On the space-time admitting some geometric structures on energy-momentum tensors

Sobre el espacio-tiempo admitiendo algunas estructuras geométricas en tensores de energía-momento

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Abstract. This paper presents a study of a general relativistic perfect fluid space-time admitting various types of curvature restrictions on energy-momentum tensors and brings out the conditions for which fluids of the space–time are sometimes phantom barrier and some other times quintessence barrier. The existence of a space–time where fluids behave as phantom barrier is ensured by an example.

Key words and phrases. General relativistic perfect fluid space-time, Einstein’s field equation, energy-momentum tensor, semi-symmetric energy-momentum tensor.

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Resumen. Este artículo presenta un estudio del tiempo-espacio fluido perfecto relativista general admitiendo varios tipos de restricciones de curvatura en los tensores de energía-momento y saca a relucir las condiciones para las cuales los fluidos del espacio-tiempo son a veces barrera fantasma y otras veces barrera de quintaesencia. La existencia de un espacio-tiempo donde los líquidos se comportan como barrera fantasma es garantizado por un ejemplo.

Palabras y frases clave. Espacio-tiempo fluido general relativista perfecto, campo de Einstein, tensor energía-momento, tensor semi-simétrico de energía-momento.
1. Introduction

Recently, in tune with Yano and Sawaki [16], Baishya and Roy Chowdhury [4] introduced and studied quasi-conformal curvature tensors in the frame of $N(k, \mu)$-manifolds. The generalized quasi-conformal curvature tensor is defined for $n$ dimensional manifolds as

$$W(X,Y)Z = \frac{n-2}{n} [1 + (n-1)(a-b) - (1 + (n-1)(a+b))c] C(X,Y)Z + [1 - b + (n-1)a] E(X,Y)Z + (n-1)(b-a) P(X,Y)Z + \frac{n-2}{n} (c-1)(1 + (n-1)(a+b)) \hat{C}(X,Y)Z$$

for all $X,Y,Z \in \chi(M)$, the set of all vector fields of the manifold $M$, where scalars $a, b, c$ are real constants. The beauty of such curvature tensors lies on the fact that it has the flavour of

(i) Riemannian curvature tensors $R$ if the scalar triple $(a, b, c) \equiv (0, 0, 0)$,

(ii) Conformal curvature tensors $C$ [9] if $(a, b, c) \equiv (-\frac{1}{n-2}, -\frac{1}{n-2}, 1)$,

(iii) Conharmonic curvature tensors $\hat{C}$ [10] if $(a, b, c) \equiv (-\frac{1}{n-2}, -\frac{1}{n-2}, 0)$,

(iv) Concircular curvature tensors $E$ [15, p. 84] if $(a, b, c) \equiv (0, 0, 1)$,

(v) Projective curvature tensors $P$ [15, p. 84] if $(a, b, c) \equiv (-\frac{1}{n-1}, 0, 0)$,

(vi) $m$-projective curvature tensors $H$ [12] if $(a, b, c) \equiv (-\frac{1}{2n-2}, -\frac{1}{2n-2}, 0)$.

Note that (1) can also be written as

$$W(X,Y)Z = R(X,Y)Z + a[S(Y,Z)X - S(X,Z)Y] + b[g(Y,Z)QX - g(X,Z)QY] - \frac{cn}{n} \left(\frac{1}{n-1} + a + b\right) [g(Y,Z)X - g(X,Z)Y].$$

The space-time under various curvature restrictions is a subject of vast literature, e.g., [1, 5, 7, 8] and the references there in.

In analogy with [14], an energy-momentum tensor $T$ of type $(0,2)$ is said to be semi-symmetric type if

$$W(X,Y) \cdot T = 0$$

holds where $W(X,Y)$ acts on $T$ as a derivation.

The paper is structured as follows. Section 2 is concerned with general relativistic perfect fluid space-time (briefly GRPFS) obeying Einstein’s equation with $W(X,Y) \cdot T = 0$. It is observed that a fluid of such space-time
always behaves as phantom barrier for each of the restrictions \( E(X,Y) \cdot T = 0 \) and \( H(X,Y) \cdot T = 0 \) whereas the same behaves either as a phantom barrier or quintessence barrier for each of the restrictions \( R(X,Y) \cdot T = 0 \) and \( \widehat{C}(X,Y) \cdot T = 0 \). A detailed study of \( GRPFS \) obeying Einstein’s equation admitting \( ((X \wedge_S Y) \cdot W) = 0 \) and \( (X \wedge_T Y) = 0 \), where the endomorphism is defined as \( (X \wedge_B Y)Z = B(Y,Z)X - B(X,Z)Y \), has been carried out in Section 3 and 4 respectively, with similar types of results as in Section 2. Finally, we give an example in Section 5 of a fluid whose character is phantom barrier.

2. \( GRPFS \) with semi-symmetric type energy momentum tensors

Einstein’s equation can be written as

\[
S = kT + \frac{r}{2}g,
\]

where \( k \) is the gravitational constant and \( r \) is the scalar curvature. Let \( (M^4, g) \) be a \( GRPFS \) with (3). Now (3) implies that

\[
T(W(X,Y)U,V) + T(U,W(X,Y)V) = 0.
\]

In view of (4) and (5), we have

\[
((W(X,Y) \cdot S)U,V) = \frac{r}{2}((W(X,Y) \cdot g)U,V) = 0.
\]

In consequence of the above, we have the following:

**Proposition 2.1.** A general relativistic space-time with a semi-symmetric type energy-momentum tensor is Ricci semi-symmetric type and vice-versa.

By [5, Theorem 1, p 1029], we can easily bring out the following:

**Proposition 2.2.** A general relativistic space-time with a covariant constant energy-momentum tensor is Ricci semi-symmetric type.

By virtue of a result of Aikawa and Matsuyama [2] if a tensor field \( L \) is recurrent or birecurrent, then \( R(X,Y) \cdot L = 0 \). Hence we have the following:

**Theorem 2.3.** A general relativistic space-time with a recurrent or birecurrent energy-momentum tensor is always Ricci semi-symmetric type.

Next we consider a perfect fluid space-time whose energy-momentum tensor is semi-symmetric type. An energy-momentum tensor is said to describe a perfect fluid [11] if

\[
T(X,Y) = (\sigma + \rho)A(X)A(Y) + \rho g(X,Y),
\]

\( \sigma \) and \( \rho \) being the pressure and energy density respectively.

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where $\sigma$ is the energy density, $\rho$ is the isotropic pressure of the fluid and $A$ is a non-zero 1-form such that $g(X, \theta) = A(X)$ for all $X$, $\theta$ being the velocity vector field of the fluid which is a time-like vector that is,

$$g(\theta, \theta) = A(\theta) = -1.$$  

By virtue of (4) and (6), we get

$$S(X,Y) = \frac{k(\sigma - \rho)}{2}g(X,Y) + k(\sigma + \rho)A(X)A(Y). \quad (7)$$

In view of (2) and (7), we have

$$W(X,Y)Z = R(X,Y)Z + k \left[ \frac{a + b}{2} - \frac{c(\sigma - 3\rho)}{4} \left( \frac{1}{3} a + b \right) \right] [g(Y,Z)X - g(X,Z)Y]$$

$$+ ak(\sigma + \rho)A(Z)[A(Y)X - A(X)Y]$$

$$+ bk(\sigma + \rho)[g(Y,Z)A(X) - g(X,Z)A(Y)]\theta. \quad (8)$$

As consequences of (5) and (6), it follows that

$$0 = (\sigma + \rho)[A(W(X,Y)U)A(V) + A(W(X,Y)V)A(U)]$$

$$+ \rho[g(W(X,Y)U,V) + g(W(X,Y)V,U)] \quad (9)$$

which yields

$$-\sigma A(W(X,Y)U) + \rho g(W(X,Y)\theta, U) = 0 \quad (10)$$

for $V = \theta$ which in turn on contraction gives

$$0 = k \left[ \frac{\sigma + 3\rho}{2} \{ \sigma(1 - a + 3b) + \rho(1 - b + 3a) \} \right]$$

$$- (\sigma - 3\rho) \left\{ (a\sigma + \rho b) - \frac{c(\sigma + \rho)(1 + 3a + 3b)}{4} \right\}. \quad (11)$$

From (11), one can easily bring out the following table by substituting the triple $(a, b, c)$ by $(0, 0, 0), (-\frac{1}{2}, -\frac{1}{2}, 1)$, etc.

<table>
<thead>
<tr>
<th>Curvature restrictions</th>
<th>Relations between $\sigma$ &amp; $\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(X,Y) \cdot T = 0$</td>
<td>$\sigma + \rho = 0$ or $\sigma + 3\rho = 0$</td>
</tr>
<tr>
<td>$C(X,Y) \cdot T = 0$</td>
<td>$\sigma, \rho$ are independent</td>
</tr>
<tr>
<td>$C'(X,Y) \cdot T = 0$</td>
<td>$\sigma + \rho = 0$ or $\sigma - 3\rho = 0$</td>
</tr>
<tr>
<td>$E(X,Y) \cdot T = 0$</td>
<td>$\sigma + \rho = 0$</td>
</tr>
<tr>
<td>$P(X,Y) \cdot T = 0$</td>
<td>$\sigma + \rho = 0$ or $\sigma = 0$</td>
</tr>
<tr>
<td>$H(X,Y) \cdot T = 0$</td>
<td>$\sigma + \rho = 0$</td>
</tr>
</tbody>
</table>
Now $\sigma + \rho = 0$ means that the fluid behaves as a cosmological constant [13]. This is also termed as phantom barrier [6]. In cosmology, a choice $\sigma = -\rho$ leads to a rapid expansion of the space-time which is now termed as inflation. Also $\sigma + 3\rho = 0$ or $(\sigma - 3\rho = 0)$ is known as the quintessence barrier. Here the strong energy condition begins to be violated. The present observations indicate that our universe is in quintessence era [3]. Thus from the above discussion we can state the following:

**Theorem 2.4.** Let $(M^4, g)$ be a GRPFS obeying Einstein’s equation admitting $C(X, Y) \cdot T = 0$. Then the density of the matter and pressure are independent.

**Theorem 2.5.** The behavior of fluids in GRPFS obeying Einstein’s equation is always phantom barrier for each of the restrictions $E(X, Y) \cdot T = 0$ and $H(X, Y) \cdot T = 0$.

**Theorem 2.6.** The behavior of a fluid in GRPFS obeying Einstein’s equation is either phantom barrier or quintessence barrier for each of the restrictions $R(X, Y) \cdot T = 0$ or $\hat{C}(X, Y) \cdot T = 0$.

### 3. GRPFS satisfying $(X \wedge_S Y) \cdot W)(Z, U)V = 0$

Let us consider a perfect fluid space-time satisfying $(X \wedge_S Y) \cdot W)(Z, U)V = 0$, i.e.,

\[
0 = (X \wedge_S Y)W(Z, U)V + W((X \wedge_S Y)Z, U)V + W(Z, (X \wedge_S Y)U)V + W(Z, U)(X \wedge_S Y)V,
\]

where the endomorphism $(X \wedge_S Y)Z$ is defined as

\[
(X \wedge_S Y)Z = S(Y, Z)X - S(X, Z)Y.
\]

In view of (13), (12) becomes

\[
\]

Replacing $Y$ and $V$ by $\theta$, we get

\[
0 = (\sigma - \rho)g(X, W(Z, U)\theta) - (\sigma + 3\rho)A(W(Z, U)X),
\]

which on contraction yields

\[
0 = \frac{k(\sigma + 3\rho)}{2}\left[[(\sigma + 3\rho)(1 - a + 3b) + (\sigma - \rho)(1 - b + 3a)]
\right. \\
\left. - (\sigma - 3\rho)(\sigma - \rho)(1 + 3a + 3b)\right].
\]

Consequently, from (16) one can easily bring out the following:
Curvature restrictions & Relations between $\sigma$ & $\rho$

| $((X \wedge S Y) \cdot R)(Z, U)V = 0$ | $\sigma + \rho = 0$ or $\sigma + 3\rho = 0$ |
| $((X \wedge S Y) \cdot C)(Z, U)V = 0$ | $\sigma, \rho$ are independent |
| $((X \wedge S Y) \cdot C)(Z, U)V = 0$ | $\sigma + \rho = 0$ or $\sigma - 3\rho = 0$ |
| $((X \wedge S Y) \cdot E)(Z, U)V = 0$ | $\sigma + \rho = 0$ |
| $((X \wedge S Y) \cdot P)(Z, U)V = 0$ | $\sigma + \rho = 0$ or $\sigma + 3\rho = 0$ |
| $((X \wedge S Y) \cdot H)(Z, U)V = 0$ | $\sigma + \rho = 0$ |

**Theorem 3.1.** The behavior of fluids in GRPFS obeying Einstein’s equation is always phantom barrier for each of the restrictions $(X \wedge S Y) \cdot E = 0$ and $(X \wedge S Y) \cdot H = 0$.

**Theorem 3.2.** The behavior of fluids in GRPFS obeying Einstein’s equation is either phantom barrier or quintessence barrier for each of the restrictions $(X \wedge S Y) \cdot R = 0$, $(X \wedge S Y) \cdot C = 0$, and $(X \wedge S Y) \cdot P = 0$.

4. The Perfect fluid space-time satisfying

Let us consider the perfect fluid space-time satisfying $((X \wedge T Y) \cdot W) = 0$.

i.e., $0 = (X \wedge T Y) W(Z, U)V + W((X \wedge T Y)Z, U)V + W(Z, (X \wedge T Y)U)V + W(Z, U)(X \wedge T Y)V$,

where the endomorphism $(X \wedge T Y)Z$ is defined as

$$ (X \wedge T Y)Z = T(Y, Z)X - T(X, Z)Y. \quad (18) $$

In consequence of (18), (17) becomes


In view of (6) and (19), we have

$$ 0 = (\sigma + \rho)[A(W(X, U)V)A(Y)A(Z) - A(W(Y, U)V)A(X)A(Z)$$

$$ + A(W(Z, X)V)A(Y)A(U) - A(W(Z, Y)V)A(X)A(U)$$

$$ + A(W(Z, U)V)A(Y)A(V) - A(W(Z, U)V)A(X)A(V)]$$

$$ + \rho[W(Z, U,V)A(X) - W(Z, U,V)A(X)A(Y)$$

$$ + g(Y, Z)A(W(X, U)V) - g(X, Z)A(W(Y, U)V)$$

$$ + g(Y, U)A(W(Z, X)V) - g(X, U)A(W(Z, Y)V)$$

$$ + g(Y, V)A(W(Z, U)V) - g(X, V)A(W(Z, U)V)]. \quad (20) $$
Replacing $Y$ and $V$ by $\theta$ in (20), we get

$$\sigma A(W(Z,U)X) + \rho g(W(Z,U)\theta, X) = 0$$

which on contraction gives

$$0 = -\frac{k(\sigma + 3\rho)}{2} [(\sigma(1 - a + 3b) - \rho(1 - b + 3a))] + k(\sigma - 3\rho)[(a\sigma - b\rho)$$

$$- \frac{c}{4}(\sigma - \rho)(1 + 3a + 3b)].$$

From the above one can easily bring out the following:

<table>
<thead>
<tr>
<th>Curvature Restrictions</th>
<th>Relations between $\sigma$ &amp; $\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$((X \wedge_T Y) \cdot R)(Z,U)V = 0$</td>
<td>$\sigma - \rho = 0$ or $\sigma + 3\rho = 0$</td>
</tr>
<tr>
<td>$((X \wedge_T Y) \cdot C)(Z,U)V = 0$</td>
<td>$\sigma$, $\rho$ are independent</td>
</tr>
<tr>
<td>$((X \wedge_T Y) \cdot C)(Z,U)V = 0$</td>
<td>$\sigma - \rho = 0$ or $\sigma - 3\rho = 0$</td>
</tr>
<tr>
<td>$((X \wedge_T Y) \cdot E)(Z,U)V = 0$</td>
<td>$\sigma - \rho = 0$ or $\sigma + \rho = 0$</td>
</tr>
<tr>
<td>$((X \wedge_T Y) \cdot P)(Z,U)V = 0$</td>
<td>$\sigma + \rho = 0$ or $\sigma = 0$</td>
</tr>
<tr>
<td>$((X \wedge_T Y) \cdot H)(Z,U)V = 0$</td>
<td>$\sigma - \rho = 0$ or $\sigma + \rho = 0$</td>
</tr>
</tbody>
</table>

**Theorem 4.1.** The behavior of fluids in GRPFS obeying Einstein’s equation is always phantom barrier for each of the restrictions $((X \wedge_T Y) \cdot E)(Z,U)V = 0$ (for $\sigma \neq \rho$), $(X \wedge_T Y) \cdot H = 0$ for $\sigma \neq \rho$ and $(X \wedge_T Y) \cdot P = 0$ for $\sigma \neq 0$.

**Theorem 4.2.** The behavior of fluids in GRPFS obeying Einstein’s equation is always quintessence barrier for each of the restrictions $((X \wedge_T Y) \cdot R)(Z,U)V = 0$ for $\sigma \neq \rho$ and $(X \wedge_S Y) \cdot C = 0$ for $\sigma \neq 0$.

5. An example of a fluid whose character is phantom barrier

**Example 5.1.** Let $(\mathbb{R}^4, g)$ be a 4-dimensional Lorentzian space endowed with the Lorentzian metric $g$ given by

$$ds^2 = g_{ij}dx^i dx^j = (1 + 2e^{x^1})[(dx^1)^2 + (dx^3)^2 + (dx^2)^2 - (dx^4)^2],$$

$(i, j = 1, 2, 3, 4)$. The only non-vanishing components of the Christoffel symbols and the Ricci tensors (up to symmetry) are

$$\Gamma^1_{11} = \Gamma^1_{44} = \Gamma^2_{12} = \Gamma^3_{13} = -\Gamma^4_{14} = -\Gamma^1_{22} = -\Gamma^1_{33} = \frac{e^{x^1}}{1 + 2e^{x^1}},$$

$$R_{1221} = R_{1331} = -R_{1441} = \frac{e^{x^1}}{1 + 2e^{x^1}},$$

$$R_{2442} = R_{3443} = -R_{2332} = \frac{e^{2x^1}}{1 + 2e^{x^1}},$$

$$\frac{1}{3}(1 + 2e^{x^1})R_{11} = R_{22} = R_{33} = -R_{44} = \frac{e^{x^1}}{1 + 2e^{x^1}}.$$
The scalar curvature $r$ of the resulting space $(\mathbb{R}^4, g)$ is $r = -\frac{6e^{x^1}(1+e^{x^1})}{(1+2e^{x^1})^3}$.

Now using the above results, we may have

\[
E_{1221} = E_{1331} = -E_{1441} = \frac{e^{x^1}(3 + e^{x^1})}{2 + 4e^{x^1}},
\]

\[
E_{2442} = E_{3443} = -E_{2332} = -\frac{e^{x^1}(1 + 3e^{x^1})}{2 + 4e^{x^1}},
\]

\[
\dot{C}_{1221} = \dot{C}_{1331} = -\dot{C}_{1441} = \frac{e^{x^1}(3 + e^{x^1})}{2 + 4e^{x^1}},
\]

\[
\dot{C}_{2332} = -\dot{C}_{2442} = -\dot{C}_{3443} = \frac{e^{x^1}(1 + 3e^{x^1})}{2 + 4e^{x^1}},
\]

\[
P_{1212} = P_{1313} = -P_{1414} = -\frac{2e^{x^1}}{1 + 2e^{x^1}},
\]

\[
P_{1221} = P_{1331} = -P_{1441} = \frac{2e^{x^1}(2 + e^{x^1})}{3 + 6e^{x^1}},
\]

\[
P_{2323} = -P_{2424} = -P_{3434} = -\frac{e^{x^1}(1 + 5e^{x^1})}{3 + 6e^{x^1}},
\]

\[
P_{2332} = -P_{2442} = -P_{3443} = \frac{e^{x^1}(1 + 5e^{x^1})}{3 + 6e^{x^1}},
\]

\[
H_{1221} = H_{1331} = -H_{1441} = \frac{e^{x^1}(5 + e^{x^1})}{3 + 6e^{x^1}},
\]

\[
H_{2332} = -H_{2442} = -H_{3443} = \frac{e^{x^1}(5 + e^{x^1})}{3 + 6e^{x^1}}.
\]

Assuming the associate vector field $\theta$ in the direction of $x^4$, we have

\[
T_{11} = \rho = T_{22} = T_{33}, \quad T_{44} = \sigma.
\]

As consequences of the above relations, we can easily bring out the following:
\[(R \circ T)_{1414} = -\frac{(\sigma + \rho)e^{x_1}}{(1 + 2e^{x_1})^2},\]
\[(R \circ T)_{2424} = (R \circ T)_{3434} = -\frac{(\sigma + \rho)e^{x_1}}{(1 + 2e^{x_1})^2},\]
\[(E \circ T)_{1414} = -\frac{(\sigma + \rho)e^{x_1}(3 + e^{x_1})}{2(1 + 2e^{x_1})^2},\]
\[(E \circ T)_{2424} = (E \circ T)_{3434} = -\frac{(\sigma + \rho)e^{x_1}(1 + 3e^{x_1})}{2(1 + 2e^{x_1})^2},\]
\[(\hat{C} \circ T)_{1414} = -\frac{(\sigma + \rho)e^{x_1}(3 + e^{x_1})}{(1 + 2e^{x_1})^2},\]
\[(\hat{C} \circ T)_{2424} = (\hat{C} \circ T)_{3434} = -\frac{(\sigma + \rho)e^{x_1}(1 + 3e^{x_1})}{(1 + 2e^{x_1})^2},\]
\[(P \circ T)_{1414} = -\frac{2e^{x_1}(\sigma + \rho)}{(1 + 2e^{x_1})^2}, \quad (P \circ T)_{1441} = \frac{2e^{x_1}(\sigma + \rho)(2 + e^{x_1})}{3(1 + 2e^{x_1})^2},\]
\[(P \circ T)_{2424} = -(P \circ T)_{2442} = (P \circ T)_{3434} = -(P \circ T)_{3443}\]
\[= -\frac{(1 + 5e^{x_1})(\sigma + \rho)e^{x_1}}{3(1 + 2e^{x_1})^2},\]
\[(H \circ T)_{1414} = \frac{e^{x_1}(\sigma + \rho)(5 + e^{x_1})}{3(1 + 2e^{x_1})^2},\]
\[(H \circ T)_{2424} = (H \circ T)_{3434} = -\frac{(1 + 5e^{x_1})(\sigma + \rho)e^{x_1}}{3(1 + 2e^{x_1})^2}.\]

This leads to the following

**Theorem 5.2.** Let \( (\mathbb{R}^4, g) \) be a 4-dimensional Lorentzian space endowed with the Lorentzian metric \( g \) given by
\[ds^2 = g_{ij}dx^i dx^j = (1 + 2e^{x_1})[(dx_1)^2 + (dx_3)^2 + (dx_2)^2 - (dx_4)^2],\]
(i, j = 1, 2, 3, 4). Then the behavior of fluids in general relativistic perfect fluid space obeying Einstein's equation is always phantom barrier for each of \( R(X,Y) \cdot T = 0, \ E(X,Y) \cdot T = 0, \ \hat{C}(X,Y) \cdot T = 0, \ P(X,Y) \cdot T = 0 \) and \( H(X,Y) \cdot T = 0. \)

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