i m_i^2 K_2 \left(\frac{m_i}{T} \right) \tag{152}$$

with K_2 being the 2nd-order modified Bessel function of the second kind.

So finally we have a general, relativistic, 1-dimensional integral form for the thermal average of the cross section, this is

$$\langle \sigma_{\text{eff}} v \rangle = \frac{\int_0^\infty dp_{\text{eff}} p_{\text{eff}}^2 W_{\text{eff}} K_1 \left(\frac{\sqrt{4p_{\text{eff}}^2 + 4m_1^2}}{T} \right)}{m_1^4 T \left[\sum_i \frac{g_i}{g_1} \frac{m_i^2}{m_1^2} K_2 \left(\frac{m_i}{T} \right) \right]^2}; \tag{153}$$

this form was obtained by Edsjö and Gondolo in Ref. [50] and includes coannihilations (within the effective annihilation rate). It also reduces to the formula obtained in Ref. [52] for the case without coannihilations. Another advantage of this form is that the effective annihilation rate can be calculated in advance, thus speeding computational calculations.

It is also useful for computational processes to rewrite

$$Y_{eq} = \frac{n_{eq}}{s} = \frac{45x^2}{4\pi^4 h_{\text{eff}}(T)} \sum_i g_i \left(\frac{m_i}{m_1} \right)^2 K_2 \left(x \frac{m_i}{m_1} \right). \tag{154}$$

Finally the toolbox that will allow us to find a bridge between reality and parameters in our model is completed. In order to obtain the relic density one integrates eq. 134 from $x = 0$ to $x = m_{DM}/T_0$ (with T_0 being the photon temperature of the universe today); the result of this integration is notated Y_0. Hence

$$\Omega_{DM} = \frac{\rho_{DM}^0}{\rho_{\text{crit}}} = \frac{m_{DM} s_0 Y_0}{\rho_{\text{crit}}}, \tag{155}$$

where the universe critical density is given by $\rho_{\text{crit}} = 3H^2/8\pi G$, s_0 here notes the entropy density today. This equation is crucial, as it connects the experimental observations (left-hand side) with the parameters of the model (right-hand side). This imposes certain limits in the parameters of our model in order to agree with nature.

Potential Minima

Now, it is neccesary to guarantee that the postulated VEVs are stable; otherwise particles will decay further and hence will have no chance to acquire a stable mass and interact. Stability is only found when the potential reaches a minimum when evaluated on VEVs. Hence it is mandatory to find the the minimum conditions for the potential.

In order to do that, the potential (eq. 113) is expanded using the field components defined in tab. 8. Then it is partially derived with respect to each field component, just as when one finds the gradient in calculus. After that, the gradient is evaluated at VEVs (see tab. 9), where it has its minima by definiton. Finally, in order to adjust μ parameters to guarantee it is a minimum, one equals each component from the grandient to zero and solves for different μ, this yields the following tadpole conditions:

$$\mu_1 \to N_\eta^2(-(\lambda_{16}+\lambda_6)) - 2\lambda_1 N_\rho^2 - \lambda_8 N_{\chi_3^0}^2 - \lambda_7 v^2, \quad (156a)$$

$$\mu_2 \to -2\lambda_2 N_\eta^2 - N_\rho^2(\lambda_{16}+\lambda_6) - \lambda_{11} N_{\chi_3^0}^2 - \lambda_{10} v^2, \quad (156b)$$

$$\mu_3 \to -\lambda_{10} N_\eta^2 - \lambda_7 N_\rho^2 - \lambda_{13} N_{\chi_3^0}^2 - 2\lambda_3 v^2, \quad (156c)$$

$$\mu_4 \to -\lambda_{11} N_\eta^2 - \lambda_8 N_\rho^2 - 2\lambda_4 N_{\chi_3^0}^2 - \lambda_{13} v^2. \quad (156d)$$

This conditions over different μ guarantee that the potential is minimized when symmetry is broken.

Higgs sector Mass Matrix and Mass Eigenstates

With the minimum conditions defined in 156, the mass matrix defined in eq. 77 takes the form

$$\mathcal{M} = \begin{pmatrix} \mathcal{A}_{2\times2} & & & & \\ & \mathcal{A}_{2\times2} & & & \\ & & \mathcal{B}_{4\times4} & & \\ & & & 0_{4\times4} & \\ & & & & \mathcal{C}_{2\times2} \end{pmatrix}, \quad (157)$$

where $\mathcal{A}_{2\times2}$ corresponds to the mass matrix of the charged real sector i.e. ρ_1, η_1 and to the charged complex sector i.e. ρ_2, η_2;

$$\mathcal{A}_{2\times2} = \begin{pmatrix} -N_\eta^2\lambda_{16} & N_\eta N_\rho \lambda_{16} \\ N_\eta N_\rho \lambda_{16} & -N_\rho^2\lambda_{16} \end{pmatrix}. \quad (158)$$

$\mathcal{B}_{4\times4}$ corresponds to the neutral real sector i. e. ρ_3, η_3, ϕ_1, $\chi^0_{3_1}$;

$$\mathcal{B}_{4\times4} = \begin{pmatrix} 4N_\rho^2\lambda_1 & 2N_\eta N_\rho(\lambda_{16}+\lambda_6) & 2N_\rho v\lambda_7 & 2N_\rho N_{\chi_3^0}\lambda_8 \\ 2N_\eta N_\rho(\lambda_{16}+\lambda_6) & 4N_\eta^2\lambda_2 & 2N_\eta v\lambda_{10} & 2N_\eta N_{\chi_3^0}\lambda_{11} \\ 2N_\rho v\lambda_7 & 2N_\eta v\lambda_{10} & 4v^2\lambda_3 & 2N_{\chi_3^0} v\lambda_{13} \\ 2N_\rho N_{\chi_3^0}\lambda_8 & 2N_\eta N_{\chi_3^0}\lambda_{11} & 2N_{\chi_3^0} v\lambda_{13} & 4N_{\chi_3^0}^2\lambda_4 \end{pmatrix}. \tag{159}$$

Finally, $\mathcal{C}_{2\times2}$ corresponds to the dark matter sector i. e. $\eta^0_{3_1}$, $\eta^0_{3_2}$;

$$\mathcal{C}_{2\times2} = \begin{pmatrix} \mu_5+\lambda_{12}N_\eta^2+\lambda_9N_\rho^2 & \\ +\lambda_{15}N_{\chi_3^0}^2+\lambda_{14}v^2 & 0 \\ & \mu_5+\lambda_{12}N_\eta^2+\lambda_9N_\rho^2 \\ 0 & +\lambda_{15}N_{\chi_3^0}^2+\lambda_{14}v^2 \end{pmatrix}. \tag{160}$$

In order to find the physical mass states it is mandatory to perform a basis transformation that diagonalizes the mass matrix; this is given by

$$\mathcal{M}_{diag} = V\mathcal{M}V^\dagger, \tag{161}$$

where, thanks to the fact that the mass matrix found in eq. 157 is diagonal by blocks, it can be diagonalized by a matrix of the form

$$V = \begin{pmatrix} \alpha_{2\times2} & & & & \\ & \alpha_{2\times2} & & & \\ & & \beta_{4\times4} & & \\ & & & 0_{4\times4} & \\ & & & & 1_{2\times2} \end{pmatrix}, \tag{162}$$

where each sub-matrix diagonalizes a block of the mass matrix of eq. 157; those submatrixes are

$$\alpha_{2\times2} = \begin{pmatrix} \frac{N_\rho}{N_\eta} & 1 \\ -\frac{N_\eta}{N_\rho} & 1 \end{pmatrix} \tag{163}$$

and

$$\beta_{4\times4} = \begin{pmatrix} 1 & 0 & \frac{N_\rho v\lambda_7}{A+B-C} & -\frac{N_\rho N_{\chi_3^0}\lambda_8}{-A+B+C} \\ 0 & 1 & \frac{N_\eta v\lambda_{10}}{A+B+C} & \frac{N_\eta N_{\chi_3^0}\lambda_{11}}{A-B+C} \\ -\frac{N_\rho v\lambda_7}{A+B-C} & -\frac{N_\eta v\lambda_{10}}{A+B+C} & 1 & 0 \\ \frac{N_\rho N_{\chi_3^0}\lambda_8}{-A+B+C} & -\frac{N_\eta N_{\chi_3^0}\lambda_{11}}{A-B+C} & 0 & 1 \end{pmatrix}, \tag{164}$$

where

$$A \to \lambda_2 N_\eta^2 + \lambda_1 N_\rho^2 - \lambda_4 N_{\chi_3^0}^2 - \lambda_3 v^2, \tag{165}$$

$$B \to \sqrt{\lambda_4^2 N_{\chi_3^0}^4 + N_{\chi_3^0}^2 v^2 \left(\lambda_{13}^2 - 2\lambda_3\lambda_4\right) + \lambda_3^2 v^4}, \tag{166}$$

$$C \to \sqrt{\lambda_2^2 N_\eta^4 + N_\eta^2 N_\rho^2 \left((\lambda_{16} + \lambda_6)^2 - 2\lambda_1\lambda_2\right) + \lambda_1^2 N_\rho^4}. \tag{167}$$

The diagonalization of $\mathcal{B}$ was not trivial. In order to do that it was decomposed in 2×2 blocks. In the first step of the diagonalization process off-diagonal blocks, which correspond to mixings between the light and heavy Higgs sectors, were ignored; on-diagonal blocks were diagonalized. Then, off-diagonal blocks were taken into account but treated as perturbations; this perturbed version of $\mathcal{B}$ was diagonalized. The transformation which diagonalizes the perturbed version of $\mathcal{B}$ is $\beta_{4\times4}$.

FERMION MASSES

Yukawa Lagrangian

Following the same reasoning explained in Chapter 4 the Yukawa lagrangian is built, but now with the particle content from tabs. 3, 4 and the Higgs sector given in tab. 8. Under this considerations the Yukawa lagrangian is

$$\begin{aligned}
-\mathcal{L}_{yuk} &= h^e_{ab}\,\bar{=}_{aL}\rho e_{bR} + h^\nu_{ab}\,\bar{=}_{aL}\tilde{\eta}\nu_{bR} + h^T\overline{T}_L\chi_3^0 T_R \\
&+ h^J_{\alpha\beta}\overline{J}_{\alpha L}\chi_3^{0*}J_{\beta R} + h^u_a\overline{Q}_{3L}\tilde{\eta}u_{aR} + h^T_a\overline{T}_L\eta_3^0 u_{aR} + h^d_a\overline{Q}_{3L}\rho d_{aR} \\
&+ h^d_{\alpha a}\overline{Q}_{\alpha L}\eta d_{aR} + h^J_{\alpha a}\overline{J}_{\alpha L}\eta_3^{0\,*}d_{aR} + h'^\nu_{ab}\overline{\nu}^c_{aR}\phi\nu_{bR} \\
&\qquad + h^u_{\alpha a}\overline{Q}_{\alpha L}\tilde{\rho}u_{aR} + h^N_{ab}\overline{(N_{aR})^c}\eta_3^0\nu_{bR} + \text{h.c.}
\end{aligned} \tag{168}$$

Fermions will acquire mass when the Higgs field breaks symmetry. When this happens, the lagrangian from eq. (168) becomes

$$\begin{aligned}
\langle -\mathcal{L}_{yuk}\rangle &= h^e_{ab}N_\rho\bar{e}_{aL}e_{bR} + h^\nu_{ab}N_\eta\bar{\nu}_{aL}\nu_{bR} + h^T N_{\chi_3^0}\overline{T}_L T_R \\
&+ h^J_{\alpha\beta}N_{\chi_3^0}\overline{J}_{\alpha L}J_{\beta R} + h^u_a N_\eta\bar{u}_{3L}u_{aR} + h^d_a N_\rho\bar{d}_{3L}d_{aR} \\
&\qquad + h^d_{\alpha a}N_\eta\bar{d}_{\alpha L}d_{aR} + h'^\nu_{ab}V\bar{\nu}^c_{aR}\nu_{bR} + h^u_{\alpha a}N_\rho\bar{u}_{\alpha L}u_{aR} + \text{h.c.}
\end{aligned} \tag{169}$$

Clearly, terms like

$$h^e_{ab}N_\rho\bar{e}_{aL}e_{bR} \tag{170}$$

will give the mass from the particle when $a = b$. But when $a \neq b$ they do not have a clear physical meaning. By going to mass eigenstate base the different $h_{ab} = 0$ if $i \neq j$ i.e. Yukawa couplings are diagonalized. Also the masses in this new base are the physical ones.

In order to see clearer which matrixes are meant to be diagonalized terms in eq. 168 are rewritten in a matricial form. Using $\alpha, \beta = 1, 2$ and $a, b = 1, 2, 3$, for up-like quarks we have

$$\begin{pmatrix} \overline{u_{\alpha L}} & \overline{u_{3L}} & \overline{T_L} \end{pmatrix} \begin{pmatrix} h^u_{\alpha a} N_\rho & 0_{2\times 1} \\ h^u_a N_\eta & 0 \\ 0_{1\times 3} & h^T N_{\chi_3^0} \end{pmatrix} \begin{pmatrix} u_{aR} \\ T_R \end{pmatrix}, \tag{171}$$

for down-like quarks

$$\begin{pmatrix} \overline{d_{\alpha L}} & \overline{d_{3L}} & \overline{J_{\alpha L}} \end{pmatrix} \begin{pmatrix} h^d_{\alpha a} N_\eta & 0_{2\times 2} \\ h^d_a N_\rho & 0_{1\times 2} \\ 0_{2\times 3} & h^J_{\alpha\beta} N_{\chi_3^0} \end{pmatrix} \begin{pmatrix} d_{aR} \\ J_{\beta R} \end{pmatrix}, \tag{172}$$

for charged leptons

$$h^e_{ab} N_\rho \overline{e}_{aL} e_{bR}, \tag{173}$$

and for neutrinos

$$\begin{pmatrix} \overline{\nu_{aL}} & \overline{\nu^c_{aR}} & \overline{N^c_{aR}} \end{pmatrix} \begin{pmatrix} 0_{3\times 3} & h^\nu_{ab} N_\eta & 0_{3\times 3} \\ h^{\nu\dagger}_{ab} N_\eta & h'^\nu_{ab} V & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \end{pmatrix} \begin{pmatrix} \nu^c_{bL} \\ \nu_{bR} \\ N_{aR} \end{pmatrix}. \tag{174}$$

It is important to notice some important facts from equations presented above.

First of all, notice how in equations 171 and 172 the form of the mixing matrix gives a solution to the mass hierarchy problem as follows: The third generation couples to a different Higgs field; this field is proposed to be heavy when compared to ρ, so that explains that there is a difference in the coupling constant. Also, by this structure, when 171 and 172 are diagonalized if all entries are of the same order eigenvalues will differ by orders of magnitude, hence giving an explanation to the mass hierarchy problem.

Also notice that exotic quarks J_α and T (with $P = -1$ i.e. dark matter quarks) acquire mass by coupling with the field χ_3^0 which is also proposed to be very heavy. This will explain why dark matter particles have not been observed in particle accelerators yet.

Finally, notice how with this representation in order to vanish anomalies we introduced ν_{aR} and N^c_{aR}, they allow the construction of both Dirac and Majorana mass terms for neutrinos. This allows to retake the advantages of the seesaw mechanism; it is also remarkable how the mixing matrix found on equation 174 naturally has the form of the type-I seesaw mechanism, which was the explained in Chapter 4.

However notice that N^c_{aR} is desconected from other neutrinos and also appears to have zero mass. This is because with the proposed Higgs sector is impossible to build mass or mixing terms with N^c_{aR}. However, notice that it can acquire mass through radiative corrections. To see this take from eq. 168 the term

$$h^N_{ab}\overline{(N_{aR})^c}\eta^0_3\nu_{bR} \tag{175}$$

which is zero when symmetry is broken. Now considering radiative corrections $\langle\eta^0_3\rangle \to 0+\eta^0_3$ it becomes

$$0+h^N_{ab}\overline{(N_{aR})^c}\eta^0_3\nu_{bR} \tag{176}$$

the second is clearly a mixed term. This connects N^c_{aR} with the other neutrinos. Hence when eq. 174 is diagonalized under radiative corrections we will have mass and mixings for N^c_{aR}.

GAUGE FIELDS

Following the same ideas of the standard model we introduce a gauge field for each group generator, so the covariant derivate for this model is

$$D_\mu = \partial_\mu + ig_1A^i_\mu T_i + i\frac{g_2}{2}B_\mu Y + i\frac{g_3}{2}C_\mu X + i\frac{g_4}{2}E_\mu T_{B-L}. \tag{177}$$

Also, analogus to the standard model we have that the terms involving the gauge fields are

$$\mathcal{L}_{gauge} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}C_{\mu\nu}C^{\mu\nu} - \frac{1}{4}E_{\mu\nu}E^{\mu\nu}, \tag{178}$$

$$\begin{aligned}\mathcal{L}_{kin\ lep} = {} & \overline{\psi}_{aL}i\not{D}\psi_{aL} + \overline{(N_{aR})^c}i\not{D}(N_{aR})^c \\ & + \overline{e}_{aR}i\not{D}e_{aR} + \overline{\nu}_{aR}i\not{D}\nu_{aR},\end{aligned} \tag{179}$$

$$\begin{aligned}\mathcal{L}_{kin\ quark} = {} & \overline{Q_{\alpha L}}i\not{D}Q_{\alpha L} + \overline{Q_{3L}}i\not{D}Q_{3L} + \overline{J_{\alpha L}}i\not{D}J_{\alpha L} + \overline{T_L}i\not{D}T_L \\ & + \overline{u_{aR}}i\not{D}u_{aR} + \overline{d_{aR}}i\not{D}d_{aR} + \overline{J_{\alpha R}}i\not{D}J_{\alpha R} + \overline{T_R}i\not{D}T_R\end{aligned} \tag{180}$$

$$\begin{aligned}\mathcal{L}_{kin\ Higgs} = {} & (D_\mu\eta)^\dagger(D^\mu\eta) + (D_\mu\rho)^\dagger(D^\mu\rho) \\ & + (D_\mu\eta^0_3)^\dagger(D^\mu\eta^0_3) + (D_\mu\chi^0_3)^\dagger(D^\mu\chi^0_3) \\ & + (D_\mu\phi)^\dagger(D^\mu\phi),\end{aligned} \tag{181}$$

where the field tensors for the gauge fields are defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\mu, \tag{182a}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{182b}$$

$$C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu, \tag{182c}$$

$$E_{\mu\nu} = \partial_\mu E_\nu - \partial_\nu E_\mu. \tag{182d}$$

As with the standard model expanding this terms will determine the different kinds of interactions between particles in the model. This is out of the scope of this work, however, doing this, and performing calculations for the newly predicted kinds of events will be an interesting future work and will also determine limits for the different parameters of this model.

However, we elaborate more on eq. 181 because the masses of the gauge bosons of the model will be deducted from it. For our model to be consistent we must find the gauge bosons of the standard model i. e. W^+_μ, W^-_μ, Z, A_μ. Our model breaks from $SU(2)_L \otimes U(1)_Y \otimes U(1)'_X \otimes U(1)_{B-L}$ which has six generators to $U(1)_Q$ which has one generator. This implies that we must find six gauge bosons, from them five must be massive and one must be massless thanks to the Goldstone theorem.

So, first we calculate the covariant derivatives for the Higgs fields

$$D_\mu\langle\eta\rangle = \begin{pmatrix} g_1 N_\eta W^+_\mu \\ \frac{1}{2} i N_\eta \left(-2A_{3\mu} g_1 + B_\mu g_2 + \frac{C_\mu g_3}{3}\right) \end{pmatrix}, \tag{183}$$

$$D_\mu\langle\rho\rangle = \begin{pmatrix} g_1 N_\rho W^+_\mu \\ \frac{1}{2} i N_\rho \left(-2A_{3\mu} g_1 + B_\mu g_2 + \frac{2C_\mu g_3}{3}\right) \end{pmatrix}, \tag{184}$$

$$D_\mu\langle\eta^0_3\rangle = 0, \tag{185}$$

$$D_\mu\langle\chi^0_3\rangle = -\frac{1}{6} i C_\mu g_3 N_{\chi^0_3}, \tag{186}$$

$$D_\mu\langle\phi\rangle = i E_\mu g_4 V, \tag{187}$$

where, as in the standard model, $W^\pm_\mu = (A_{1\mu} \mp iA_{2\mu})/\sqrt{2}$. Plugging this in eq. 181 and ignoring pure kinetic terms i. e. terms of the form $\partial_m u\phi\partial^\mu\phi$ renders

$$\begin{aligned} \mathcal{L}_{mass} = \frac{1}{36} \Big(& 36A_{3\mu}A^\mu_3 g_1^2 \left(N_\eta^2 + N_\rho^2\right) - 12A_{3\mu} g_1 \left(3B^\mu g_2 \left(N_\eta^2 + N_\rho^2\right) + C^\mu g_3 \left(N_\eta^2 + 2N_\rho^2\right)\right) \\ & + 9B_\mu B^\mu g_2^2 \left(N_\eta^2 + N_\rho^2\right) + 6B_\mu C^\mu g_2 g_3 \left(N_\eta^2 + 2N_\rho^2\right) \\ & + C_\mu C^\mu g_3^2 \left(N_\eta^2 + 4N_\rho^2 + N_{\chi^0_3}^2\right) + 36E_\mu E^\mu g_4^2 V^2 + 36 g_1^2 W^-_\mu W^+_\mu \left(N_\eta^2 + N_\rho^2\right)\Big) \end{aligned} \tag{188}$$

Notice that the last two terms give the masses for W^+_μ, W^+_μ and E_μ i. e.

$$M^2_{W^\pm} = g_1^2 \left(N_\eta^2 + N_\rho^2\right), \tag{189}$$

$$M^2_E = g_4^2 V^2. \tag{190}$$

Writing the rest of the terms of eq. 188 in matricial form allows us to see the matrix that we must diagonalize in order to find the masses of the other gauge bosons. So they are written as

$$\begin{pmatrix} A_{3\mu} & B_\mu & C_\mu \end{pmatrix} M'_{gauge} \begin{pmatrix} A^\mu_3 \\ B^\mu \\ C^\mu \end{pmatrix} \tag{191}$$

with

$$M'_{gauge} = \begin{pmatrix} g_1^2 a & -\frac{1}{2}g_1 g_2 a & -\frac{1}{6}g_1 g_3 b \\ -\frac{1}{2}g_1 g_2 a & \frac{1}{4}g_2^2 a & \frac{1}{12}g_2 g_3 b \\ -\frac{1}{6}g_1 g_3 b & \frac{1}{12}g_2 g_3 b & \frac{1}{36}g_3^2 c \end{pmatrix}, \tag{192}$$

$$a = \left(N_\eta^2 + N_\rho^2\right), \tag{193}$$

$$b = \left(N_\eta^2 + 2N_\rho^2\right), \tag{194}$$

$$c = \left(N_\eta^2 + 4N_\rho^2 + N_{\chi_3^0}^2\right). \tag{195}$$

M'_{gauge} can be diagonalized by the transformation

$$P^{-1}M'_{gauge}P = M_{gauge}^{diag}, \tag{196}$$

where

$$P = \begin{pmatrix} \frac{g_2}{2g_1} & \frac{g_1(f-e)}{3bdg_3} & -\frac{g_1(e+f)}{3bdg_3} \\ 1 & \frac{g_2(e-f)}{6bdg_3} & \frac{g_2(e+f)}{6bdg_3} \\ 0 & 1 & 1 \end{pmatrix}, \tag{197}$$

with

$$d = 4g_1^2 + g_2^2, \tag{198}$$

$$e = 9ad - cg_3^2, \tag{199}$$

$$f = \sqrt{81a^2d^2 + 18(2b^2 - ac)dg_3^2 + c^2g_3^4}. \tag{200}$$

When M'_{gauge} is diagonalized we find the masses for the rest of the gauge bosons

$$M_Z^2 = \frac{1}{72}\left(9ad + cg_3^2 + \sqrt{81a^2d^2 + 18dg_3^2(2b^2 - ac) + c^2g_3^4}\right), \tag{201}$$

$$M_{Z'}^2 = \frac{1}{72}\left(9ad + cg_3^2 - \sqrt{81a^2d^2 + 18dg_3^2(2b^2 - ac) + c^2g_3^4}\right), \tag{202}$$

$$M_A^2 = 0. \tag{203}$$

So from equations 189, 190, 201, 202 we found that five gauge bosons are massive, and from eq. 203 we had a massless boson. This is consistent with what we expected. This results also recover what was predicted by the standard model, hence proving consistency of this model.

6 CONCLUSIONS

During the course of this work, we have reviewed the fundamentals of group theory behind particle physics. We have also studied how standard model is formulated and also the flaws it presents when compared with reality.

We proposed an extension of the standard model using the groups $U(1)'_X$ and $U(1)_{B-L}$. We found the adequate representation that makes this model anomaly free, hence guaranteeing that theories constructed with this model are renormalizable. This is key when constructing new theories, because otherwise there will be no way of making finite calculations, hence measurable predictions.

Along the particles of the anomaly free representation, with the Higgs sector proposed in Chapter 5 we were able to adress the problems which motivated this work.

First of all, the $U(1)'_X$ symmetry allowed the construction of Yukawa terms which effectively implemented an horizontal symmetry on particles. When symmetry broke, this caused different generations of particles to couple in different ways to different higgs particles; this fact gave a natural explanation to the mass hierarchy problem: there is an inherent difference between generation couplings.

It is also interesting how exotic particles are coupled to the heaviest Higgs fields, hence explaining why they have not been observed on any experiments to date. It is also remarkable how by defining operator P (which is possible thanks to the $B-L$ symmetry), which allows to distinguish between barionic and dark matter, this ultra heavy quarks are dark matter candidates. This provides a way to explain why dark matter particles account to roughly 24% of the universe composition; it is not more abundant than barionic matter per se, but is much more massive.

We also studied a way in which dark matter could decay into barionic matter. Following ideas from [55, 50, 52] we reviewed a way in which calculations could be made in order to tune the parameters of the model to match it with reality and hence explain stability of dark matter particles.

Finally, it was found that it was possible within this model to construct mass terms for neutrinos. Also, the neutrino mass matrix was found to naturally have a form that resembled the neutrino mass matrix of type-I seesaw mechanism. This last fact is amazing, because it means that this model naturally explains why observed neutrino masses are tiny when compared to the rest of particles.

Interesting future works based on this model will be determining masses for gauge bosons, determining mixing matrixes, and studying interactions between particles as dictated by the expansion of kinetic terms of the lagrangian. Also, studying 1-loop corrections, performing numeric calculations for different cross sections and comparing them with reality will limit the parameter space for the model. Doing this as a first step with known processes will then allow for experimental predictions on new processes proposed by this model, thus allowing another way to compare it with reality.

A

ALTERNATIVE ANOMALY CALCULATION

Here we present an alternative method for anomaly calculation for $SU(N)$ groups. This method is based on group theory properties and is developed in [56].

The Young tableaux method [18] characterizes the irreducible representations of a $SU(N)$ group by a set of $N-1$ numbers q_i [56]. The quantity $q_i - 1$ is the number of columns with i blocks in a Young tableaux for a certain representation. For example, for the fundamental representation of $SU(N)$ the Young tableaux will be a single box, hence the set of characterizing $\{q_i\} = \{q_1 = 2, q_2 = 1, q_3 = 1, ..., q_{N-1} = 1\}$.

The dimension of any representation can be expressed in terms of q_i [56] i. e.

$$D(q) = \prod_{j=1}^{N-1} \left[\frac{1}{j!} \prod_{k=1}^{N-1} \left[\sum_{i=k-j+1} q_i \right] \right] . \tag{204}$$

And the anomaly can also be calculated in terms of q_i [56] i. e.

$$A(q) = D(q) \sum_{i,j,k}^{N-1} a_{ijk} q_i q_j q_k, \tag{205}$$

where

$$a_{ijk} = \frac{2(N-3)!}{(N+2)!} i(N-j)(N-k). \tag{206}$$

Thanks to this, and to [56, Eqns. 11] for completely antisymmetric representations, the results from [56, Tabla I, II and III] are found. This confirms that the chosen representations that accomodate leptons and quarks in this model make it anomaly free (this is because leptons contribute to anomalies with a total of -3, which cancels the $+3$ contribution to the anomaly made by quarks).

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