UNIVERSIDAD NACIONAL DE COLOMBIA

Stochastic optimization of strategic mine planning of a hypothetical copper deposit through a parameterizable algorithm

José Mario Pareja Zapata

Universidad Nacional de Colombia

Facultad de Minas, Departamento de Materiales y Minerales

Medellín, Colombia

2016

Stochastic optimization of strategic mine planning of a hypothetical copper deposit through a parameterizable algorithm

José Mario Pareja Zapata

Thesis submitted as a partial fulfilment of the requirements of the degree:

Master of Engineering-Mineral Resources

Director:

Magister Giovanni Franco Sepulveda

Research Line:

Long Term Mine Planning Optimization

Research Group:

Grupo de investigación en Planeamiento Minero (GIPLAMIN)

Universidad Nacional de Colombia

Facultad de Minas, Departamento de Materiales y Minerales

Medellín, Colombia

2016

A mis padres, hermanos y en especial a mi novia, Marcela, gracias por tu apoyo incondicional.

Acknowledgements

It is important to mention the feedback and help provided by members from GIPLAMIN (the Spanish acronym for research group of mine planning), likewise, the support provided from PhD Luis Montiel from McGill university research laboratory COSMO. Also, Gurobi optimization software for allowing free academic license. Finally, to all the engineers and experts, members of Stackoverflow community and Stack Exchange team, for stablishing such a professional Q&A place for programmers and computer science engineers.

Resumen

Para realizar la planificación de una mina de superficie es necesario partir de una evaluación inicial del recurso mineral. La evaluación del secuenciamiento de una mina a cielo abierto es un paso clave en el proceso de planeación de las actividades de extracción de una empresa minera. Los enfoques tradicionales aplicados para definir el límite máximo de la fosa consideran un único modelo estimado, que se desvía de una evaluación real del activo mineral. En los últimos años, se propusieron nuevos enfoques, de modo que los beneficios de apartarse de la visión del mundo determinística, donde cada variable es estática y modelada desde un promedio aritmético, hasta una evaluación estocástica que permite comprender el riesgo asociado a la planificación minera a largo plazo. Los enfoques de optimización exacta se estudiaron debido a el rol crucial de la planificación minera en los analísis financieros, sin embargo se consideran las implicaciones asociadas con estos métodos y se propone un enfoque metaheurístico para resolver el caso de estudio.

Palabras clave: Planeación minera a largo plazo, Secuenciamiento de minería de superficie, Optimización estocástica.

Abstract

To perform a surface mine planning it is necessary to start from an initial evaluation of the mineral resource. The open pit schedule evaluation is a key step in the process of planning the extraction activities of a mining company. Traditional approaches applied to define the ultimate pit limit consider a single estimated model, which deviates from a real assessment of the mineral asset. Over the recent years, new approaches were proposed, so that the benefits of departing from deterministic world view, where every variable are static and modeled from an arithmetic average, to a stochastic evaluation which allows understanding the risk associated to the open pit long term mine planning. Exact optimization approaches were studied due the major roll of mine planning to financial analytics, however the implications associated with these methods are considered and a metaheuristic approach is proposed to solve the case of study.

Keywords: Long term mine planning, Open pit scheduling, Stochastic optimization.

Content

List of figures

List of tables

Introduction

Mining is the process of extracting a naturally occurring material from the earth to derive a profit(Newman, Rubio, Caro, Weintraub, & Eurek, 2010). Companies require the minerals and metal products to have the capacity to operate on a regular basis. The high demand of this materials have created the obligation to develop extraction techniques. Surface mining accounts for a significant proportion of the current mineral production (Sattarvand, 2009). Surface mining have advantages like large production equipment size, short preproduction development period, high ore recovery and less labour requirements. In this thesis, the research is focus on open pit mining method.

Developing an open pit mine is a hard and complex task that must be planned and strategically programmed to get highest possible profit, while accounting for the constraints that the mining company is subjected. There are procedures that are traditionally follow in the process of mine planning. Initially, the minerals domain have to be outlined, then it is discretized into mining blocks and its ore content is estimated. Mine planners must decide which blocks should be exploited, when and whether they should be processed or not.

At this point, the traditional procedures would continue to calculate economic pit limits through Lerchs-Grossmann graphic algorithm (Lerchs & Grossmann, 1965), and then schedule the extraction based on mine and processing plant constraints, maximizing the net present value of the mine. However, the parameters (cutoff, block mineral content, CAPEX, OPEX, etc.), used at the first analysis are assumed or approximated, driving production targets that may not be met as planned, leading to suboptimal management of cash flows. This generates changes to short term plans that deviate from long term production plans and forecasts, all of which lead to unfulfilled expectations(F. R. Albor Consuegra & Dimitrakopoulos, 2009).

Throughout the last 50 years the amount of research that has been devoted have accomplished substantial progress, mainly because high grade deposits are unusual, market and operational conditions are becoming dynamic and uncertain, and environmental politics get tougher; bringing on that, as previously said, the mine plans may not be reached as expected. However the industry is still withdrawn to receive these developments.

The past 2 decades have been dedicated to improve the method that traditionally have been used, applying new optimization techniques that are able to manage the risk and uncertainty from the input parameters. At this thesis it is studied and applied an already developed procedure to create mine plans under the consideration of geological uncertainty.

1. Literature Review

The execution of tasks related to surface mining results in deformation of the topography so that a pit is dug for the purpose of extracting ore. Through the life of the mine the pit must get deeper and deeper until the economic boundaries are reach. An organized mining company must value their assets before compromising at investing and construction phases, and the mineral deposit is its most valuable resource. Therefore, after the deposit have been studied enough to be modeled and estimated the mine planners must decide how much of the ore can be processed to maximize how much money the company could produce based on the capacity of truck fleet and processing plant.

Traditional methods have been applied since mid-sixties and many mines have been hit by the variability between the predicted amount of ore that should have been extracted and the reality after it have been processed. At this chapter some of these methods are going to be review and how they could be simplified applying mathematical programming.

1.1 Ultimate pit limits for Open pit Design

Calculating the ultimate pit limits is a task that should led to maximum extraction of ore without compromising economical interest of the mine company. This can be simplified as the determination of optimum contour that holds a volume of mineral while considering the maximum operational slope angle. The ultimate pit limit problem has been solved using the Lerchs-Grossmann graph theoretic algorithm(Lerchs & Grossmann, 1965), Picard's network flow method(Picard, 1976) and Hochbaum's Maximum flow method(Hochbaum, 2008), minimizing the time consumption and computational capacities. However, nowadays we can solve the problem applying Mix integer linear programming with efficient optimization algorithms like branch and bound with heuristic methods and parallel computing within a commercial optimizer called GUROBI(Gurobi Optimization Inc., 2015).

1.1.1 Lerchs-Grossmann

This wasn't the first method used to solve the open pit mining problem, but it was a big accomplishment, thus, its understanding is a must-have for any mine planner. Lerchs-Grossmann importance is such that it have been used by the mining industry over the last four decades. "They associate a directed node-weighted graph, called the mine graph, with the three-dimensional grid of blocks. They note that the maximum profit open-pit mine contour corresponds to a maximum closure in the graph. A closure in the graph is a subset of the nodes such that if a node belongs to this set then all its successors also belong to the set, and the closure is maximal if the sum of node-weights is maximum"(Amankwah, 2011).

To close the directed graph a set of vertices must be defined such that, if ***i*** is a member of this set and ***(i, j)*** is an arc of the graph, then ***j*** must also be a member of this set. The Lerchs-Grossmann method defines some concepts that describes relationships between the blocks. First, a dummy block is added to the bottom and it is called root. When the directed graph is connected (there are no breaks in it) and there are no cycles (circular block dependencies), it is called a tree. A tree ***T*** with a connection to the root it is called a rooted tree. It is called as a branch T_i the set of arcs in a tree, which supports the rooted tree. The branches are characterized by the orientation of the arcs, so they can be called as plus (pointing towards T_i), and minus (pointing away T_i).

Arcs can be considered as weak or strong, a plus arc is strong if it supports a weight (sum of net value) that is strictly positive; a minus arc is strong if it supports a weight that is equal to zero or negative; arcs that are not strong are weak. If exist at least one strong arc on the single path of the tree ***T*** that joins a node ***i*** to the root, it is said that it is a strong Node. Finally, a tree is normalized if the root is common to all strong arcs. The maximum closure of a normalized tree is the set of its strong nodes.

1.1.2 Picard's Maximum Flow Algorithm

Picard (1976), solves the maximum flow problem adding to the mine graph a source node and a sink node, therefore finds the maximum closure. In a given directed graph G = (V, A), where V is the set of nodes and A the set of arcs, Picard's formulates the maximum closure problem as a ***0 - 1*** mathematical programming, as shown in Equation 1

$$Max\ Z = \sum_{i \in V} p_i x_i$$

Subject to:

$$x_i \leq x_j \qquad \forall\ (i,j) \in\ A$$

$$x_i \in \{0,1\} \qquad i \in V$$

Equation 1. Picard (1976) Linear Programming Formulation.

Where p_i is a weight associated to node ***i***, known as the net profit of the block; and ***x*** is a binary variable, which could be equal to 1 if the block is in the closure or 0 if otherwise. The restriction $x_i <= x_j$ is the one that determines the precedencies constrain. Picard studied the equivalency of that previous restriction to the relation $a_{ij}\, x_i\, (x_j - 1) = 0$, where a_{ij} is the element ***(i, j)*** of the incident matrix of the graph G, that is, $a_{ij} = 1$ if ***(i, j)*** ∈ ***A*** and $a_{ij} = 0$ if otherwise(Amankwah, 2011). Based on this, Picard reformulates the Equation 1 problem as shown in Equation 2, and then since $a_{ij}\, x_i\, (x_j - 1) \leq 0$ for all $x_i \in \{0, 1\}$ and $x_j \in \{0, 1\}$, Picard deduces that Equation 2 is equivalent to Equation 3 (Picard, 1976)

$$Max\ Z = \sum_{i \in V} p_i x_i$$

Subject to:

$$\sum_{i \in V} \sum_{j \in V} a_{ij} x_i (x_j - 1) = 0$$

$$x_i \in \{0,1\} \qquad i \in V$$

Equation 2. LP formulation with first constrain relaxation (Picard, 1976).

$$Max\ Z = \sum_{i \in V} p_i x_i + \lambda \sum_{i \in V} \sum_{j \in V} a_{ij} x_i (x_j - 1)$$

Subject to:

$$x_i \in \{0,1\} \qquad i \in V$$

Equation 3. LP formulation with Multiplier Relaxation (Picard, 1976).

where λ is a positive number large enough to ensure that an optimal solution of Equation 3 satisfies that $\sum_{i \in V} \sum_{j \in V} a_{ij} x_i (x_j - 1) = 0$. Then, Picard replaces the maximization problem by a minimization problem and it is equivalent to finding a minimum cut in a related network.

1.2 Open Pit Scheduling Problem

Traditionally, it have been considered that the scheduling problem only must be solve after finding the economical pit limits or maximum closure, however this statement have changed since direct block scheduling techniques and efficient optimizers have been developed. The open-pit mine production scheduling problem can be defined as discovering the sequence in which rock blocks should be removed from the deposit as a certain material type in order to maximise the total discounted profit from the mine subject to a variety of physical and economic constraints(Sattarvand, 2009). The optimum schedule plays an important role in mine planning, and it should be at constant review at all stages of the life of an open-pit.

The scheduling problem can be formulated as a mix integer linear programing problem (MILP). However, in real applications this formulation is too large, in terms of both the number of variables and the number of constraints, to solve by any available commercial MILP software (Caccetta & Hill, 2003). When this problem is reach, a possible option is to solve the optimization problem sequentially period to period, or to develop special methods that are able to produce an acceptable sub optimal solution. Exact optimization methods (LP, MILP, etc.), can guarantee an optimal solution, however there are this kind of problems can not be solved in polynomial time, it grows exponentially with the size of the model.

In consequence, alternative methods have been studied through the past 4 decades, as Caccetta & Hill (1999), resumes: “Several heuristic approaches have appeared in the literature including methods based on Lagrangian relaxation (Caccetta et al., 1998); parameterisation (Matheron, 1975; Francois-Bongarcon and Guibal, 1984; Dagdelen and Johnson, 1986); dynamic programming (Tolwinski and Underwood, 1996); MILP (Gershon, 1983; Dagdelen and Johnson, 1986; Caccetta et al., 1998; Ramazan et al., 2005); simulated annealing and genetic algorithms (Denby and Schofield, 1995) and neural networks (Denby et al.,1991)”(Weintraub, Romeroes, Bjørndal, & Epstein, 2007)

1.2.1 Mix Integer Linear Programming formulation for Open Pit Scheduling

Caccetta & Hill (1999), modeled the open pit scheduling problem as Mix Integer Linear Programming formulation that right now could be solve using the available optimization software as shown in Equation 4. However this could be troublesome or commonly not solvable for real size mine planning problems.

$$Max\ Z = \sum_{t=2}^{T}\sum_{i=1}^{N}\left(C_i^{t-1} - C_i^{t}\right)X_i^{t-1} + \sum_{i=1}^{N} C_i^T X_i^T$$

Subject to:

$$\sum_{i\in O} ton_i X_i^1 = m^1 \qquad \textbf{(1)}$$

$$\sum_{i\in O} ton_i\left(X_i^t - X_i^{t-1}\right) = m^t\,, \quad t = 2,3,\ldots,T \qquad \textbf{(2)}$$

$$\sum_{i\in W} ton_i X_i^1 = u_W^1 \qquad \textbf{(3)}$$

$$\sum_{i\in W} ton_i\left(X_i^t - X_i^{t-1}\right) = u_W^t\,, \quad t = 2,3,\ldots,T \qquad \textbf{(4)}$$

$$X_i^{t-1} \le X_i^t, \quad t = 2,3,\ldots,T \qquad \textbf{(5)}$$

$$X_i^t \le X_j^t, \quad t = 2,3,\ldots,T;\ \forall j \in S_i;\ i = 1,2,\ldots,N \qquad \textbf{(6)}$$

$$\ell_O^t \le m^t \le u_O^t, \quad t = 1,2,\ldots,T \qquad \textbf{(7)}$$

$$X_i^t \in \{0,1\}\ ;\ t = 1,2,\ldots,T$$

Equation 4. MILP formulation for NPV maximization (Caccetta & Hill, 1999).

Where $\boldsymbol{T}$ is the number of periods over which the mine is being scheduled; N is the total number of blocks in the orebody; $\boldsymbol{C_i^t}$ is the profit (in NPV sense) resulting from mining the block $\boldsymbol{i}$ in the period $\boldsymbol{t}$; $\boldsymbol{O}$ is the set of ore blocks; $\boldsymbol{W}$ is the set of waste blocks; $\boldsymbol{ton_i}$ is the tonnage of block $\boldsymbol{i}$;$\boldsymbol{m^t}$ is the tonnage of ore milled in period t; $\boldsymbol{S_i}$ is the set of blocks that must be removed prior the mining of block $\boldsymbol{i}$; $\boldsymbol{X^t_i}$ is a binary variable that establish if the block $\boldsymbol{i}$ is mined in periods 1 to $\boldsymbol{t}$; $\boldsymbol{\ell_O^t}$ is the lower bound of the amount of ore that is milled in period $\boldsymbol{t}$; $\boldsymbol{u^t_O}$ is the upper bound of the amount of ore that is milled in period $\boldsymbol{t}$; $\boldsymbol{u^t_W}$ is the upper bound of the amount of waste that is milled in period $\boldsymbol{t}$. The constraints 1, 2 and 8 ensure that the milling capacities are hold. Constraints 3 and 4 ensure that the tonnage of waste removed does not exceed the prescribed upper bounds. Constraint 5 ensure that a block is removed in one period only. Constraint 6 is the precedence set of blocks that must be removed prior an ore block ***i***.

According to Kumral (2012), cited by (Sari, 2014), the production scheduling can be formulated as shown in Equation 5, as long as the cut-off value is previously defined.

$$Max\ f(x) = \sum_{i=1}^{T} \sum_{j=1}^{N} V_{ij}(m) x_{ij}$$

Subject to:

$$x_{Li} \geq x_{ji}, \quad i = 1,2,\dots,T;\ j = 1,2,\dots,N\ ; \forall L \in L_j \qquad \textbf{(8)}$$

$$\sum_{j=1}^{N} (r_j + v_j) x_{ij} - C \leq 0, \quad i = 1,2,\dots,T \qquad \textbf{(9)}$$

$$\sum_{j=1}^{N} (r_j - A_i) x_{ij} \leq 0, \quad i = 1,2,\dots,T \qquad \textbf{(10)}$$

$$H_L \sum_{j=1}^{N} x_{ij} r_j \leq \sum_{j=1}^{N} c_j x_{ij} r_j \leq H_u \sum_{j=1}^{N} x_{ij} r_j\ , \quad i = 1,2,\dots,T \qquad \textbf{(11)}$$

$$\sum_{i=1}^{\boldsymbol{T}} x_{ij} \leq 1, \quad j = 1,2,\dots,N \qquad \textbf{(12)}$$

$$x_{ij} \in \{0,1\}, \quad i = 1,2,\dots,T; j = 1,2,\dots,N$$

Equation 5. Simplified MILP model for NPV maximization.(Kumral, 2012)

Where ***N*** is the number of blocks considered for extraction; ***T*** is the number of periods of extraction; ***m*** is a binary parameter that defines if the block grade is greater than or equal to cut-off value; V_{ij} is the net discounted (NPV sense) profit of the block; r_j is the ore amount for block ***j***; A_i is the mineral processing capacity for period ***i***; v_j is the waste amount in the block ***j***; ***C*** is the mining capacity; L_j is the set of precedence blocks that must be removed before mining block ***j***; H_L and H_U are the lower and upper bounds for blending restrictions; C_j is the grade of the block ***j***. Constrain 8 ensures that for every extracted block ***j***, its precedence set of block L_j have been selected for extraction. Constrains 9 and 10 ensure that the mining and processing capacities are hold. Constrain 11 ensures that the blending quality for each period is between the upper and lower bounds. Constrain 12 ensures that any block is extracted only one time during LoM.

According to Johnson (1969) cited by Sattarvand (2009), proposed to solve this problem by decomposing of the large multi-period production planning model into a master problem and a set of sub-problems that are exactly similar to UPL problem. After solving all sub-problems by well-known UPL algorithms such as Lerchs-Grossmann's algorithm, solving the master problem would be relatively simple. Although this method produces optimum solutions for each period individually, however, it does not optimize the problem totally.

D. Espinoza et al (2012) said that "researchers have used Lagrangian Relaxation, e.g.,Dagdelen & Johnson (1986), in order to maximize net present value subject to constraints on production and processing. Akaike & Dagdelen (1999) extend this work by iteratively altering the values of the Lagrangian multipliers until the solution to the relaxed problem meets the original side constraints, if possible. Kawahata (2006), includes a variable cutoff grade. This research has been successful at solving some instances, though authors also report difficulty in obtaining convergence, or even determining a feasible solution for the monolithic problem".

Espinoza et al (2012), studied the implementation of AMPL optimizer(Fourer, Gay, Hill, Kernighan, & T Bell Laboratories, 1990) to solve a formulation similar to Equation 5, as shown in Equation 6, applied to different standard block models (Marvin, Newman, Maclaughlin, etc.), demonstrating that considerable size models could have a near optimal solution for multiple destination mine plan, however these block models are not real size problem.

$$Max\ f(x) = \sum_{b\in\mathfrak{B}}\sum_{d\in\mathfrak{D}}\sum_{t\in T} P_{bdt}y_{bdt}$$

Subject to:

$$\sum_{r\leq t} x_{br} \leq \sum_{r\leq t} x_{b'r} \quad , \quad \forall b \in \mathfrak{B};\ b' \in \mathfrak{B}_b; t \in T \qquad \textbf{(13)}$$

$$x_{bt} = \sum_{d\in\mathfrak{D}} y_{bdt}\ , \quad \forall b \in \mathfrak{B}, t \in T \qquad \textbf{(14)}$$

$$\sum_{t\in T} x_{bt} \leq 1, \ \ \forall b \in \mathfrak{B}, t \in T \qquad \textbf{(15)}$$

$$\underline{R}_{rt} \leq \sum_{b\in\mathfrak{B}}\sum_{d\in\mathfrak{D}} q_{brd}y_{dbt} \leq \overline{R}_{rt} \qquad \textbf{(16)}$$

$$\underline{a} \leq A_y \leq \overline{a} \qquad \textbf{(17)}$$

$$y_{bdt} \in [0,1] \quad \forall b \in \mathfrak{B};\ d \in \mathfrak{D}; t \in T$$

$$x_{bt} \in \{0,1\}, \ \ \forall b \in \mathfrak{B}, t \in T$$

Equation 6. MILP model for multiple destinations.

Where $\boldsymbol{p_{bdt}}$ is the profit for a block ***b*** in NPV sense, $\boldsymbol{x_{bt}}$ is a binary variable which equals 1 if block ***b*** is extracted in time period ***t***, and 0 otherwise, a second variable, y_{bdt}, which equals the amount of block ***b*** sent to destination ***d***. Constraint 13 is the precedence requirements for all blocks and time periods. Constraint 14 ensures that the extraction and processing variable values are consistent. That is, if a block is not extracted, its contents cannot be sent to any destination, and if a block is extracted, the entirety of its contents must be sent somewhere. Constraints 15 restrict a block to be extracted at most once over the horizon. Constraints 16 require that no more operational resource than available is used for extraction purposes. Constraints 17 represent general side constraints, it can model cases in which mining operations are governed by more than simply “common sense”, sequencing, and operational resource constraints (Mineral content that is considered as a penalty, minimal operational width, etc.) in the form of knapsacks. Note that because ***x*** can be written as a function of ***y*** (see (14)), it is not included the former variable in this constraint.

In Addition, Kumral (2011) cited by Sari (2014) stated that a mineral processing operation is installed according to ore consistency in the sense of certain specifications of ore material (e.g. grade, impurities and grindability). Also, Kumral (2011) suggested, when the objective function of the formulation is established to maximize the NPV of LoM, the ore extraction will be set at a lower order of importance since high grade blocks are going to be selected at initial periods because they produce a greater NPV and then, extra cost is added to the process because the difficult to reproduce these early high grade periods of extraction is raised. Therefore, it is necessary to study a different approach that when it is solve the ore and waste production rates are hold stable.

1.2.2 Metaheuristic Approach for Open Pit Mine Scheduling

1.2.2.1 Why Metaheuristics?

“The consequences of the computational complexity for a great many real world problems are fundamental. Exact method for scheduling problems “become computationally impracticable for problems of realistic size, either because the model grows too large, or because the solution procedures are too lengthy, or both, and heuristics provide the only viable scheduling techniques for large projects" (Cooper, 1976)"(Collet & Rennard, 2006). Therefore, instead of the usually used exact methods, heuristics or metaheuristics (nature-inspired algorithms) should be developed to solve this problems. According to Voß et al (2012), cited by Collet et al (2006)) "a metaheuristic is an iterative master process that guides and modifies the operations of subordinate heuristics to efficiently produce high-quality solutions. It may manipulate a complete (or incomplete) single solution or a collection of solutions at each iteration. The subordinate heuristics may be high (or low) level procedures, or a simple local search, or just a construction method".

1.2.2.2 Simulated Annealing

Initially Kirkpatrick et al (1983), developed this optimization technique based on an analogy in condensed matter physics, annealing is a thermal treatment technique in which a metallic or glass material is heated up sufficiently and then cooled gradually down to rearrange in a new configuration, where it is probable to reach a lower energy level (crystallization) at the internal structure of the solid. Metropolis et al (1953) proposed a simple algorithm to simulate the behavior of a collection of atoms at a given temperature. At each iteration, a small random move is applied to an atom and the difference of energy ΔE is computed. If ΔE ≤ 0 the new state is always accepted. If Δ > E 0 the new state is accepted according to a probability defined by Equation 7.

$$p(\Delta E) = e^{-\Delta E / k_B T}$$

Equation 7. Metropolis Criterion for Perturbation probability of acceptance.

Simulated Annealing Algorithm steps could be described briefly as:

1. Build a Seed or Initial Solution β
2. Evaluate the Objective value of β
3. Select a neighbor solution θ through a perturbation mechanism
4. If Objective value of θ is greater(for maximization problem) that objective value of β, select θ as the new state solution

5. Else select β according to the probability function of Equation 7 where T is the current temperature of the system.
6. Update the system temperature
7. Repeat step 3
8. Finish when stop condition is reached (system is frozen).

According to Thomas (1996) cited by Sattarvand (2009), “initial temperature and the cooling rate are the critical factors in the success of SA process. Excessively low starting temperature makes the process to converge too quickly and a sub-optimal solution might be produced. In contrast, extremely high initial temperature would cause spending a long time on poor initial solutions. Similarly, rapidly cooling of the system potentially gets locked around a local-optimum solution and produces a sub-optimal consequence. On the other hand, disproportionately slow cooling rate unnecessarily rises the computation time”.

1.2.2.3 Simulated Annealing Applied to Open pit Scheduling

Kumral & Dowd (2005) proposed a methodology to develop a production schedule by using SA Algorithm to improve any suboptimal schedule, that fits better to the objective of an open pit mine company, but achieving sub-optimality. Kumral et al (2005) affirmed that “there is no universally admitted scheduling approach because all methods, more or less, suffer from some shortcomings such as assumptions, period by period scheduling and computer time. This shortcomings lead to sub-optimality”.

The objective function was construct to solve a multi-objective minimization problem, divided in 3 main components, deviation from the required tonnage, penalty and opportunity cost for each content variable and content variability of each content variable

The first component of the objective function ensures that the ore quantity extracted in each period satisfy mill or plant capacity, minimizing an implicit cost associated to deviation from the required tonnage. The second component is modeled to control the cost associated to the average content of the variable under consideration (metal grade or equivalent), if the extracted ore is not between the upper and lower tolerance limits that the company sets. The last component is meant to minimize the cost associated to the content variance of the variable through LoM. Finally, the objective function is equal to the sum of every single objective/cost using a different weights for each component previously explain, Kumral et al (2005) emphasized that the selection of weights (or priorities) is critical and they depend on the ore body, sales contract, ore market structure and plant characteristics. Additionally, it is necessary to punctuate that the sum of all weights must be equal to one. Besides, two major constrains are set to ensure some common company issues. The first one ensures that the maximum number of periods of extraction is not exceeded, and another that guarantees a minimum operational area to the extraction of every ore block.

1.3 Stochastic Optimization for Open pit scheduling

The last section was emphasized in optimization techniques applied to find the best design to the open pit mine, so it must be recognized the major role for developing forecast, maximization and management of cash flows and the financial aspects that reign in the mining operation. The key input for all open pit schedule optimization methods is the orebody that have been modeled through estimation techniques. Independent of the estimation method applied, the resulted model is a representation that does not reproduce the in situ characteristics of ore content. Furthermore, the orebody is discretized to a finite number of blocks containing averaged values, propagating uncertainty to the different mining process. Mining design and production scheduling are nonlinear transfer functions in consequence, averaged grades may not provide an average profile of response uncertainty.

1.3.1 Stochastic Block Model

In general, as Dimitrakopoulos (2011) stated, the estimated orebody model is based on imperfect geological knowledge and lacks of inclusion or assessment of the related geological uncertainty. As Dimitrakopoulos (1998) and Albor (2010), highlighted, due to the smoothing effect(overestimate low-grade zones and underestimate high-grade values) present in any estimated type orebody model, as in the case of a kriged model, the histogram and variogram show lower variability than the actual data which leads to not meeting production targets and NPV forecasts.

To deal with the unknown deposit and its attributes of interest, one may generate several models (images) of the deposit based on and conditional to the same data and statistical properties (Dimitrakopoulos, 1998). This images are representations of the same orebody conditioning to include all data within the deposit, so they can be considered as equally probable, **Figure 1** illustrate the idea. Sequential geostatistical simulation methods have been widely accepted for the simulation of in situ mineral properties, and are based on an application of the Bayes Theorem (Kumral et al, 2005)(Goovaerts, 1999).

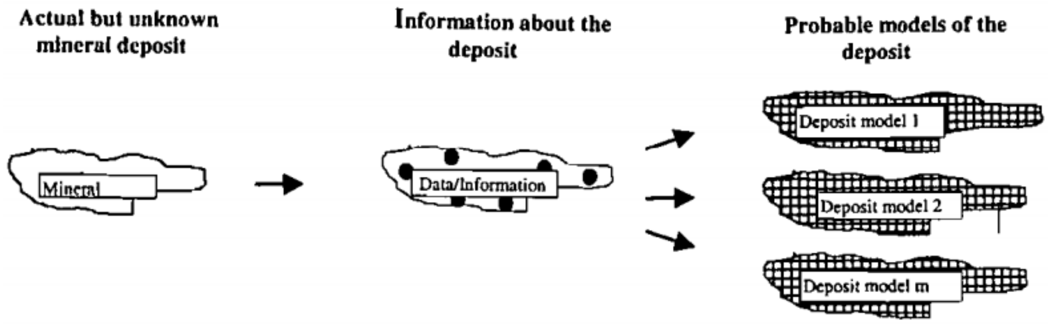

Figure 1. Conditional Geostatistic simulation scheme.(Dimitrakopoulos, 1998)

The simulation algorithms take into account both the spatial variation of actual data at sampled locations and the variation of estimates at unsampled locations. It means that stochastic simulation reproduces the sample statistics (histogram and semi-variogram model) and honors sample data at their original locations(Soltani et al, 2013). Therefore, according to Goodfellow (2014), geostatistical simulation methods are tools used to generate equally probable scenarios of a mineral deposit, where each simulation accurately reproduces the spatial statistics of the original drillhole data.

1.3.2 Integrating Uncertainty to Open Pit Scheduling Optimization

As previously was studied, applying optimization techniques to improve or solve all the related open pit problems is a major refinement of the traditional approaches developed at early state of mining research, however, it must be understand that an optimal solution is only optimal for the data input to the model, thus, for real processes like mining means that an average type input does not generate an average LoM schedule and forecast. Ravenscroft (1992) cited by Albor & Dimitrakopoulos (2009) suggested using simulated orebody models to probabilistically assess the performance of production targets as a function of the use of a given mine design and a Life of Mine production schedule. Dimitrakopoulos et al (2002), studied a typical, disseminated, low-grade, epithermal, quartz breccia-type gold deposit, hosted in intermediate– felsic volcanic rocks and sediments; and how geological uncertainty and risk in the design, planning and production expectations is accentuated by the generally low ore reserve grade and a variable. Subsequently, 50 realizations of the deposit were developed to quantify geological risk for the given mine design and long-term mine plan. This was implemented by replacing the estimated orebody model with each one of the 50 simulations and rerunning the optimization while the other mining and economic parameters are kept the same. The NPV outcome for the traditional approach was shown to be higher than the ninety-fifth quantile of the distribution, i.e. there is a 95% probability of the project returning a lower NPV than predicted by the estimated orebody model. Average in generates different average out, conventional optimization are misled and can not provide good forecast. Figures 2 and 3 shows the results of the (Dimitrakopoulos et al., 2002) case of study.

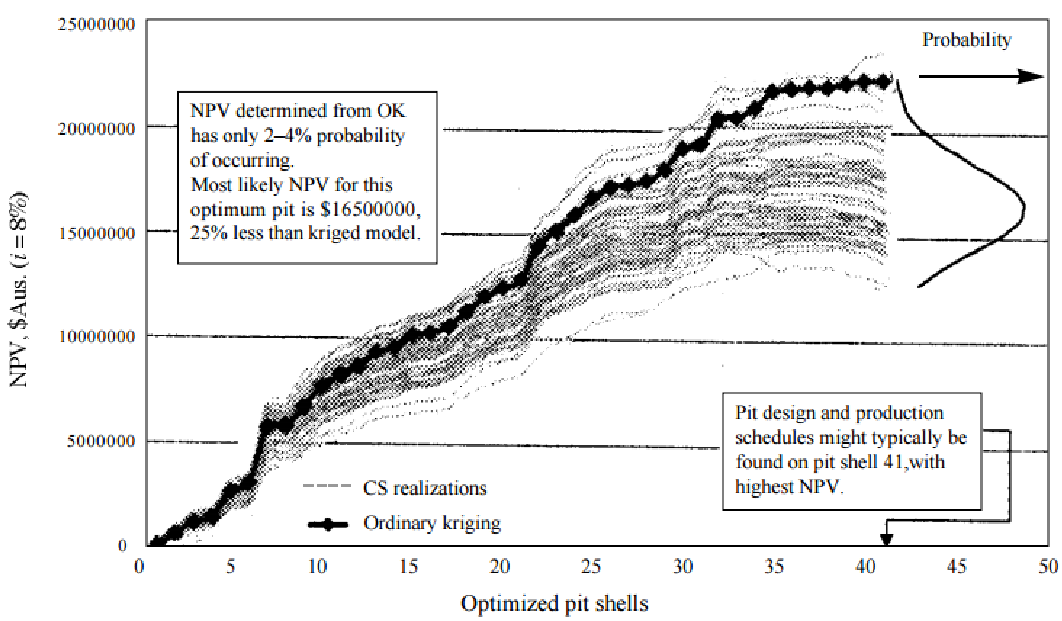

Figure 2. NPV sensitivity analysis applied on simulated orebodies.(Dimitrakopoulos et al., 2002)

The previous example shows the importance and implications of managing properly the uncertainty, but it generates the question of how could a mine planner integrate it. Godoy

& Dimitrakopoulos (2004) proposed a four stage stochastic optimiser process based on SA that can joint multiple simulated orebody representations and showed a 28% improvement in cash flows generated from the stochastic LoM schedule versus the conventional one. Leite & Dimitrakopoulos (2007) developed a three stage framework generating a final schedule, which considers geological uncertainty so as to minimise the risk of deviations from production targets. Figure 4 summarize the process to generate a robust design capable of increasing value while minimizing risks.

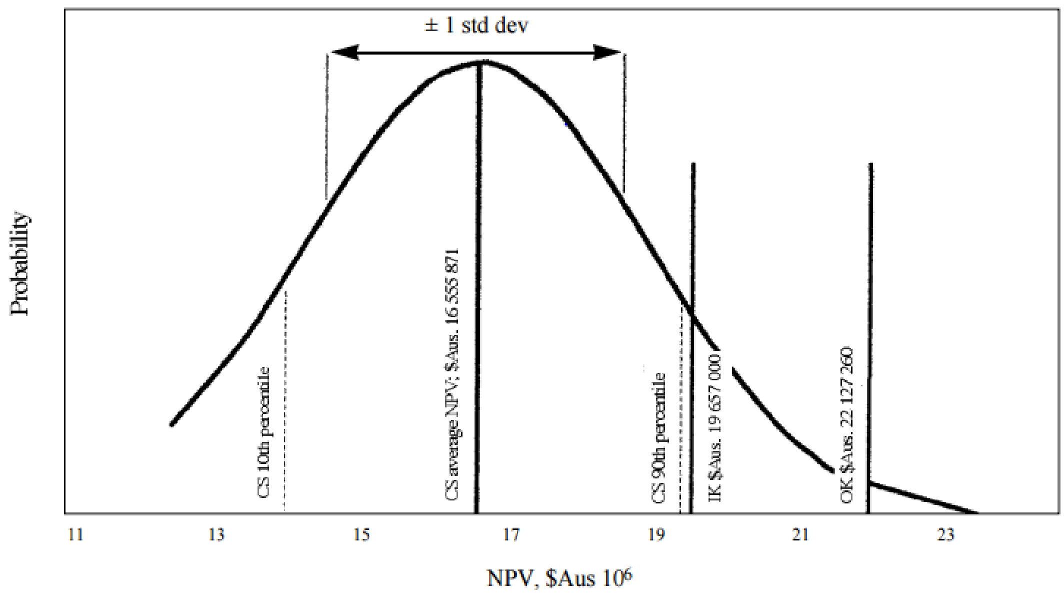

Figure 3. Probability distribution function for NPV analysis.(Dimitrakopoulos et al, 2002)

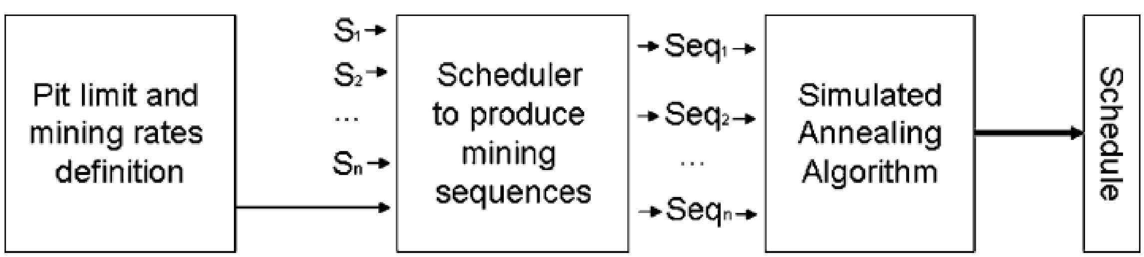

Figure 4. Three stage formulation for stochastic open pit schedule optimization.(Leite & Dimitrakopoulos, 2007)

Leite et al (2007) stated that the proposed approach steps follows as:

- Definition, through a conventional optimization approach, of the ultimate pit limits and mining rates to be used in subsequent stages.

  - Mining rates are either defined by a commonly used interactive procedure, or are preselected for mine operational reasons related to mill demand and geometric constraints. Any approach to defining mining rates can be accommodated in this stage.

- Development of a set of schedules within the predetermined pit limits that meet the ore and waste production targets defined in the previous stage; this set of schedules is developed using any scheduler and simulated orebodies one at a time.
  - The mining sequences generated are used to compute the probability that a mining block belongs to a given period of the LoM schedule. The map of such probabilities is basic input for SA in Stage 3.

- Generation of a single production schedule that minimises the risk of deviation from production targets using a SA formulation.
  - The perturbation method applied in this method was through the use of a connectivity test. A block is said to have connectivity, if at least one of the four surrounding blocks at the same level is scheduled in the same candidate period, the block just above it is scheduled in a previous or in the same period, and the block just below it is scheduled after or in the same period. If a block has connectivity it can be swapped to the candidate period.

2. Case of study

For the development of this research work, a hypothetical disseminated copper deposit was considered. The model consist of blocks of dimensions 20x20x10 meters, without subcells, with a total of 282800 blocks. For the stochastic analysis twenty orebody simulation are considered. The economic viability for each block was defined with copper price equal to 4629.71 USD/Ton. The rock density is equal to 2.7 Ton/m^3. In Table 1, it can be seen the economic parameters defined for the case study. The mining ore and waste mining rates were predefined. Table 2 shows the production targets for each mining period.

Table 1. Economic Parameters defined to Case of study.

Parameter	**Value**
Mining Cost	1 USD/Ton
Processing Cost	6.5 USD/Ton
Cost Augmentation rate	0.1 USD/level
Discount rate	10%

Table 2. Mining production Targets.

Mining Period	**Ore (10^6 Ton)**	**Waste (10^6 Ton)**
1	7.5	20.5
2	7.5	20.5
3	7.5	20.5
4	7.5	20.5
5	7.5	20.5
6	7.5	10
7	7	2

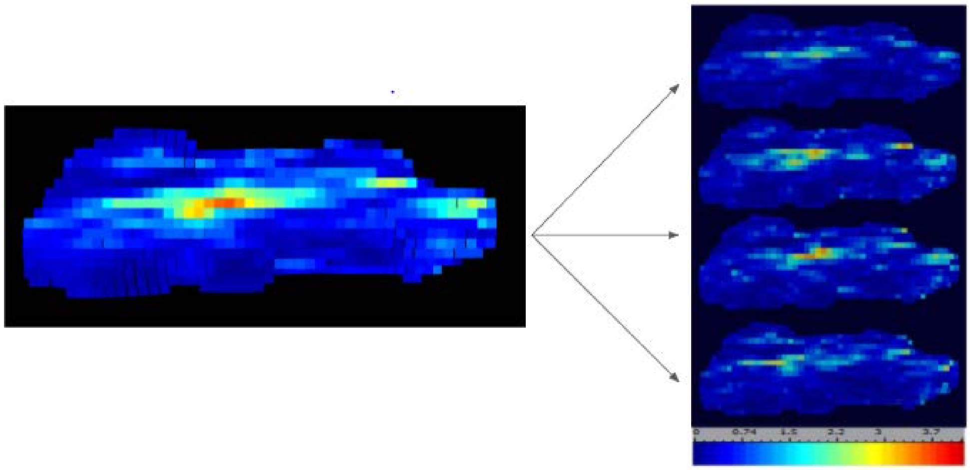

Figure 5. Estimated Model and Some orebody simulation.

2.1 Open pit Limits

In section 1.1 the open pit limits problem was studied. The importance of this stage at the long term mine planning development could be avoid if the computational capabilities allow it, however for research propose this problem is going to be solved. Industry engineers generally consider this as a standard or must have practice, because it reduces the size of the optimization problem. Nevertheless, this process carries some drawbacks, due its nature to maximize ore while minimizing waste over undiscounted cash flow and its performance is sensitive to the input values. Applying the Picard's formulation stated in section 1.1.2 and Equation 1, through GUROBI optimization software(Gurobi Optimization Inc., 2015) with Python API, Algorithm 1 shows how it was modeled for the deterministic case.

```
#Load Input Information:
val = net_value_All_blocks_sim1
edges = Precedencies_oreblocks
lof = range(7)

#Create Model:
m = Model()
n = len(val) # number of blocks

# Decision variable for each block
x={}
for i in range(n):
    x[i] = m.addVar(vtype=GRB.BINARY, name="x%d" %(i))
```

```
m.update()

# Set objective
obj = quicksum(val[i]*x[i] for i in range(n))
m.setObjective(obj,GRB.MAXIMIZE)

#Load Precedence Constraints:
for edge in edges:
    u = edge[0]
    v = edge[1]
    m.addConstr(x[u]<=x[v])
m.optimize()
```

Algorithm 1. Picard's formulation modeled through GUROBI Python API

The net value for each block calculation was determined using Equation 7. To calculate the precedencies for each mineralized block a 45° cone was projected and all the blocks that were contained were paired with its corresponded ore block, i.e. Figure 6 shows a possible case of blocks that must be extracted prior the ore block is mined, so the resulting edge array will contain the sets [ore,1],[ore,2],…[ore,5]. The cut-off is the minimum copper grade which allows a extraction without differing on loss.

$$netvalueBlock_i = \begin{cases} Tonnage * ((price * recovery * Cu_i) - ((level_i * AugmentationRate) + MiningCost) + Proc.Cost)) & \boldsymbol{if\ Cu_i > Cu_{cut-off}} \\ -1 * Tonnage * ((level_i * AugmentationRate) + MiningCost) & \boldsymbol{if\ Cu_i < Cu_{cut-off}} \end{cases}$$

Equation 7. Net value for ore and waste blocks.

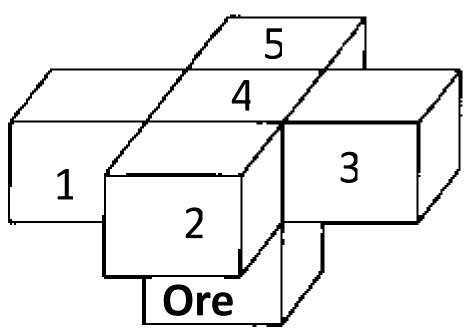

Figure 6. Precedencies possible case for ore block.

The resulting open pit shell or maximum contour for orebody simulation 1 is shown in the Figure 7, and its value of 534.46 million dollars, 67.28 million tons of ore and 222.1 million tons of waste.

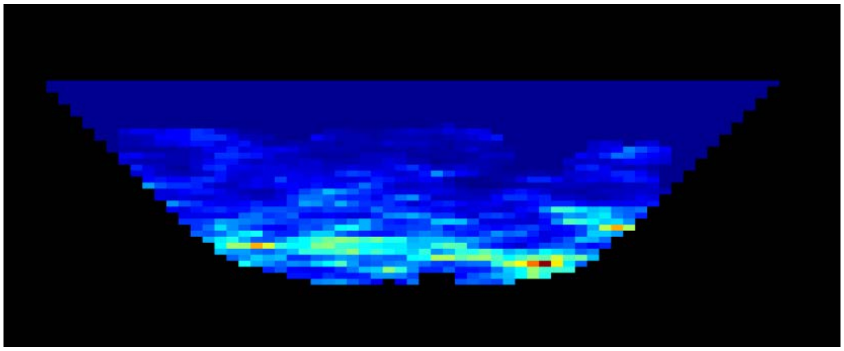

Figure 7. Ultimate pit limits for Simulation 1.

2.2 Deterministic Open pit scheduling

For comparison purpose, it was determined that a deterministic schedule should be developed with traditional or common industry practices. Miningmath SimSched was the selected software to schedule the E-type orebody, the same operational parameters were used, as Table 1 and 2 shows. Similarly, Figure 8 overviews the parameters on the software.

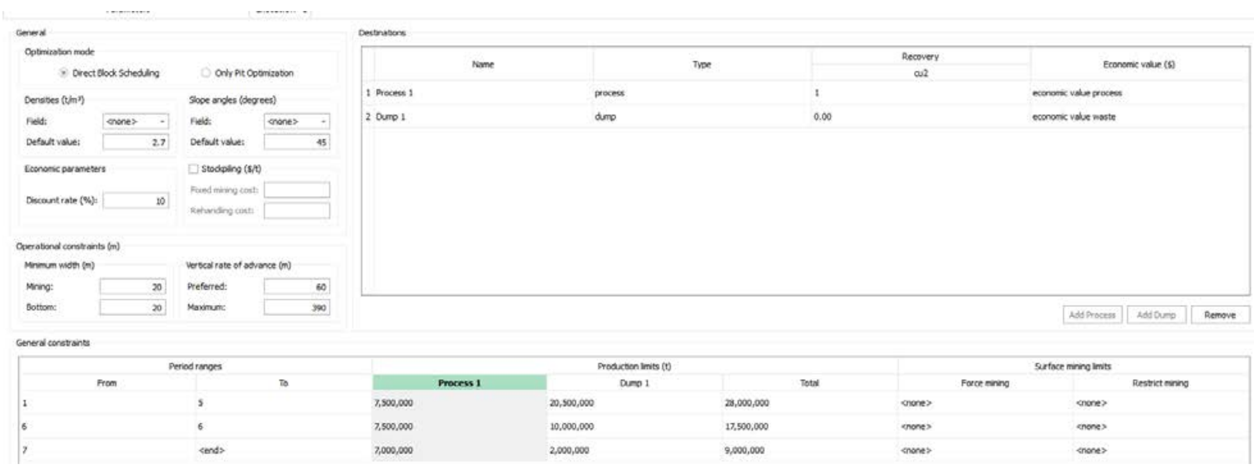

Figure 8. Overview of used parameters for MiningMath SimSched Direct Block Schedule Algorithm.

A cross section from the final deterministic schedule is shown in the Figure 9, which cumulative NPV reached a value of 158.34 Million dollars, the total production deviation was 11.7 million Tons of ore overall LoM, and it can be seen at Figure 11.

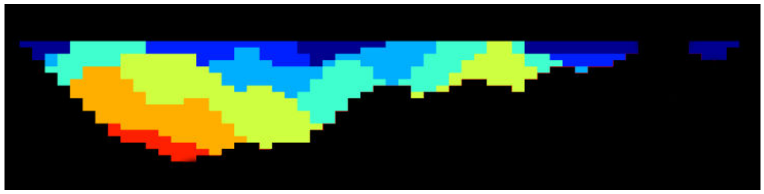

Figure 9. Cross section for deterministic schedule.

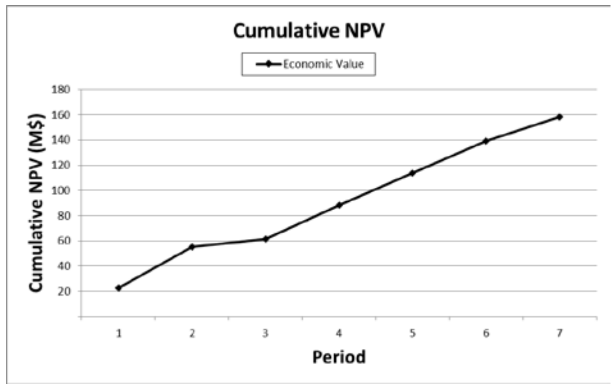

Figure 10. Cumulative NPV for deterministic schedule.

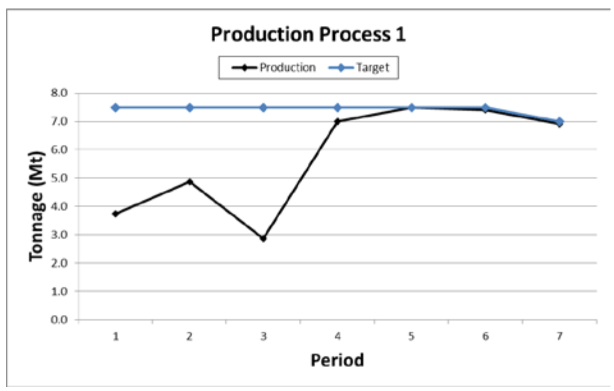

Figure 11. Tonnage of ore produced and production targets overall LoM.

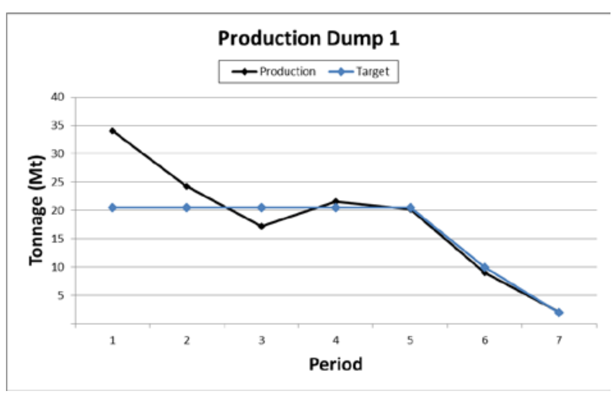

Figure 12. Tonnage of waste produced and production targets overall LoM.

2.3 Stochastic Open pit scheduling optimization

As stated earlier in section 1.3.2, (Leite & Dimitrakopoulos, 2007) approach is divide in a three stage process. This was the selected methodology for the current case of study, it is a simplified version of Godoy et al (2004) that has proved benefits.

Stage 1: the mining rates that must be accomplished every period of the LoM are necessary input for the development of open pit schedules. Table 2 shows the predefined mining rates by the equipment capacity for this case of study.

Stage 2: a set of schedules were generated according the twenty orebody simulations. The schedule optimization formulation proposed at this stage is a period to period programming approach based on MILP formulation from Kumral's NPV maximization formulation (Equation 5) using Gurobi python API(Gurobi Optimization Inc., 2015). The reason to use this formulation is because Equation 5 formulation memory consumption is over 16 Gigabytes (maximum available RAM at GIPLAMIN laboratory). Milawa Algorithm (Whittle, 1999) and MILP formulation have been used to develop initial schedules and the effects of this decision were not significant. In Addition, others researchers like Albor (2010) and Sari (2014), found out that the implications to the final stochastic Schedule were minimum. Algorithm 2 shows the proposed period to period formulation for NPV schedule maximization.

```
#Load Input Information:
val = net_value_All_blocks_sim_i
edges = Precedencies_oreblocks_i
tonore = Tonnage_for_ore_blocks
tonwaste = Tonnage_for_waste_blocks
discrate = discount_rate_array_7periods
lof = range(7)

#Create Model:
m = Model()
n = len(val) # number of blocks

# Decision variable for each block
x={}
for i in range(n):
    x[i] = m.addVar(vtype=GRB.BINARY, name="x%d" %(i))
m.update()
```

```
# Set objective
obj = quicksum(val[i]*x[i]*discrate[0] for i in range(n))
m.setObjective(obj,GRB.MAXIMIZE)

#Load Precedence Constraints:
for edge in edges:
    u = edge[0]
    v = edge[1]
    m.addConstr(x[u]<=x[v])

#Add ore and waste constraints for period 1:
m.addConstr(quicksum(tonore[i]*x[i] for i in
range(n))<=7500000,name="core")
m.addConstr(quicksum(tonwaste[i]*x[i] for i in
range(n))<=20500000,name="cwaste")

m.optimize()

#user must create a function for solution storage named print solution
sol=print_solution()
#store optimized period 1 in final schedule
sch=[]
sch.append(sol)

#function to replace the value of every block extracted in period1 to zero,
this avoid the influence of the blocks in next periods.
def updatesch():
    for i in sol:
        val[i]=0
        tonore[i]=0
        tonwaste[i]=0

updatesch()
#function to change discount rate according the period N=2,3,…,7
def new_obj(N):
    objt=quicksum(val[i]*x[i]*discrate[N] for i in range(n))
    m.setObjective(objt,GRB.MAXIMIZE)

#calculte max NPV extraction for period 2 to 5 because they keep the same
ore-waste constraints
new_obj(1)
m.optimize()
sol=print_solution()
sch.append(sol)
updatesch()
new_obj(2)
m.optimize()
sol=print_solution()
sch.append(sol)
updatesch()
new_obj(3)
m.optimize()
sol=print_solution()
sch.append(sol)
updatesch()
new_obj(4)
m.optimize()
sol=print_solution()
sch.append(sol)
```

```
#Change waste constraint for period 6, ore constraint is the same
m.addConstr(quicksum(tonwaste[i]*x[i] for i in range(n))
<=10000000,name="cwaste")
updatesch()
new_obj(5)
m.optimize()
sol=print_solution()
sch.append(sol)

#Change ore-waste contraints for period 7
m.addConstr(quicksum(tonwaste[i]*x[i] for i in
range(n))<=2000000,name="cwaste")
m.addConstr(quicksum(tonore[i]*x[i] for i in
range(n))<=7000000,name="core")

updatesch()
new_obj(6)
m.optimize()
sol=print_solution()
sch.append(sol)
```

Algorithm 2. Proposed period to period programing approach for open pit scheduling modeled through Gurobi's Python API.

In Appendix 1 is shown the resulting schedules for every simulated orebody. So, these result will be implemented to calculate the seed or initial input for the Stage 3. Figure 6 shows how number of blocks change according the probability to belong to a given mining period. To calculate the seed for the SA algorithm, the blocks with probability of 100% were frozen, and this did not constrain the set of candidate blocks for swapping in the Stage 3. A total of 1524 mineralized blocks were frozen and initially 8429 for swapping.

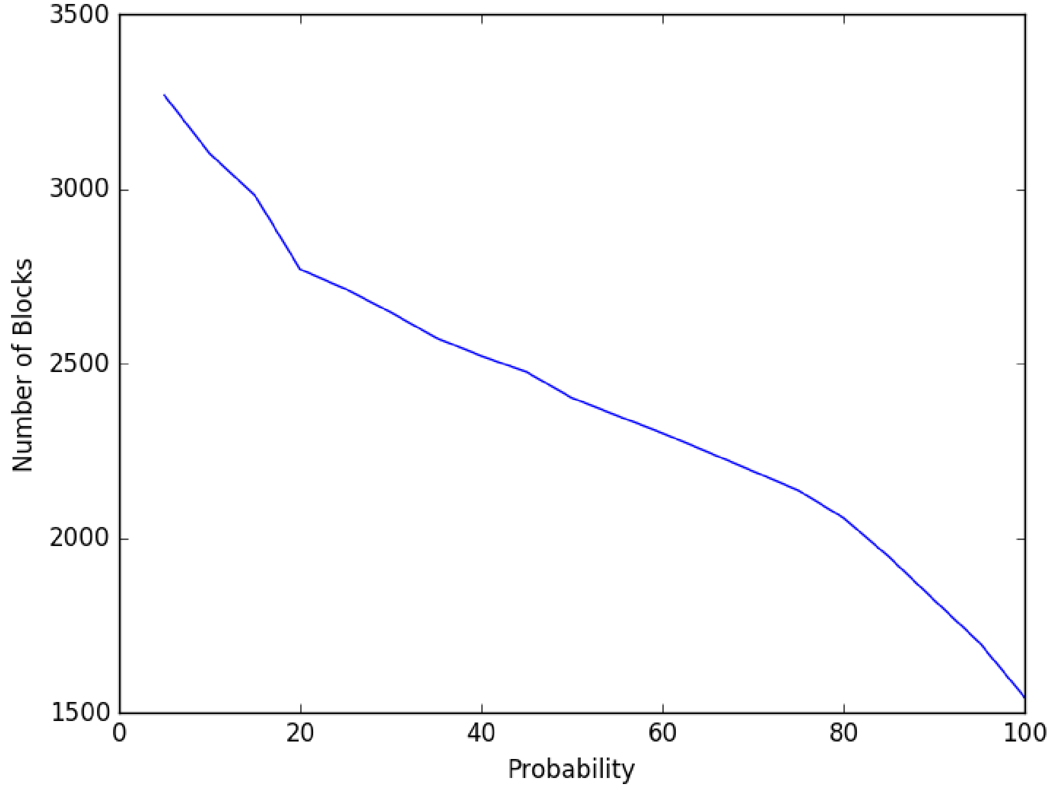

Figure 13. Frozen mineralized blocks according probability.

Stage 3: The selection of the initial mining sequence or seed to start the stochastic optimization process has influence to the achieving time of the final stochastic schedule. Freezing blocks with low probability could led to local minimum. As section 1.2.2.2 explained, to continue the SA algorithm, it is needed to define a perturbation strategy, so that the improvement for new solution is probable.

The perturbation method: In the transition mechanism, a solution is perturbed by swapping or adding blocks that do not belong the set of frozen blocks. The selection of a candidate block is random, so a block could be already in a previous perturbed state or not. This defines the type of transition mechanism, allowing a block to be added or moved to the next or previous period of extraction, if current block period is at the boundaries (Initial or final period), or moved to a randomly selected period. This stochasticity enable the algorithm to test a wider neighborhood of possible solution.

Simulated Annealing Algorithm: the objective function is used to measure the difference between a candidate perturbed schedule and the state schedule. The Equation 8 shows the applied objective function for this research, it is design to minimize the ore and waste deviation from production targets over all simulated orebodies. In addition, a geological discount rate factor is introduce to improve the schedule because early periods are more penalize for deviation of production targets. According to Godoy et al (2004), if a mining sequence achieves that objective for all the equally probable simulated orebody models, there is a 100% chance that the production targets will be met, given the knowledge of the orebody as represented in the simulations. Algorithm 3 the SA optimization was developed to perfectly understand how the decision of accept or reject a perturbation was taken. There were two major stop constraints, the freezing temperature of the system and the maximum numbers of perturbation without change. The Cooling schedule was design to only reduce the current temperature if a perturbation was accepted.

$$Min\ Z = \sum_{t=1}^{T}\left(\sum_{s=1}^{S}|O_t^*(s) - O_t(s)| + \sum_{s=1}^{S}|w_t^*(s) - w_t(s)|\right)G_t$$

Equation 8. Objective function for stochastic optimization.

```
#Function of Simulated Annealing
def  simulatedannealing(seed):
    counter
    T=1
    k=1
    xi=seed
    state=xi

    while T>0:
#Function “move” perturb the current state solution and delivers its objective value (vax)
        xi,vax = move(state, T)
#Objective Function was modeled in Function “OVTon” which inputs are a Schedule and the ore and was production target array
        vastate=OVTon(state,target)
        delta=vax-vastate
        if(vax < vastate):
            state = xi
            vastate=vax
            counter=0
```

```
#Function “update_temperature” reduce the system temperature
            T = update_temperature(T, k)
        else:
#Metropolis Criterion for perturbation acceptance
            p=np.exp(-1*delta/T)
            if np.random.random()<p:
                state=xi
                vastate=vax
                counter=0
                T = update_temperature(T, k)
#if Metropolis Criterion rejects, a counter variable constraint the algorithm to break the process
            else:
                counter+=1
#”k” variable is to understand how many perturbation were made before acceptance or the process is finished
        k += 1
        if counter==100:
            break
    return state
```

Algorithm 3. Simulated annealing formulation programed in Python

Many tries were made before determining the final schedule, thus the suboptimal nature of SA algorithm. Using the best so far schedules a risk analysis overall the equiprobable simulated orebodies was realized, so it can be selected the best option as LoM. Metal production and cumulative NPV were the variable analysed. Figure 14 shows a cross section of the sequence of extraction for both considered solutions. The first schedule presented an average deviation from production targets of 7.5 Million Tons of ore (overall LoM), an expected NPV of 267.6 Million Dollars. On the other hand. The second considered solution average deviation from production target is 12.34 Million Tons of ore (overall LoM) and an expected NPV 257.77 Million Dollars.

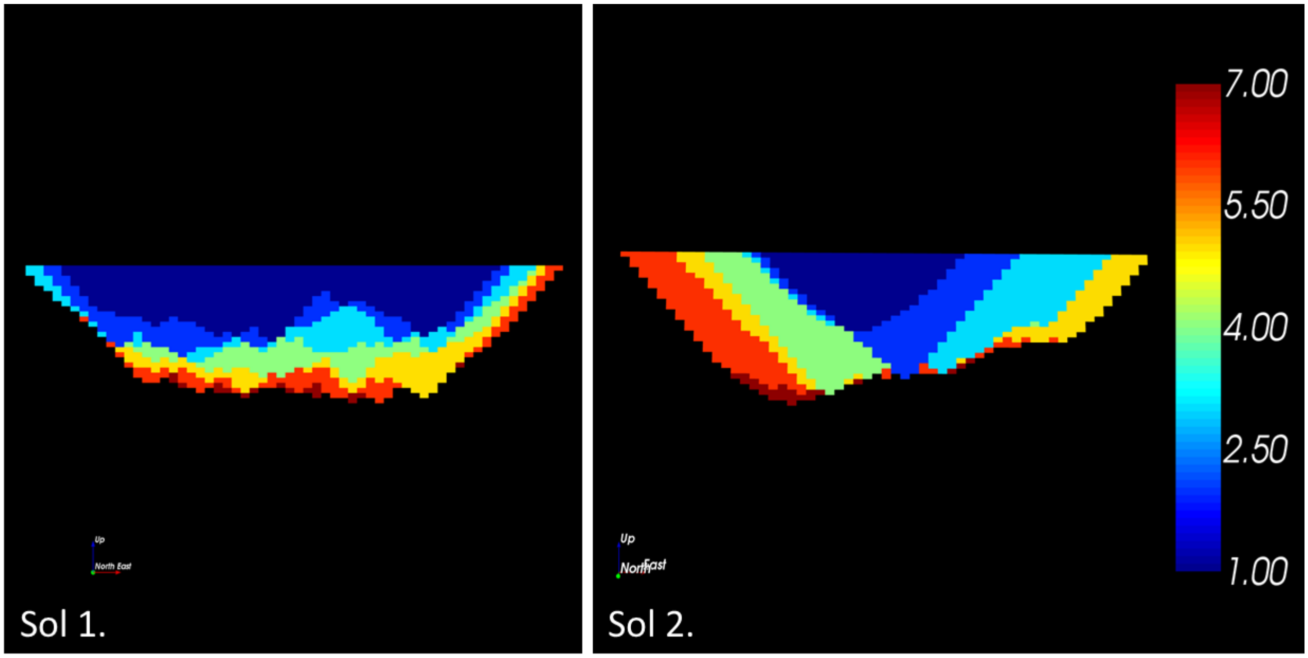

Figure 14. Sequence of extraction for first considered solutions.

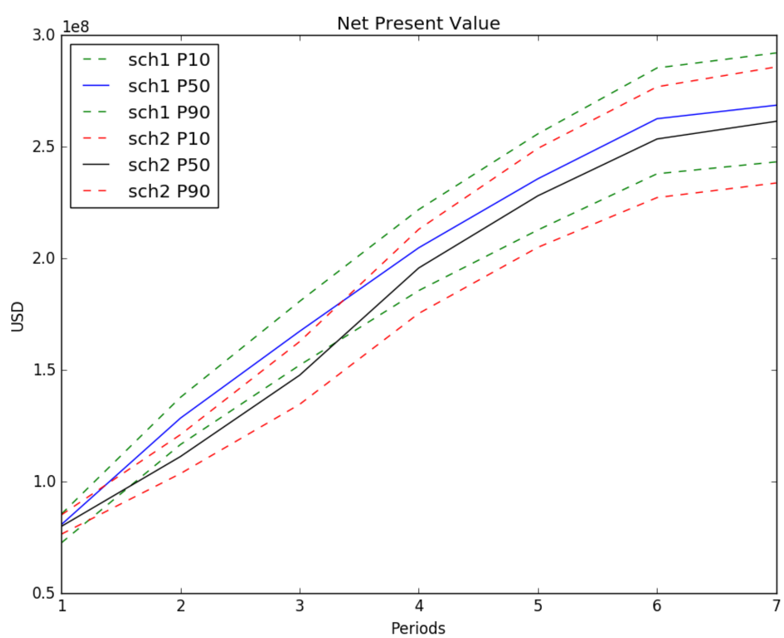

Figure 15. Cumulative NPV of proposed Solution 1 and 2.

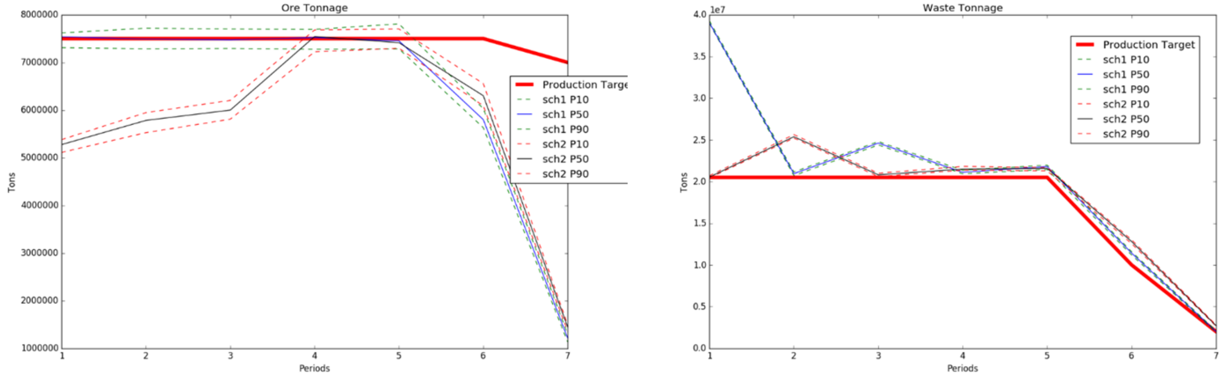

Figure 16. Ore and waste tonnage for solutions 1 and 2.

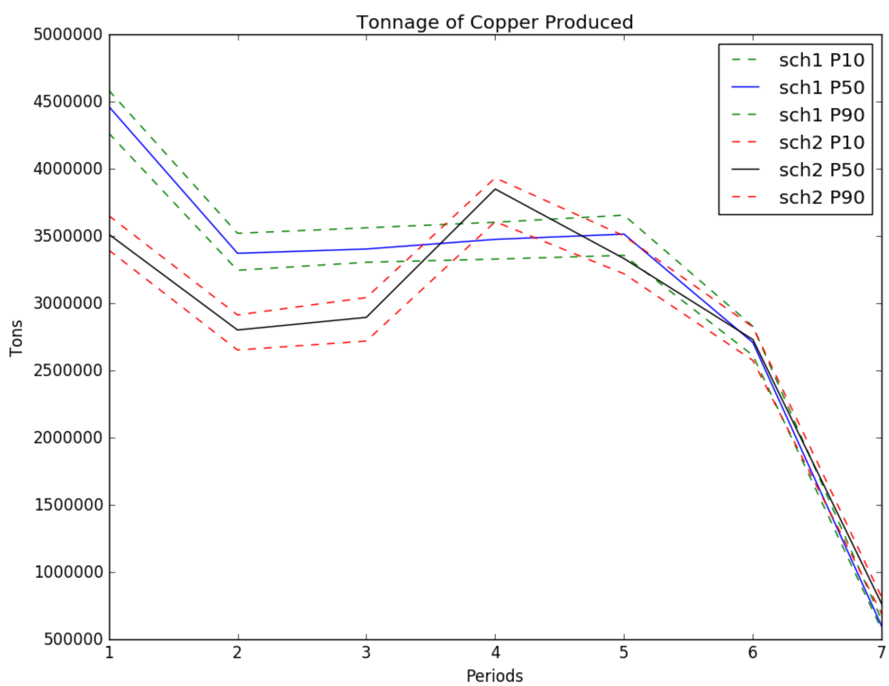

Figure 17. Copper Production of considered solutions 1 and 2.

3. Conclusions and recommendations

3.1 Conclusions

The present study explores the impacts of open pit methods through mining research history. The Lerchs-Grossman Algorithm was overviewed, and the linear programing approaches were applied, understanding the impacts for mine planning between each other. Many schedule approaches were stated, however the deterministic nature of the problem could lead to unprecise solution. While a continuous series of assumptions are made and average values are established, the risk spectrum gets wider, therefore if a company is able to assess that risk, any phase at the mine design process would get added value for future procedures for financial analysis. Covering the impacts of traditional methods, through classical studies, a new field of research is open to improve the long term plans. The stochastic optimization is able to manage the in-sitv geology variabilities that affect directly the profitability of the mining company, in addition, the proposed method showed an increased NPV up to 69% in compared to the traditional schedule. By using this methodology one can evaluate any mine project so a conditional value at risk can be measure before investments and the possible losses are assessed based on a confidence level.

Finally, it is concluded that due the sub optimality of SA algorithm solution depends on mine planner and stakeholders, who guide the best so far solutions according their particular interest. A wide field of modifications for objective value and constraints from SA algorithm is opened to test for improvement at the already found proposed solutions.

3.2 Recommendations

A particular methodology was studied to find a solution for the open pit schedule problem, however, a comparison between industry standard process and the proposed approach should be done to properly show the benefits of the research. The geostatistical estimation and simulation process is a must have for oncoming studies.

Bibliography

Akaike, A., & Dagdelen, K. (1999). A strategic production scheduling method for an open pit mine. In C. Dardano, M. Francisco, & J. Proud (Eds.), *Proceedings of the 28thApplications of Computers and Operations in the Mineral Industries Conference (APCOM)* (pp. 729–738). Golden, CO.

Albor Consuegra, F. R. (2010). *Exploring stochastic optimization in open pit mine design*. Retrieved from http://ezproxy.library.usyd.edu.au/login?url=http://search.proquest.com/docview/817552477?accountid=14757%5Cnhttp://dd8gh5yx7k.search.serialssolutions.com/?ctx_ver=Z39.88-2004&ctx_enc=info:ofi/enc:UTF-8&rfr_id=info:sid/ProQuest+Dissertations+&+Theses+Global

Albor Consuegra, F. R., & Dimitrakopoulos, R. (2009). Stochastic mine design optimisation based on simulated annealing: pit limits, production schedules, multiple orebody scenarios and sensitivity analysis. *Mining Technology : IMM Transactions Section A*, *118*(2), 79–90. https://doi.org/10.1179/037178409X12541250836860

Amankwah, H. (2011). *Mathematical Optimization Models and Methods for Open-Pit Mining*. *Methods*.

Caccetta, L., & Hill, S. P. (1999). Optimization Techniques for open pit mine scheduling.pdf. *International Congress on Modelling and Simulation*.

Caccetta, L., & Hill, S. P. (2003). An application of branch and cut to open pit mine scheduling. In *Journal of Global Optimization* (Vol. 27, pp. 349–365). https://doi.org/10.1023/A:1024835022186

Collet, P., & Rennard, J.-P. (2006). Stochastic Optimization Algorithms. Retrieved March 21, 2015, from http://arxiv.org/ftp/arxiv/papers/0704/0704.3780.pdf

Dagdelen, K., & Johnson, T. B. (1986). Optimum Open Pit Mine Production Scheduling By Lagrangian Parameterization. *Application of Computers and Operations Research in the Mineral Industry - 19th International Symposium.*, 127–142.

Dimitrakopoulos, R. (1998). Conditional simulation algorithms for modelling orebody uncertainty in open pit optimisation. *International Journal of Surface Mining, Reclamation and Environment*, *12*(4), 173–179. https://doi.org/10.1080/09208118908944041

Dimitrakopoulos, R. (2011). Stochastic optimization for strategic mine planning: A decade of developments. *Journal of Mining Science*, *47*(2), 138–150.

Dimitrakopoulos, R., Farrelly, C. T., & Godoy, M. (2002). Moving forward from traditional optimization. *AIME Transactions*, *111*(April), A82–A88.

Espinoza, D., Goycoolea, M., Moreno, E., & Newman, A. (2012). MineLib: A Library of Open Pit Mining Problems.

Espinoza, D., Goycoolea, Moreno, & Newman. (2012). MineLib file format specification V1.0.

Fourer, R., Gay, D. M., Hill, M., Kernighan, B. W., & T Bell Laboratories. (1990). AMPL : A Mathematical Programming Language. *Management Science*, *36*, 519–554. https://doi.org/10.1007/BF01783416

Godoy, M., & Dimitrakopoulos, R. (2004). Managing risk and waste mining in long-term production scheduling of open-pit mines. *SME Transactions*, *316*(3), 43–50.

Goodfellow, R. (2014). *Unified Modelling and Simultaneous Optimization of Open Pit Mining Complexes with Supply Uncertainty*.

Goovaerts, P. (1999). Geostatistics for Natural Resources Evaluation. *Journal of Environment Quality*, *28*(3), 1044. https://doi.org/10.2134/jeq1999.00472425002800030046x

Gurobi Optimization Inc. (2015). Gurobi Optimizer Reference Manual. Retrieved from http://www.gurobi.com

Hochbaum, D. S. (2008). The Pseudoflow Algorithm: A New Algorithm for the Maximum-Flow Problem. *Operations Research*, *56*(4), 992–1009. https://doi.org/10.1287/opre.1080.0524

Johnson, T. (1969). Optimum production scheduling. In *Proceedings of 8th APCOM symp., Salt Lake City, Utah* (pp. 539 – 562).

Kawahata, K. (2006). *A new algorithm to solve large scale mine production scheduling problems by using the Lagrangian relaxation method*. s, Colorado School of Mines.

Kirkpatrick, S., Gelatt, C. D., & Vecchi, M. P. (1983). Optimization by simulated annealing. *Science (New York, N.Y.)*, *220*(4598), 671–80. https://doi.org/10.1126/science.220.4598.671

Kumral, M. (2011). Incorporating geometallurgical information into mine production scheduling. *Journal of the Operational Research Society*, *62*(1), 60–68. https://doi.org/10.1057/jors.2009.174

Kumral, M. (2012). Production planning of mines: Optimisation of block sequencing and destination. *International Journal of Mining, Reclamation and Environment*, *26*(2), 93–103. https://doi.org/10.1080/17480930.2011.644474

Kumral, M., & Dowd, P. A. (2005). A simulated annealing approach to mine production scheduling. *Journal of the Operational Research Society*, *56*(8), 922–930. Retrieved from http://www.jstor.org/stable/4102063?seq=1&cid=pdf-reference#page_scan_tab_contents

Leite, a., & Dimitrakopoulos, R. (2007). Stochastic optimisation model for open pit mine planning: application and risk analysis at copper deposit. *Mining Technology : IMM Transactions Section A*, *116*(3), 109–118. https://doi.org/10.1179/174328607X228848

Lerchs, H., & Grossmann, I. F. (1965). Optimum Design of Open-Pit Mines. *Transactions, Canadian Institute of Mining*, *68*(1), 17–24.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. (1953). Equation of state calculations by fast computing machines. *Journal Chemical Physics*, *21*(6), 1087–1092. https://doi.org/http://dx.doi.org/10.1063/1.1699114

Newman, A. M., Rubio, E., Caro, R., Weintraub, A., & Eurek, K. (2010). A review of operations research in mine planning. *Interfaces*, *40*(3), 222–245. https://doi.org/10.1287/inte.1090.0492

Picard, J.-C. (1976). Maximal Closure of a Graph and Applications to Combinatorial Problems. *Management Science*, *22*(11), 1268–1272. https://doi.org/10.1287/mnsc.22.11.1268

Ravenscroft, P. J. (1992). Risk analysis for mine scheduling by conditional simulation. *Trans. Inst. Min. Metall. (Sec. A: Min. Industry)*, *101*. https://doi.org/10.1016/0148-9062(93)90969-K

Sari, Y. A. (2014). *Mine Production Scheduling through Heuristic Memory Based , Improved Simulated Annealing*.

Sattarvand, J. (2009). Long-term Open pit Planning by Ant Colony Optimization, 125. Retrieved from http://publications.rwth-aachen.de/record/62878/files/3906.pdf

Soltani, F., Afzal, P., & Asghari, O. (2013). Sequential Gaussian Simulation in the Sungun Cu Porphyry Deposit and Comparing the Stationary Reproduction with Ordinary Kriging. *Universal Journal of Geoscience*, 106–113. https://doi.org/10.13189/ujg.2013.010210

Thomas, G. S. (1996). Optimisation and Scheduling of Open Pits via Genetic Algorithms and Simulated Annealing. In *1st International Symposium of Mine Simulation via Internet*.

Voß, S., Martello, S., Osman, I. H., & Roucairol, C. (2012). *Meta-Heuristics: Advances and Trends in Local Search Paradigms for Optimization*. Springer US. Retrieved from https://books.google.com.co/books?id=6CzoBwAAQBAJ

Weintraub, A., Romeroes, C., Bjørndal, T., & Epstein, R. (2007). *Handbook Of Operations Research In Natural Resources*. https://doi.org/10.1007/978-0-387-71815-6

Whittle, J. (1999). A decade of open pit mine planning and optimization - The craft of turning algorithms into packages, 15–24. Retrieved from http://espace.library.uq.edu.au/view/UQ:149936#.VjgPA9EmIsc.mendeley

A. Appendix: Sequences of extraction for orebody simulations

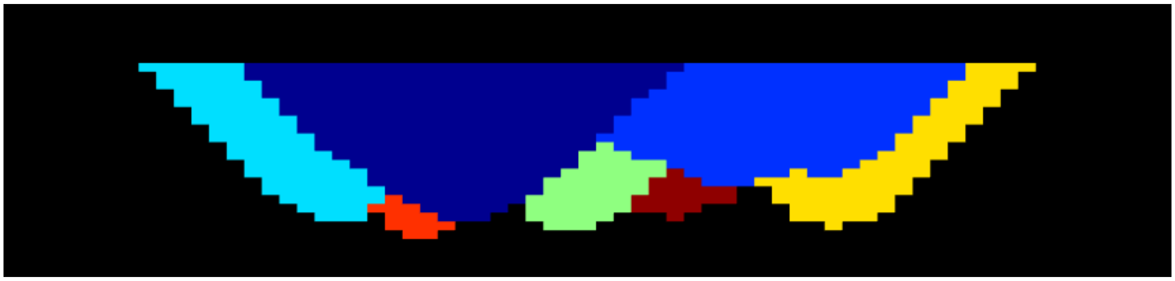

Figure A1. Dynamic programing Schedule for orebody 1 simulation.

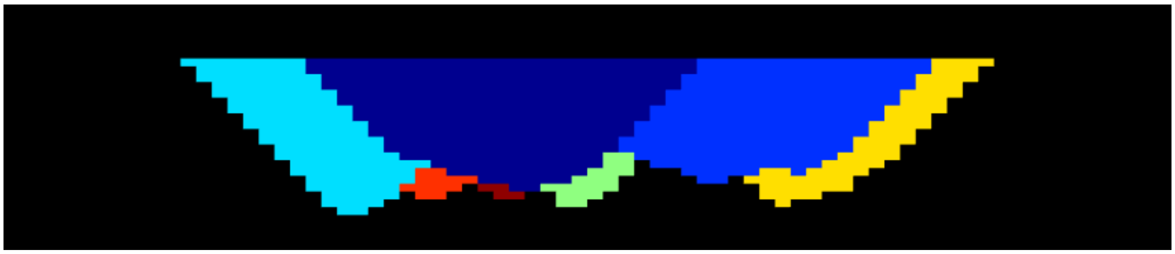

Figure A2. Dynamic programing Schedule for orebody 2 simulation.

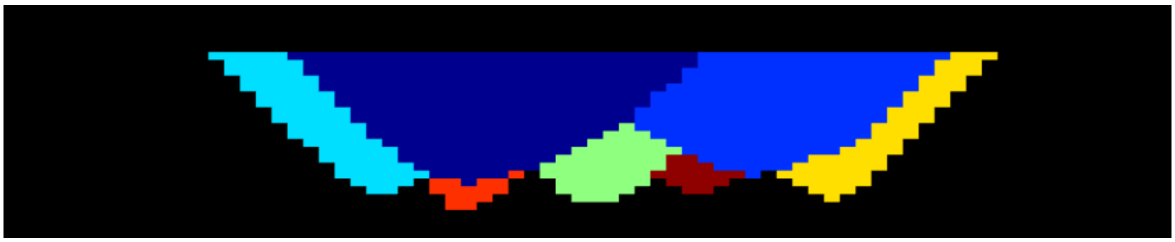

Figure A3. Dynamic programing Schedule for orebody 2 simulation.

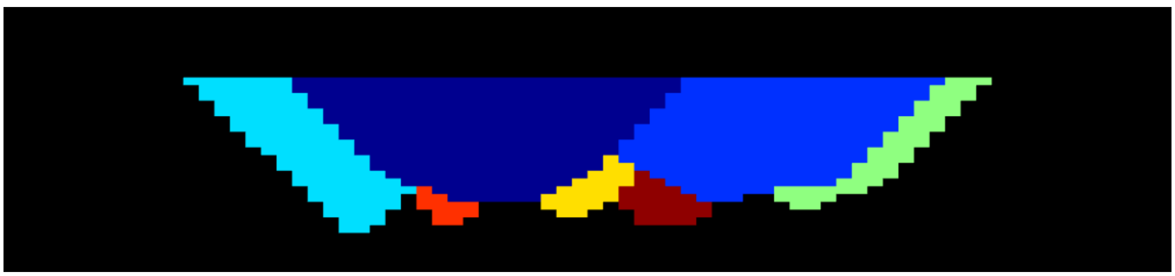

Figure A4. Dynamic programing Schedule for orebody 4 simulation.

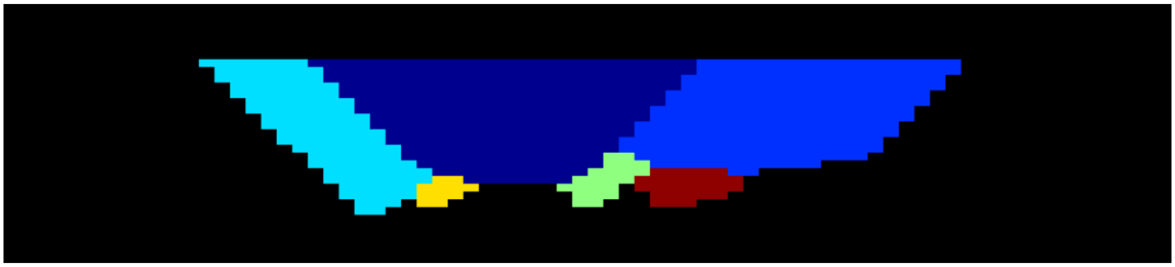

Figure A5. Dynamic programing Schedule for orebody 5 simulation.

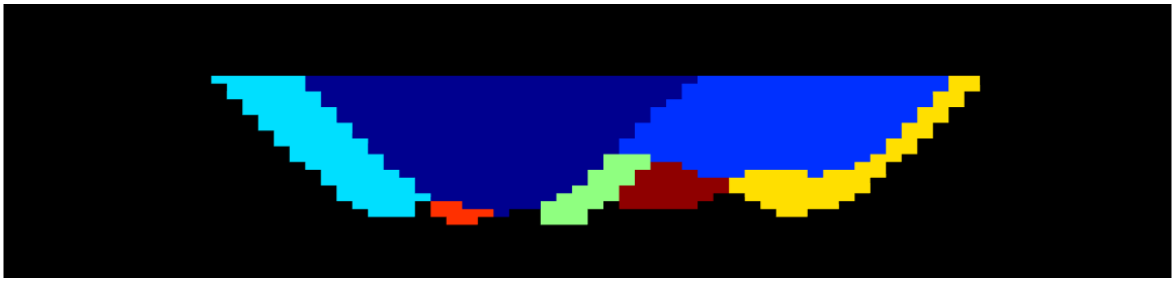

Figure A6. Dynamic programing Schedule for orebody 6 simulation.

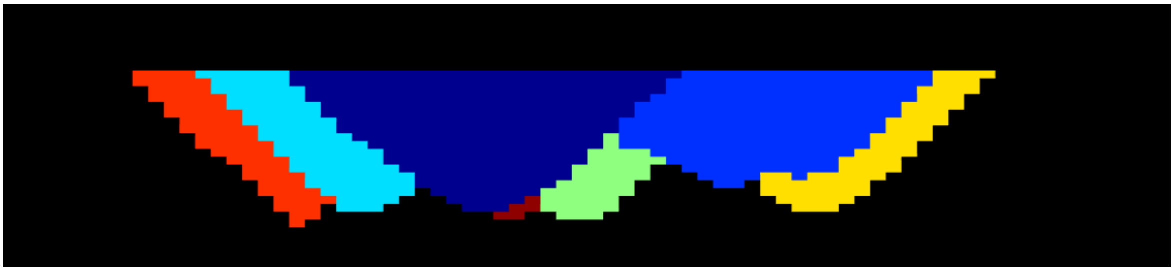

Figure A7. Dynamic programing Schedule for orebody 7 simulation.

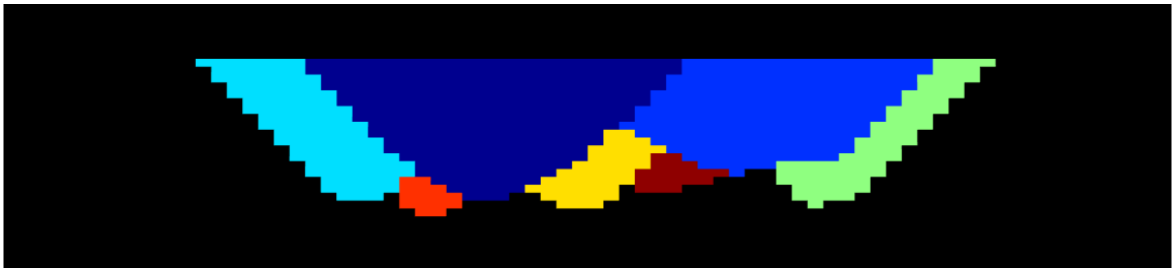

Figure A8. Dynamic programing Schedule for orebody 20 simulation.

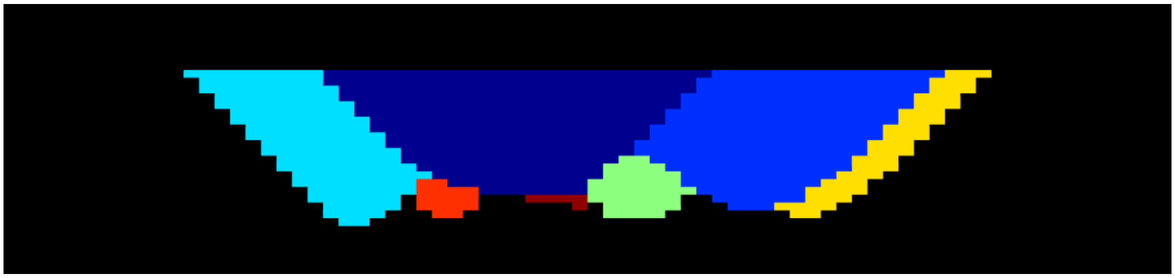

Figure A9. Dynamic programing Schedule for orebody 9 simulation.

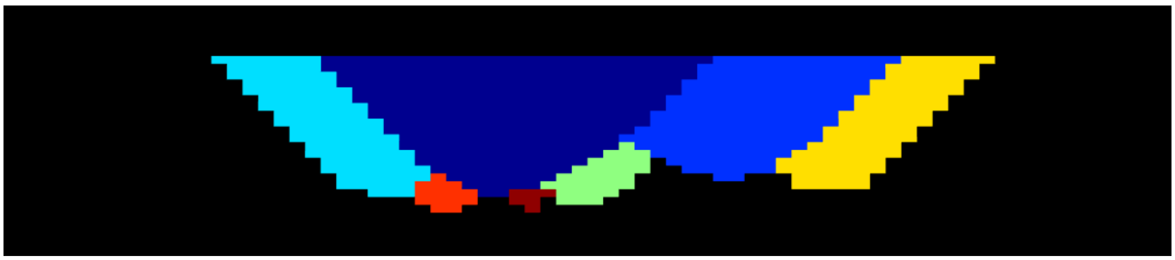

Figure A10. Dynamic programing Schedule for orebody 10 simulation.

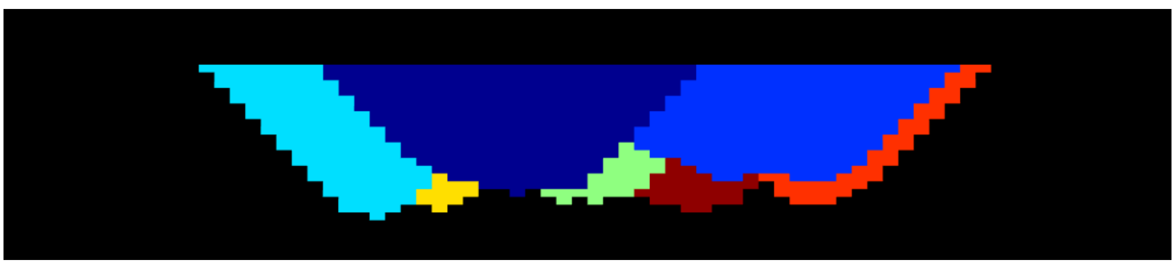

Figure A11. Dynamic programing Schedule for orebody 11 simulation.

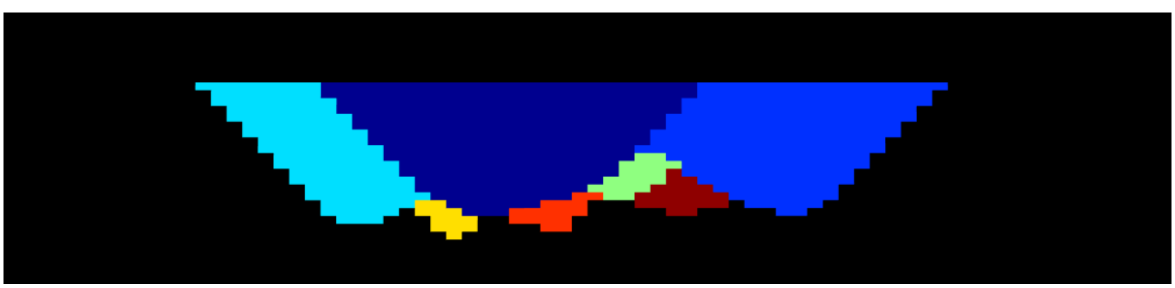

Figure A12. Dynamic programing Schedule for orebody 12 simulation.

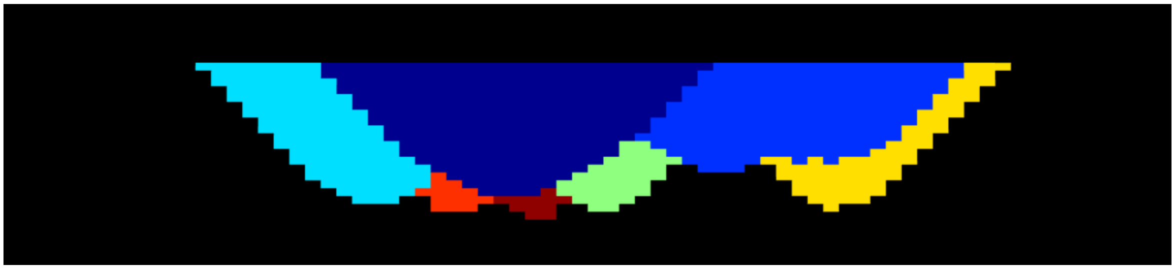

Figure A13. Dynamic programing Schedule for orebody 13 simulation.

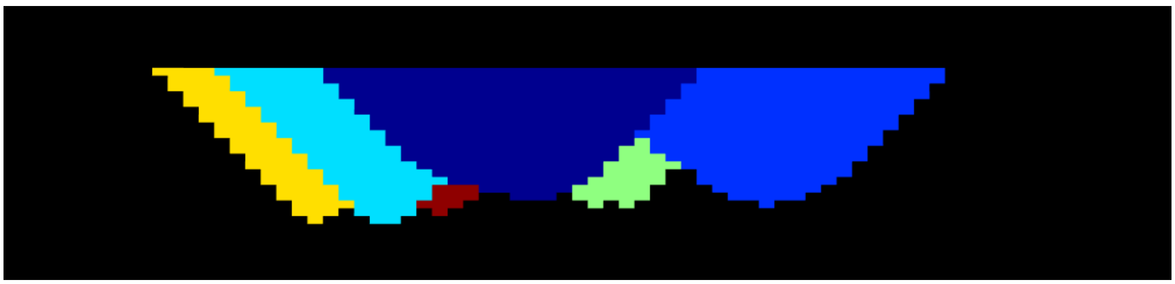

Figure A14. Dynamic programing Schedule for orebody 14 simulation.

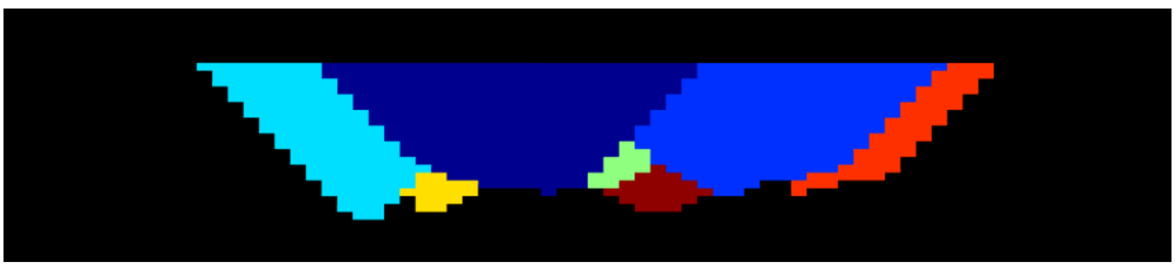

Figure A15. Dynamic programing Schedule for orebody 15 simulation.

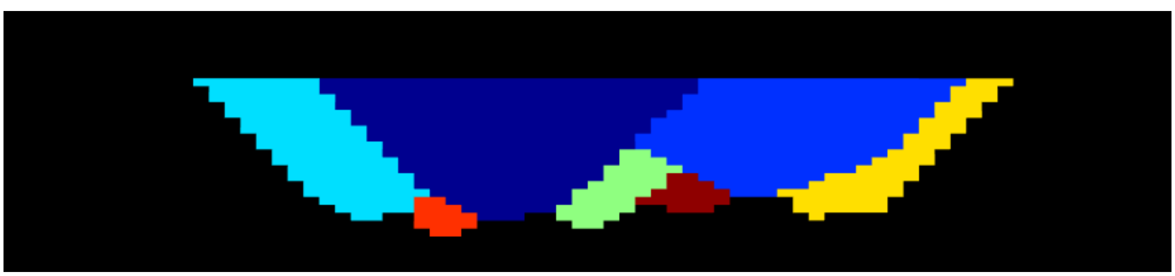

Figure A16. Dynamic programing Schedule for orebody 16 simulation.

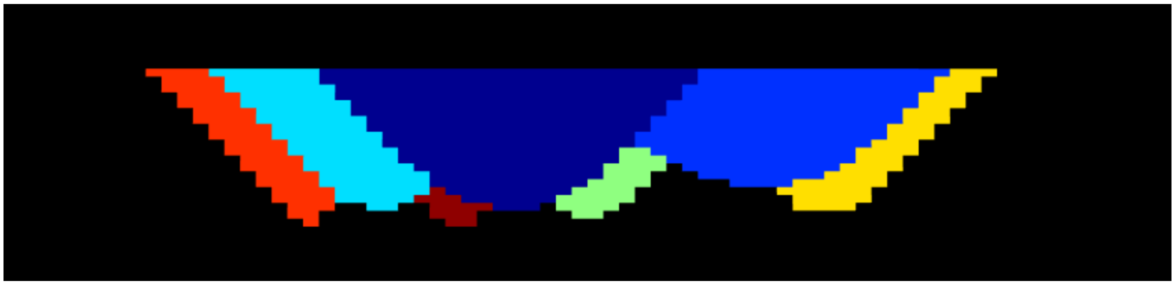

Figure A17. Dynamic programing Schedule for orebody 17 simulation.

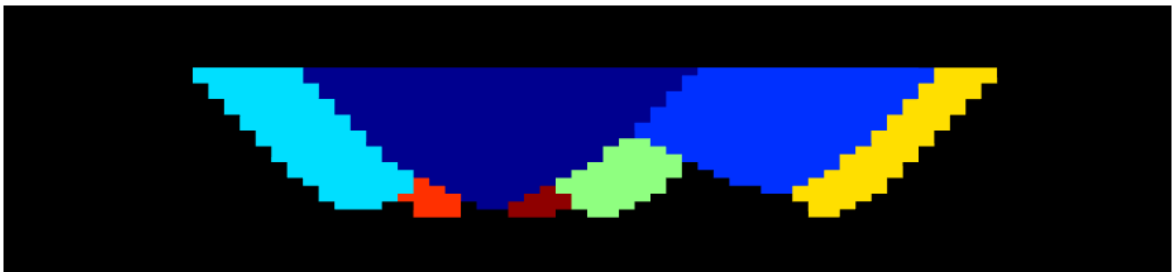

Figure A18. Dynamic programing Schedule for orebody 18 simulation.

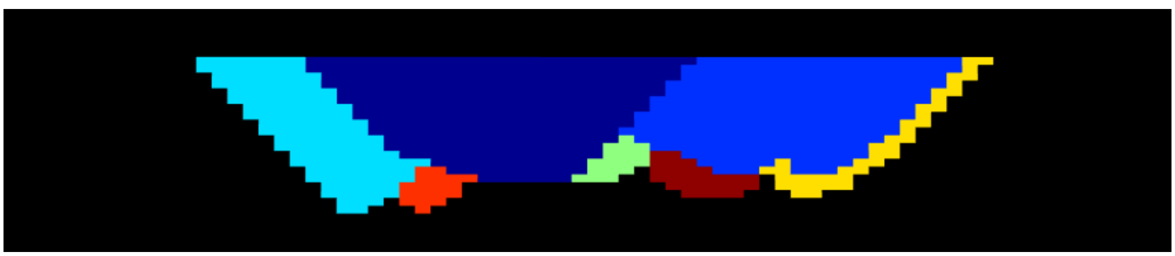

Figure A19. Dynamic programing Schedule for orebody 19 simulation.

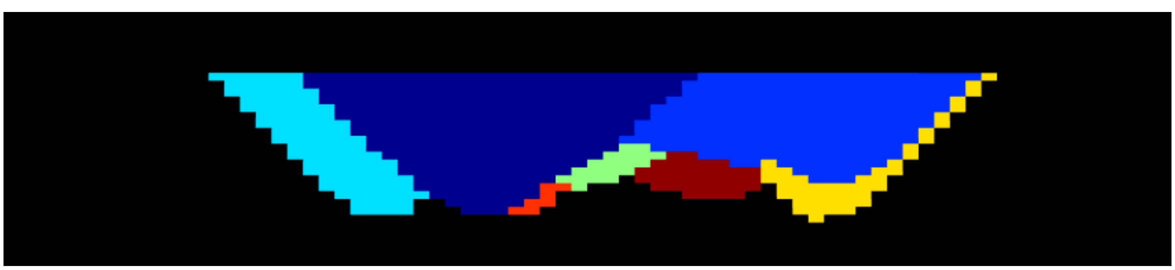

Figure A20. Dynamic programing Schedule for orebody 20 simulation.