8. CONCLUSIONES Y RECOMENDACIONES

Considerando el modelo de flujo de fluidos mediante un análisis composicional y las ecuaciones de esfuerzo – deformación – presión se desarrolló un simulador de flujo de fluidos acoplado a deformación geomecánica para yacimientos naturalmente fracturados, que puede ser utilizado en diferentes escenarios de simulación, especialmente, en yacimiento complejos tanto a nivel estructural como a nivel de los fluidos, como Cupiagua.

Comparando los modelos reportados en literatura, el simulador desarrollado, no tiene restricciones con respecto al tipo de fluidos utilizados en la simulación, debido a que, al hacer un análisis composicional de los fluidos, el simulador permite trabajar con fluidos que sean monofásicos o bifásicos. Al realizar un análisis composicional de los fluidos, se obtienen resultados cada vez más reales acerca del comportamiento de los yacimientos de petróleo y gas.

Realizando algunas modificaciones en los archivos de entrada, valores de compresibilidades, porosidades y permeabilidades, el simulador puede ser usado para predecir el comportamiento de un yacimiento homogéneo o naturalmente fracturado, este último tiene una conducta más elástica que los yacimientos homogéneos y por tanto debe ser estudiado en forma más cuidadosa. Lo anterior representa una ventaja sobre algunos simuladores comerciales que cuentan con el módulo geomecánico.

La respuesta del simulador, corrobora la importancia del trabajo acoplado, en el cual un yacimiento se deforma debido a la respuesta mecánica del macizo rocoso y a la hidráulica de los fluidos. Lo anterior constituye una teoría de superposición de deformaciones inducidas por un comportamiento constitutivo mecánico y un comportamiento constitutivo hidráulico.

La respuesta del simulador permite concluir que, en general, el esfuerzo efectivo aumenta en el tiempo como respuesta a los cambios de permeabilidad en las fracturas principalmente. A partir de cierto tiempo, el valor del esfuerzo efectivo tiende a estabilizarse.

Los resultados que se obtienen con el simulador muestran el comportamiento de la presión del fluido en cada uno de los medios continuos, matriz y fractura. El comportamiento general, que se obtiene para la presión de fluido en matriz muestra que la presión aumenta con el tiempo para los nodos que se ubican por encima del nodo abierto a producción, mientras en los nodos inferiores la presión del fluido en matriz disminuye en el tiempo como respuesta a la producción de los fluidos. Para la presión del fluido que se encuentra en la fractura el comportamiento es diferente. En la fractura el fluido siempre tiende a aumentar su presión como respuesta a los diferentes fenómenos que ocurren: disminución de la permeabilidad de fractura, caudal de producción constante y flujo de fluidos desde la matriz a la fractura.

La diferencia en los valores que toman las variables para los yacimientos naturalmente fracturados y para el caso que se supuso homogéneo, es muy pequeña debido a que los valores de las compresibilidades para ambos medio se dejaron constantes en todos los casos de simulación. Para tener resultados más reales, es necesario ser cuidadosos con los valores de compresibilidades que se elijan, ya que con estos valores se obtienen los parámetros de Biot, y una mala elección de estos parámetros puede resultar en interpretaciones erróneas.

Cualquier cambio en las propiedades mecánicas de la roca tiene un efecto significativo en la respuesta del simulador en diferentes variables, principalmente en el valor del esfuerzo efectivo. Para obtener valores confiables de las variables de interés es recomendable tener valores que se aproximen de las propiedades mecánicas de la roca a la cual se le va a realizar el estudio, ya que estas propiedades presentan gran influencia en la respuesta del simulador.

El efecto del cambio en las propiedades mecánicas en la permeabilidad hace que ésta presente un comportamiento diferente cuando se considera el domino interno más rígido que el dominio externo que en el caso contrario. Cuando el domino interno es más rígido, la permeabilidad en vez de disminuir con profundidad aumenta, debido a que la roca no puede deformarse y su repuesta a esta restricción es aumentando la permeabilidad. Cuando ocurre lo contrario, es decir, cuando el módulo de Young en el dominio interno es menor que el módulo de Young en el dominio externo; la permeabilidad disminuye con profundidad como respuesta al aumento en el esfuerzo efectivo promedio.

El caudal con el que se trabaja para todos los casos de simulación mostrados se tomó como constante, sin embargo, esto no es cierto pues en un yacimiento, dado que el caudal de producción varía en toda la vida productiva de un pozo debido a la disminución de la presión que ocurre en el yacimiento. Por esta razón, se recomienda tener la historia de producción para ser implementada en el simulador y así tener una respuesta más real de la presión del fluido en el yacimiento y de la deformación de la roca en el tiempo. De igual forma, es importante tener la información de producción, para utilizar el simulador para predecir el comportamiento de las diferentes variables en el tiempo.

De la misma manera, es recomendable ubicar otros pozos que estén produciendo en diferentes lugares dentro del yacimiento, para tener una respuesta más real y conforme con lo que puede estar ocurriendo.

La ubicación de otros pozos dentro del yacimiento a diferentes tasas requiere de un análisis más profundo que considere un modelo adecuado para la producción de los fluidos, y que además tenga en cuenta los diferentes procesos que se llevan a cabo en un campo, como operaciones de reacondicionamiento de pozos, inyección de fluidos al yacimiento, cambios de pozos productores a inyectores, entre otros.

En los nodos de simulación donde se encuentra el pozo se debe realizar, en un futuro, una refinación de la malla. Es decir, pasar de una malla cartesiana a una malla híbrida que permita considerar el flujo radial en el pozo mediante una malla cilíndrica, y a una distancia del pozo establecida, trabajar con una malla cartesiana. Lo anterior permitiría visualizar de una manera más acertada lo que está ocurriendo en las zonas aledañas al pozo.

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ANEXOS

ANEXO 1. MODELO COMPOSICIONAL DE FLUJO DE FLUIDOS EN COORDENADAS CARTESIANAS PARA MATRIZ Y FRACTURA

Para encontrar el modelo de flujo de fluidos composicionales en coordenadas cartesianas se parte de siete relaciones básicas:

- Conservación de masa de fluidos
- Conservación de masa de sólido
- Ley de Darcy
- Ecuaciones de compresibilidad de Zimmerman
- Ecuación de Estado
- Ecuación de Presión
- Equilibrio de Fases y Ecuación de Composición

A1.1 Conservación de masa de fluidos:

Denotando el número de moles del componente i en el petróleo que se encuentra en la matriz como n_{moi}, y el número total de moles de los componentes del petróleo como n_{mo}, para N componentes en la mezcla se cumple que:

$$\sum_{i=1}^{N} n_{moi} = n_{mo} \tag{A1.1}$$

De igual manera, el número de moles del componente i en el petróleo que se encuentra en la fractura como n_{foi}, y el número total de moles de los componentes del petróleo como n_{fo}, para N componentes en la mezcla se tiene:

$$\sum_{i=1}^{N} n_{foi} = n_{fo} \tag{A1.2}$$

La fracción molar del componente *i* en el petróleo que se encuentra en la matriz, x_{mi} se define como:

$$x_{mi} = \frac{n_{moi}}{n_{mo}} = \frac{n_{moi}}{\sum_{i=1}^{N} n_{moi}} \tag{A1.3}$$

De la misma manera, para la fractura:

$$x_{fi} = \frac{n_{foi}}{n_{fo}} = \frac{n_{foi}}{\sum_{i=1}^{N} n_{foi}} \tag{A1.4}$$

El peso molecular del petróleo que se encuentran dentro de la matriz (M_{mo}), y su densidad (ρ_{mo}), están definidos por las siguientes expresiones:

$$M_{mo} = \frac{m_{mo}}{n_{mo}} \tag{A1.5}$$

$$\rho_{mo} = \frac{m_{mo}}{V_{mo}} \tag{A1.6}$$

De la misma manera que para la matriz, para la fractura:

$$M_{fo} = \frac{m_{fo}}{n_{fo}} \tag{A1.7}$$

$$\rho_{fo} = \frac{m_{fo}}{V_{fo}} \tag{A1.8}$$

La velocidad volumétrica se define como:

$$u_{mo} = \frac{Volumen\_de\_petroleo\_en\_matriz}{Area(porosa\_matriz) * Tiempo} = \frac{V_{mo}}{A_{mp} * \Delta t} \tag{A1.9}$$

$$u_{mo}\phi_m = \frac{V_{mo}}{A_T * \Delta t} \tag{A1.10}$$

El número de moles del componente i en el petróleo en la matriz por unidad de área, por unidad de tiempo está dado por la expresión:

$$\left(\frac{N_{o.}\ moles\ componente\ i\ en\ el\ petroleo\ en\ la\ matriz}{Area(Total) * \Delta t} \right) = \frac{n_{moi}}{A_T * \Delta t} = \frac{n_{moi}}{n_{mo}} \frac{1}{\frac{m_{mo}}{n_{mo}}} \frac{m_{mo}}{V_{mo}} \frac{V_{mo}}{A_T * \Delta t}$$

$$= x_{mi} \frac{\rho_{mo}}{M_{mo}} u_{mo}\phi_m \tag{A1.11}$$

De igual manera, para el gas:

$$\left(\frac{N_{o.}\ moles\ componente\ i\ en\ el\ gas\ en\ la\ matriz}{Area(Total) * \Delta t} \right) = y_{mi} \frac{\rho_{mg}}{M_{mg}} u_{mg}\phi_m \tag{A1.12}$$

De forma similar, para la fractura:

$$\left(\frac{N_{o.}\ moles\ componente\ i\ en\ el\ petroleo\ en\ la\ fractura}{Area(Total)\ *\Delta t}\right) = x_{fi}\frac{\rho_{fo}}{M_{fo}}u_{fo}\phi_f \qquad (A1.13)$$

$$\left(\frac{N_{o.}\ moles\ componente\ i\ en\ el\ gas\ en\ la\ fractura}{Area(Total)\ *\Delta t}\right) = y_{fi}\frac{\rho_{fg}}{M_{fg}}u_{fg}\phi_f \qquad (A1.14)$$

Balance de Masa:

$$\left(N^o\ moles\ que\ entran\ de\ i\right)_{\Delta t} - \left(N^o\ moles\ que\ salen\ de\ i\right)_{\Delta t} \pm \begin{pmatrix} N^o\ moles\ que\ entran/salen \\ fuentes/sumideros\ de\ i \end{pmatrix}_{\Delta t}$$
$$= \left(N^o\ moles\ por\ acumulacion\ y/o\ agotamiento\ de\ i\right)_{\Delta t} \qquad (A1.15)$$

A continuación, se toma cada uno de los términos de la ecuación de balance de masa:

$$\left(N^o\ moles\ que\ entran\ de\ i\right)_{\Delta t} = \left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{mox}\phi_m\right)\Delta y\Delta z\Delta t + \left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moy}\phi_m\right)\Delta x\Delta z\Delta t +$$
$$\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moz}\phi_m\right)\Delta x\Delta y\Delta t + \left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgx}\phi_m\right)\Delta y\Delta z\Delta t + \left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgy}\phi_m\right)\Delta x\Delta z\Delta t +$$
$$\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgz}\phi_m\right)\Delta x\Delta y\Delta t \qquad (A1.16)$$

$$\left(N^o\ moles\ que\ salen\ de\ i\right)_{\Delta t} = \left\{\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{mox}\phi_m\right) + \Delta\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{mox}\phi_m\right)\right\}\Delta y\Delta z\Delta t +$$
$$\left\{\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moy}\phi_m\right) + \Delta\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moy}\phi_m\right)\right\}\Delta x\Delta z\Delta t + \left\{\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moz}\phi_m\right) + \Delta\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moz}\phi_m\right)\right\}\Delta x\Delta y\Delta t$$
$$+\left\{\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgx}\phi_m\right) + \Delta\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{gx}\phi_m\right)\right\}\Delta y\Delta z\Delta t + \left\{\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgy}\phi_m\right) + \Delta\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{gy}\phi_m\right)\right\}\Delta x\Delta z\Delta t +$$
$$\left\{\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgz}\phi_m\right) + \Delta\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{gz}\phi_m\right)\right\}\Delta x\Delta y\Delta t \qquad (A1.17)$$

$$\begin{pmatrix} N^{o} \ moles \ que \ entran/salen \\ fuentes/sumideros \ de \ i \end{pmatrix}_{\Delta t} = \tilde{q}_{mi}\Delta x\Delta y\Delta z\Delta t \qquad \text{(A1.18)}$$

$$\begin{pmatrix} N^{o} \ moles \ por \ acumulacion \ y/o \\ agotamiento \ de \ i \end{pmatrix}_{\Delta t} = \left(moles \ de \ i\right)_{t+\Delta t} - \left(moles \ de \ i\right)_{\Delta t} \qquad \text{(A1.19)}$$

$$moles \ de \ i \ en \ el \ petroleo \ en \ matriz = x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}V_{pm} = x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}\phi_m\Delta x\Delta y\Delta z \qquad \text{(A1.20)}$$

$$moles \ de \ i \ en \ el \ gas \ en \ matriz = y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}V_{pm} = y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\phi_m\Delta x\Delta y\Delta z \qquad \text{(A1.21)}$$

Teniendo presente las moles de i en el gas y en petróleo que se encuentra en la matriz, dadas por las ecuaciones (A1.20) y (A1.21) y reemplazando en la ecuación (A1.19), se llega a:

$$\begin{pmatrix} N^{o} \ moles \ por \ acumulacion \ y/o \\ agotamiento \ de \ i \end{pmatrix}_{\Delta t} = \left[\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}\phi_m + y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\phi_m\right)\Delta x\Delta y\Delta z\right]_{t+\Delta t} - \left[\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}\phi_m + y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\phi_m\right)\Delta x\Delta y\Delta z\right]_{\Delta t} \qquad \text{(A1.22)}$$

Llevando las ecuaciones (A1.16), (A1.17), (A1.18), (A1.22), a la ecuación (A1.15) y agrupando los términos semejantes la ecuación de balance de materiales queda:

$$-\Delta\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{mox}\phi_m\right)\Delta y\Delta z\Delta t - \Delta\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moy}\phi_m\right)\Delta x\Delta z\Delta t - \Delta\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moz}\phi_m\right)\Delta x\Delta y\Delta t - \Delta\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgx}\phi_m\right)\Delta y\Delta z\Delta t - \Delta\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgy}\phi_m\right)\Delta x\Delta z\Delta t - \Delta\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgz}\phi_m\right)\Delta x\Delta y\Delta t = \pm\tilde{q}_{mi}\Delta x\Delta y\Delta z\Delta t + \left[\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}\phi_m + y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\phi_m\right)V_T\right]_{t+\Delta t} - \left[\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}\phi_m + y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\phi_m\right)V_T\right]_{\Delta t}$$

Dividiendo por $\Delta x \Delta y \Delta z \Delta t$ y despreciando los términos infinitesimales y teniendo presente que $q_{mi}>0$ cuando se están produciendo fluidos $q_{mi}<0$ cuando se inyectan fluidos al yacimiento, la expresión queda como:

$$-\frac{\partial}{\partial x}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{mox}\phi_m\right)-\frac{\partial}{\partial y}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moy}\phi_m\right)-\frac{\partial}{\partial z}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}u_{moz}\phi_m\right)-\frac{\partial}{\partial x}\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgx}\phi_m\right)$$

$$-\frac{\partial}{\partial y}\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgy}\phi_m\right)-\frac{\partial}{\partial z}\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}u_{mgz}\phi_m\right)=\pm\tilde{q}_{mi}+\frac{1}{V_T}\frac{\partial}{\partial t}\left[\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}\phi_m+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\phi_m\right)V_T\right]+T$$

Esta ecuación de balance de Masa para la matriz puede ser expresada como:

$$-\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}\vec{u}_{mo}\right)-\nabla\cdot\left(y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\vec{u}_{mg}\right)=\tilde{q}_{mi}+\frac{1}{V_T}\frac{\partial}{\partial t}\left[\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}\phi_m+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\phi_m\right)V_T\right]+T \qquad \text{(A1.23)}$$

Siguiendo el mismo procedimiento se llega a la ecuación de Balance de Masa para la Fractura:

$$-\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\phi_f}{M_{fo}}\vec{u}_{fo}\right)-\nabla\cdot\left(y_{fi}\frac{\rho_{fg}\phi_f}{M_{fg}}\vec{u}_{fg}\right)=\tilde{q}_{fi}+\frac{1}{V_T}\frac{\partial}{\partial t}\left[\left(x_{fi}\frac{\rho_{fo}}{M_{fo}}S_{fo}\phi_f+y_{fi}\frac{\rho_{fg}}{M_{fg}}S_{fg}\phi_f\right)V_T\right]-T \qquad \text{(A1.24)}$$

En las ecuaciones (A1.23) y (A1.24) ρ es la densidad, ϕ es la porosidad efectiva, $\vec{u}$ es el vector velocidad, *t* es tiempo, T es la tasa de transferencia de masa de fluido de los bloques de la matriz a la fractura por unidad de volumen total, *q* es el término de fuentes/ sumideros expresado como tasa de masa por unidad de volumen total (la convención es que el signo menos representa una fuente y el signo mas representa un sumidero). Los subíndices *mo*, *fo, mg, fg* se refieren a las fases de petróleo en la matriz, petróleo en la fractura, gas en la matriz, gas en la fractura, respectivamente. Los símbolos ∇ y $\nabla\cdot$ denotan gradiente y divergencia, respectivamente.

A1.2 Conservación de Masa de Sólidos

Continuando con el procedimiento seguido para encontrar la ecuación de balance de materiales para el fluido en la matriz y en la fractura para el sólido se llega a:

$$-\nabla\cdot\left(\frac{\rho_s\left(1-\phi_t\right)}{M_s}\vec{u}_s\right)=\tilde{q}_s+\frac{1}{V_T}\frac{\partial}{\partial t}\left[\left(\frac{\rho_s\left(1-\phi_s\right)}{M_s}\right)V_T\right] \qquad \text{(A1.25)}$$

Resolviendo el término de la izquierda en la ecuación se llega a:

$$-\vec{u}_s \nabla \cdot \left(\frac{\rho_s (1-\phi_t)}{M_s} \right) - \frac{\rho_s (1-\phi_t)}{M_s} \nabla \cdot \vec{u}_s = \tilde{q}_s + \frac{1}{V_T} \frac{\partial}{\partial t} \left[\left(\frac{\rho_s (1-\phi_s)}{M_s} \right) V_T \right] \qquad \text{(A1.26)}$$

En términos de la derivada del material:

$$\left. \frac{d\left[(1-\phi_t)\rho_s / M_s\right]}{dt} \right|_{\substack{particula \\ movil}} = \left. \frac{\partial\left[(1-\phi_t)\rho_s / M_s\right]}{\partial t} \right|_{punto} + \vec{u}_s \nabla \cdot \left[(1-\phi_t)\rho_s / M_s\right]$$

Reemplazando en la ecuación (A1.26)

$$\frac{d}{dt}\left[\frac{(1-\phi_t)\rho_s}{M_s} \right]_{\substack{particula \\ movil}} - \frac{\partial}{\partial t}\left[\frac{(1-\phi_t)\rho_s}{M_s} \right]_{punto} + \frac{\rho_s (1-\phi_t)}{M_s} \nabla \cdot \vec{u}_s + \tilde{q}_s + \frac{1}{V_T} \frac{\partial}{\partial t} \left[\left(\frac{\rho_s (1-\phi_s)}{M_s} \right) V_T \right] = 0$$

Expandiendo el último término en la ecuación (A1.26)

$$\frac{1}{V_T} \frac{\partial}{\partial t} \left[\left(\frac{\rho_s (1-\phi_s)}{M_s} \right) V_T \right] = \frac{1}{V_T} \left(\frac{\rho_s (1-\phi_s)}{M_s} \right) \frac{\partial}{\partial t}(V_T) + \frac{\partial}{\partial t} \left(\frac{\rho_s (1-\phi_s)}{M_s} \right)$$

Reemplazando de nuevo en la ecuación (A1.26) se encuentra:

$$\frac{d}{dt}\left[\frac{(1-\phi_t)\rho_s}{M_s} \right]_{movil} - \frac{\partial}{\partial t}\left[\frac{(1-\phi_t)\rho_s}{M_s} \right]_{punto} + \frac{\rho_s (1-\phi_t)}{M_s} \nabla \cdot \vec{u}_s + \tilde{q}_s + \frac{1}{V_T} \left(\frac{\rho_s (1-\phi_s)}{M_s} \right) \frac{\partial}{\partial t}(V_T)$$
$$+ \frac{\partial}{\partial t} \left(\frac{\rho_s (1-\phi_s)}{M_s} \right) = 0$$

Finalmente agrupando de la manera adecuada se llega a:

$$\frac{1}{V_T} \frac{d}{dt}\left[\frac{(1-\phi_t)\rho_s}{M_s} V_T \right] + \frac{\rho_s (1-\phi_t)}{M_s} \nabla \cdot \vec{u}_s + \tilde{q}_s = 0 \qquad \text{(A1.27)}$$

Ahora se desea despejar el término $\nabla \cdot \vec{u}_s$

$$\nabla \cdot \vec{u}_s = -\frac{M_s}{\rho_s (1-\phi_t)} \frac{1}{V_T} \frac{d}{dt}\left[\frac{(1-\phi_t)\rho_s}{M_s} V_T \right] - \tilde{q}_s \frac{M_s}{\rho_s (1-\phi_t)} = 0$$

Reemplazando (A1.7) y (A1.8)

$$\nabla \cdot \vec{u}_s = -\frac{M_s}{\frac{m_s}{V_s}\frac{V_s}{V_T}} \frac{1}{V_T} \frac{d}{dt}\left[\frac{V_s m_s}{V_T V_s M_s}\right] - \frac{1}{V_T}\frac{dV_T}{dt} - \tilde{q}_s \frac{M_s}{\rho_s\left(1-\phi_t\right)}$$

$$\nabla \cdot \vec{u}_s = -M_s \frac{d}{dt}\left[\frac{1}{V_T M_s}\right] - \frac{1}{V_T}\frac{dV_T}{dt} - \tilde{q}_s \frac{M_s}{\rho_s\left(1-\phi_t\right)}$$

Tomando el peso molecular del sólido constante se llega finalmente a:

$$\nabla \cdot \vec{u}_s = -\frac{\tilde{q}_s M_s}{\rho_s\left(1-\phi_t\right)} \qquad \text{(A1.28)}$$

Si no hay flujo de sólidos entonces $q_s = 0$, por lo tanto $\nabla \cdot \vec{u}_s = 0$

A1.3 Ley de Darcy

Dado que el sólido puede estar en movimiento, la velocidad relativa del petróleo de la matriz con respecto a la del sólido puede escribirse como:

$$\vec{u}_{rmo} = \vec{u}_{mo} - \vec{u}_s = \frac{q_{mo}}{\phi_m A_T} \qquad \text{(A1.29)}$$

De la ley de Darcy

$$\frac{q_{mo}}{A_T} = -\frac{\vec{k}_m k_{rmo}}{\mu_{mo}} \nabla P_{mo}$$

La ecuación (A1.29) puede escribirse:

$$\vec{u}_{mo}\phi_m = \vec{u}_s \phi_m - \frac{\vec{k}_m k_{rmo}}{\mu_{mo}} \nabla P_{mo} \qquad \text{(A1.30)}$$

Para el gas en la matriz:

$$\vec{u}_{mg}\phi_m = \vec{u}_s \phi_m - \frac{\vec{k}_m k_{rmg}}{\mu_{mg}} \nabla P_{mg} \qquad \text{(A1.31)}$$

Para el petróleo en la fractura:

$$\vec{u}_{fo}\phi_f = \vec{u}_s \phi_f - \frac{\vec{k}_f k_{rfo}}{\mu_{fo}} \nabla P_{fo} \qquad \text{(A1.32)}$$

Para el gas en la fractura:

$$\vec{u}_{fg}\phi_f = \vec{u}_s\phi_f - \frac{\vec{k}_f k_{rfg}}{\mu_{fg}}\nabla P_{fg} \qquad \text{(A1.33)}$$

En las ecuaciones (A1.30) – (A1.33) $\vec{u}_s$ es el vector velocidad del sólido, $\vec{u}_{mo}, \vec{u}_{mg}, \vec{u}_{fo}, \vec{u}_{fg}$ son los vectores velocidad del aceite y del gas en la matriz y en la fractura respectivamente. $\vec{k}_m, \vec{k}_f$ son los tensores de permeabilidades absolutas de matriz y fracturas respectivamente. $k_{rmo}, k_{rmg}, k_{rfo}, k_{rfg}$ son las permeabilidades relativas de cada fase dentro de la matriz y dentro de la fractura. $P_{mo}, P_{mg}, P_{fo}, P_{fg}$ son las presiones en cada fase en la matriz y en la fractura.

Llevando las ecuaciones (A1.30) y (A1.32) a la ecuación (A1.23) se obtiene:

$$\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{mo}\right)+\nabla\cdot\left(y_{mi}\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\nabla P_{mg}\right)=\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}\vec{u}_s\right)+\nabla\cdot\left(y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\vec{u}_s\right)+\tilde{q}_{mi}$$
$$+\frac{1}{V_T}\frac{\partial}{\partial t}\left[\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}\phi_m + y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\phi_m\right)V_T\right]+T \qquad \text{(A1.34)}$$

Con las ecuaciones (A1.31) y (A1.33) en la ecuación (A1.24) se obtiene una ecuación similar para fracturas:

$$\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}\nabla P_{fo}\right)+\nabla\cdot\left(y_{fi}\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\nabla P_{fg}\right)=\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\phi_f}{M_{fo}}\vec{u}_s\right)+\nabla\cdot\left(y_{fi}\frac{\rho_{fg}\phi_f}{M_{fg}}\vec{u}_s\right)+\tilde{q}_{fi}$$
$$+\frac{1}{V_T}\frac{\partial}{\partial t}\left[\left(x_{fi}\frac{\rho_{fo}}{M_{fo}}S_{fo}\phi_f + y_{fi}\frac{\rho_{fg}}{M_{fg}}S_{fg}\phi_f\right)V_T\right]+T \qquad \text{(A1.35)}$$

Teniendo en cuenta los efectos capilares, se debe introducir la relación entre la presión de petróleo y la presión del gas. La presión capilar para sistemas petróleo – gas se puede escribir:

Para la matriz:

$$P_{cmog} = P_{mg} - P_{mo} \qquad \text{(A1.36)}$$

Para la fractura:

$$P_{cfog} = P_{fg} - P_{fo} \qquad \text{(A1.37)}$$

Llevando la ecuación (A1.36) a la ecuación (A1.34) se obtiene:

$$\nabla\cdot\left[\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+y_{mi}\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla P_{mg}\right]=\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}\vec{u}_s\right)+\nabla\cdot\left(y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\vec{u}_s\right)+\tilde{q}_{mi}$$
$$+\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{cmog}\right)+\frac{1}{V_T}\frac{\partial}{\partial t}\left[V_T\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)\right]+T$$

Resolviendo para el primer y el segundo término del lado derecho de la ecuación:

$$\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}\vec{u}_s\right)=\vec{u}_s\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}\right)+x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}\nabla\cdot\vec{u}_s$$

$$\nabla\cdot\left(y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\vec{u}_s\right)=\vec{u}_s\nabla\cdot\left(y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\right)+y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\nabla\cdot\vec{u}_s$$

Considerando que el divergente de la velocidad de los sólidos es igual a cero la ecuación queda expresada como:

$$\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}\vec{u}_s\right)=\vec{u}_s\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}\right)$$

$$\nabla\cdot\left(y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\vec{u}_s\right)=\vec{u}_s\nabla\cdot\left(y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\right)$$

Se llega a la siguiente ecuación para la matriz:

$$\nabla\cdot\left[\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+y_{mi}\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla P_{mg}\right]=\vec{u}_s\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}+y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\right)+\tilde{q}_{mi}$$
$$+\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{cmog}\right)+\frac{1}{V_T}\frac{\partial}{\partial t}\left[V_T\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)\right]+T \qquad \text{(A1.38)}$$

De igual manera para la fractura

$$\nabla\cdot\left[\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}+y_{fi}\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\right)\nabla P_{fg}\right]=\vec{u}_s\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\phi_f}{M_{fo}}+y_{fi}\frac{\rho_{fg}\phi_f}{M_{fg}}\right)+\tilde{q}_{fi}$$
$$+\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}\nabla P_{cfog}\right)+\frac{1}{V_T}\frac{\partial}{\partial t}\left[V_T\phi_f\left(x_{fi}\frac{\rho_{fo}}{M_{fo}}S_{fo}+y_{fi}\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)\right]-T \qquad \text{(A1.39)}$$

Expandiendo el último término de la ecuación (A1.38)

$$\frac{1}{V_T}\frac{\partial}{\partial t}\left[V_T\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)\right]=\frac{1}{V_T}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)\frac{\partial}{\partial t}\left(V_T\phi_m\right)$$
$$+\frac{1}{V_T}V_T\phi_m\frac{\partial}{\partial t}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)$$

Recordando la definición de volumen poroso:

$$\frac{1}{V_T}\frac{\partial}{\partial t}\left[V_T\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)\right]=\frac{\phi_m}{V_{pm}}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)\frac{\partial}{\partial t}\left(V_{pm}\right)$$
$$+\phi_m\frac{\partial}{\partial t}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right) \tag{A1.40}$$

Llevando (A1.40) y (A1.28) a la ecuación (A1.38) se llega a:

$$\nabla\cdot\left[\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+y_{mi}\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla P_{mg}\right]=\tilde{q}_{mi}-\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}+y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial u}{\partial t}$$
$$+\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{cmog}\right)+\frac{\phi_m}{V_{pm}}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)\frac{\partial}{\partial t}\left(V_{pm}\right) \tag{A1.41}$$
$$+\phi_m\frac{\partial}{\partial t}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)+T$$

De la misma manera para la fractura:

$$\nabla\cdot\left[\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}+y_{fi}\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\right)\nabla P_{fg}\right]=\tilde{q}_{fi}-\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\phi_f}{M_{fo}}+y_{fi}\frac{\rho_{fg}\phi_f}{M_{fg}}\right)\frac{\partial u}{\partial t} \tag{A1.42}$$
$$+\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}\nabla P_{cfog}\right)+\frac{\phi_f}{V_{pf}}\left(x_{fi}\frac{\rho_{fo}}{M_{fo}}S_{fo}+y_{fi}\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)\frac{\partial}{\partial t}\left(V_{pf}\right)+\phi_f\frac{\partial}{\partial t}\left(x_{fi}\frac{\rho_{fo}}{M_{fo}}S_{fo}+y_{fi}\frac{\rho_{fg}}{M_{fg}}S_{g}\right)-T$$

A1.4 Ecuación de Compresibilidad de Zimmerman et al.

Para describir los cambios tanto en poros de matriz como en poros de fracturas, se tiene para la matriz:

$$-\frac{dV_{pm}}{Vpm} = c_{pcm} d\bar{\sigma}_p^d = c_{pcm}\left(d\sigma_m - \beta_{pm} dP_m - \beta_{pf} dP_f\right) \quad \text{(A1.43)}$$

Para el volumen poroso de la fractura:

$$-\frac{dV_{pf}}{V_{pf}} = c_{pcf} d\bar{\sigma}_p^d = c_{pcf}\left(d\sigma_m - \beta_{pm} dP_m - \beta_{pf} dP_f\right) \quad \text{(A1.44)}$$

Para el caso multifásico que se esta considerando, se escribe la ecuación de Zimmerman en una forma más general

$$-\frac{dV_{pm}}{Vpm} = c_{pcm} d\bar{\sigma}_p^d = c_{pcm}\left(d\sigma_m - \beta_{pm} d\Omega_m - \beta_{pf} d\Omega_f\right) \quad \text{(A1.45)}$$

Donde Ω_m y Ω_f dependen de la presión del gas y de la presión capilar así:

$$\Omega_m = P_{mg} - \psi_m\left(P_{mcog}, S_{mo}\right) \quad \text{(A1.46)}$$

Y para la fractura:

$$\Omega_f = P_{fg} - \psi_f\left(P_{fcog}, S_{fo}\right) \quad \text{(A1.47)}$$

Dependiendo del caso particular para calcular la presión en el medio poroso, se obtiene diferentes formas para ψ_m y ψ_f. Se presentan los siguientes casos:

Caso 1:
$\Omega_m = P_{mg}$ es decir $\psi_m = 0$

Caso 2:
$\Omega_m = P_{mo}$ es decir $\psi_m = P_{mcog}$

Caso 3:

$$\Omega_m = S_{mo}P_{mo} + S_{mg}P_{mg}$$

$$\Omega_m = \left(S_{mo} + S_{mg}\right)P_{mg} - S_{mo}P_{mcog}$$

Caso 4:

$$\Omega_m = \frac{1}{2}\left(P_{mo} + P_{mg}\right)$$

$$\Omega_m = \frac{1}{2}\left(P_{mg} - P_{mcog} + P_{mg}\right)$$

$\Omega_m = P_{mg} - \frac{1}{2} P_{mcog}$ es decir $\psi_m = \frac{1}{2} P_{mcog}$

Las compresibilidades c_{pcm} y c_{pcf} se pueden obtener de:

$$c_{pcm} = \frac{c^s_{bc} - c^s_s}{\phi_m} = \frac{\phi_t}{\phi_m} c^s_{pc} = \frac{\phi_t}{\phi_m} c^d_{pc} = \frac{\phi_t}{\phi_m} \frac{c^s_{bc} - c^s_s}{c^d_{bc} - c^d_s}$$

$$c_{pcf} = \frac{\phi_t c^d_{pc} - \phi_m c_{pcm}}{\phi_f} = \frac{\phi_t}{\phi_f}\left(c^d_{pc} - c^s_{pc}\right)$$

Reemplazando (A1.43) en (A1.41) y considerando la ecuación (A1.46) se llega a:

$$\begin{aligned}
&\nabla \cdot \left[\left(x_{mi} \frac{\rho_{mo} \vec{k}_m k_{rmo}}{M_{mo} \mu_{mo}} + y_{mi} \frac{\rho_{mg} \vec{k}_m k_{rmg}}{M_{mg} \mu_{mg}} \right) \nabla P_{mg} \right] = \tilde{q}_{mi} - \nabla \cdot \left(x_{mi} \frac{\rho_{mo} \phi_m}{M_{mo}} + y_{mi} \frac{\rho_{mg} \phi_m}{M_{mg}} \right) \frac{\partial \bar{u}}{\partial t} + \\
&\nabla \cdot \left(x_{mi} \frac{\rho_{mo} \vec{k}_m k_{rmo}}{M_{mo} \mu_{mo}} \nabla P_{cmog} \right) - \phi_m \left(x_{mi} \frac{\rho_{mo}}{M_{mo}} S_{mo} + y_{mi} \frac{\rho_{mg}}{M_{mg}} S_{mg} \right) c_{pcm} \frac{\partial \sigma_m}{\partial t} \\
&+ \phi_m \left(x_{mi} \frac{\rho_{mo}}{M_{mo}} S_{mo} + y_{mi} \frac{\rho_{mg}}{M_{mg}} S_{mg} \right) c_{pcm} \beta_{pm} \frac{\partial P_{mg}}{\partial t} - \phi_m \left(x_{mi} \frac{\rho_{mo}}{M_{mo}} S_{mo} + y_{mi} \frac{\rho_{mg}}{M_{mg}} S_{mg} \right) c_{pcm} \beta_{pm} \frac{\partial \psi_m}{\partial t} \\
&+ \phi_m \left(x_{mi} \frac{\rho_{mo}}{M_{mo}} S_{mo} + y_{mi} \frac{\rho_{mg}}{M_{mg}} S_{mg} \right) c_{pcm} \beta_{pf} \frac{\partial P_{fg}}{\partial t} - \phi_m \left(x_{mi} \frac{\rho_{mo}}{M_{mo}} S_{mo} + y_{mi} \frac{\rho_{mg}}{M_{mg}} S_{mg} \right) c_{pcm} \beta_{pf} \frac{\partial \psi_f}{\partial t} \\
&+ \phi_m \frac{\partial}{\partial t} \left(x_{mi} \frac{\rho_{mo}}{M_{mo}} S_{mo} + y_{mi} \frac{\rho_{mg}}{M_{mg}} S_{mg} \right) + T
\end{aligned}$$

(A1.48)

De igual manera para la fractura queda:

$$\begin{aligned}
&\nabla \cdot \left[\left(x_{fi} \frac{\rho_{fo} \vec{k}_f k_{rfo}}{M_{fo} \mu_{fo}} + y_{fi} \frac{\rho_{fg} \vec{k}_f k_{rfg}}{M_{fg} \mu_{fg}} \right) \nabla P_{fg} \right] = \tilde{q}_{fi} - \nabla \cdot \left(x_{fi} \frac{\rho_{fo} \phi_f}{M_{fo}} + y_{fi} \frac{\rho_{fg} \phi_f}{M_{fg}} \right) \frac{\partial \bar{u}}{\partial t} + \nabla \cdot \left(x_{fi} \frac{\rho_{fo} \vec{k}_f k_{rfo}}{M_{fo} \mu_{fo}} \nabla P_{cfog} \right) \\
&- \phi_f \left(x_{fi} \frac{\rho_{fo}}{M_{fo}} S_{fo} + y_{fi} \frac{\rho_{fg}}{M_{fg}} S_{fg} \right) c_{pcf} \frac{\partial \sigma_m}{\partial t} + \phi_f \left(x_{fi} \frac{\rho_{fo}}{M_{fo}} S_{fo} + y_{fi} \frac{\rho_{fg}}{M_{fg}} S_{fg} \right) c_{pcf} \beta_{pm} \frac{\partial P_{mg}}{\partial t} \\
&- \phi_f \left(x_{fi} \frac{\rho_{fo}}{M_{fo}} S_{fo} + y_{fi} \frac{\rho_{fg}}{M_{fg}} S_{fg} \right) c_{pcf} \beta_{pm} \frac{\partial \psi_m}{\partial t} + \phi_f \left(x_{fi} \frac{\rho_{fo}}{M_{fo}} S_{fo} + y_{fi} \frac{\rho_{fg}}{M_{fg}} S_{fg} \right) c_{pcf} \beta_{pf} \frac{\partial P_{fg}}{\partial t} \\
&- \phi_f \left(x_{fi} \frac{\rho_{fo}}{M_{fo}} S_{fo} + y_{fi} \frac{\rho_{fg}}{M_{fg}} S_{fg} \right) c_{pcf} \beta_{pf} \frac{\partial \psi_f}{\partial t} + \phi_f \frac{\partial}{\partial t} \left(x_{fi} \frac{\rho_{fo}}{M_{fo}} S_{fo} + y_{fi} \frac{\rho_{fg}}{M_{fg}} S_g \right) - T
\end{aligned}$$

(A1.49)

A1.5 Ecuación de Estado

Con las definiciones de la ecuación de estado se puede llegar a las siguientes expresiones para matriz y fractura respectivamente:

Matriz:

$$C_{mo}\frac{\partial P_{mo}}{\partial t}=\frac{1}{\rho_{mo}}\frac{\partial \rho_{mo}}{\partial t}$$

$$C_{mg}\frac{\partial P_{mg}}{\partial t}=\frac{1}{\rho_{mg}}\frac{\partial \rho_{mg}}{\partial t}$$

Fractura:

$$C_{fo}\frac{\partial P_{fo}}{\partial t}=\frac{1}{\rho_{fo}}\frac{\partial \rho_{fo}}{\partial t}$$

$$C_{fg}\frac{\partial P_{fg}}{\partial t}=\frac{1}{\rho_{fg}}\frac{\partial \rho_{fg}}{\partial t}$$

Haciendo una expansión del antepenúltimo término de la ecuación (A1.48) se tiene:

$$\phi_m\frac{\partial}{\partial t}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)=\phi_m\frac{\partial}{\partial t}\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}\right)+\phi_m\frac{\partial}{\partial t}\left(y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)$$

Derivando los dos términos

$$\phi_m\left[\frac{\partial}{\partial t}\left(\frac{x_{mi}S_{mo}}{M_{mo}}\right)\rho_{mo}+\frac{x_{mi}S_{mo}}{M_{mo}}\frac{\partial \rho_{mo}}{\partial t}\right]+\phi_m\left[\frac{\partial}{\partial t}\left(\frac{y_{mi}S_{mg}}{M_{mg}}\right)\rho_{mg}+\frac{y_{mi}S_{mg}}{M_{mg}}\frac{\partial \rho_{mg}}{\partial t}\right]$$

Considerando las definiciones de las compresibilidades anteriores se llega a:

$$\phi_m\rho_{mo}\left[\frac{\partial}{\partial t}\left(\frac{x_{mi}S_{mo}}{M_{mo}}\right)+\frac{x_{mi}S_{mo}}{M_{mo}}C_{mo}\frac{\partial P_{mo}}{\partial t}\right]+\phi_m\rho_{mg}\left[\frac{\partial}{\partial t}\left(\frac{y_{mi}S_{mg}}{M_{mg}}\right)+\frac{y_{mi}S_{mg}}{M_{mg}}C_{mg}\frac{\partial P_{mg}}{\partial t}\right]$$

Tomando la ecuación (A1.36) en la ecuación anterior se llega a:

$$\phi_m\rho_{mo}\left[\frac{\partial}{\partial t}\left(\frac{x_{mi}S_{mo}}{M_{mo}}\right)+\frac{x_{mi}S_{mo}}{M_{mo}}C_{mo}\frac{\partial P_{mg}}{\partial t}-\frac{x_{mi}S_{mo}}{M_{mo}}C_{mo}\frac{\partial P_{cmog}}{\partial t}\right]+\phi_m\rho_{mg}\left[\frac{\partial}{\partial t}\left(\frac{y_{mi}S_{mg}}{M_{mg}}\right)+\frac{y_{mi}S_{mg}}{M_{mg}}C_{mg}\frac{\partial P_{mg}}{\partial t}\right]$$

Agrupando los términos semejantes:

$$\phi_m\left[\rho_{mo}\frac{\partial}{\partial t}\left(\frac{x_{mi}S_{mo}}{M_{mo}}\right)+\rho_{mg}\frac{\partial}{\partial t}\left(\frac{y_{mi}S_{mg}}{M_{mg}}\right)\right]+\phi_m\left[\frac{x_{mi}S_{mo}\rho_{mo}}{M_{mo}}C_{mo}+\frac{y_{mi}S_{mg}\rho_{mg}}{M_{mg}}C_{mg}\right]\frac{\partial P_{mg}}{\partial t}-\frac{x_{mi}S_{mo}\rho_{mo}\phi_m}{M_{mo}}C_{mo}\frac{\partial P_{comg}}{\partial t}$$

(A1.50)

La ecuación (A1.50) se agrega a la ecuación (A1.48) y se encuentra:

$$\nabla\cdot\left[\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+y_{mi}\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla P_{mg}\right]=\tilde{q}_{mi}-\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}+y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial\bar{u}}{\partial t}+$$
$$\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{cmog}\right)-\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\frac{\partial\sigma_m}{\partial t}$$
$$+\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pm}\frac{\partial P_{mg}}{\partial t}-\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pm}\frac{\partial\psi_m}{\partial t}$$
$$+\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial P_{fg}}{\partial t}-\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial\psi_f}{\partial t}$$
$$+\phi_m\left[\rho_{mo}\frac{\partial}{\partial t}\left(\frac{x_{mi}S_{mo}}{M_{mo}}\right)+\rho_{mg}\frac{\partial}{\partial t}\left(\frac{y_{mi}S_{mg}}{M_{mg}}\right)\right]+\phi_m\left[\frac{x_{mi}S_{mo}\rho_{mo}}{M_{mo}}C_{mo}+\frac{y_{mi}S_{mg}\rho_{mg}}{M_{mg}}C_{mg}\right]\frac{\partial P_{mg}}{\partial t}$$
$$-\frac{x_{mi}S_{mo}\rho_{mo}\phi_m}{M_{mo}}C_{mo}\frac{\partial P_{comg}}{\partial t}+T$$

Agrupando los términos semejantes la ecuación de flujo de fluidos queda como:

$$\nabla\cdot\left[\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+y_{mi}\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla P_{mg}\right]=\tilde{q}_{mi}-\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}+y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial\bar{u}}{\partial t}+$$
$$\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{cmog}\right)-\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\frac{\partial\sigma_m}{\partial t}$$
$$-\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pm}\frac{\partial\psi_m}{\partial t}+\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial P_{fg}}{\partial t}$$
$$-\phi_m\left(x_{mi}\frac{\rho_{mo}}{M_{mo}}S_{mo}+y_{mi}\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial\psi_f}{\partial t}+\phi_m\left[\rho_{mo}\frac{\partial}{\partial t}\left(\frac{x_{mi}S_{mo}}{M_{mo}}\right)+\rho_{mg}\frac{\partial}{\partial t}\left(\frac{y_{mi}S_{mg}}{M_{mg}}\right)\right]$$
$$-\frac{x_{mi}S_{mo}\rho_{mo}\phi_m}{M_{mo}}C_{mo}\frac{\partial P_{comg}}{\partial t}+\frac{\phi_m x_{mi}S_{mo}\rho_{mo}}{M_{mo}}\left[C_{mo}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}+\frac{\phi_m y_{mi}\rho_{mg}S_{mg}}{M_{mg}}\left[C_{mg}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}$$
$$+T$$

(A1.51)

Siguiendo el mismo procedimiento para la fractura se llega a la siguiente ecuación de flujo de fluidos para fractura:

$$
\begin{aligned}
&\nabla\cdot\left[\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}+y_{fi}\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\right)\nabla P_{fg}\right]=\tilde{q}_{fi}-\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\phi_f}{M_{fo}}+y_{fi}\frac{\rho_{fg}\phi_f}{M_{fg}}\right)\frac{\partial\overline{u}}{\partial t}+\\
&\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}\nabla P_{cfog}\right)-\phi_f\left(x_{fi}\frac{\rho_{fo}}{M_{fo}}S_{fo}+y_{fi}\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\frac{\partial\sigma_f}{\partial t}\\
&-\phi_f\left(x_{fi}\frac{\rho_{fo}}{M_{fo}}S_{fo}+y_{fi}\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\beta_{pm}\frac{\partial\psi_m}{\partial t}+\phi_f\left(x_{fi}\frac{\rho_{fo}}{M_{fo}}S_{fo}+y_{fi}\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\beta_{pm}\frac{\partial P_{mg}}{\partial t}\\
&-\phi_f\left(x_{fi}\frac{\rho_{fo}}{M_{fo}}S_{fo}+y_{fi}\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\beta_{pf}\frac{\partial\psi_f}{\partial t}+\phi_f\left[\rho_{fo}\frac{\partial}{\partial t}\left(\frac{x_{fi}S_{fo}}{M_{fo}}\right)+\rho_{fg}\frac{\partial}{\partial t}\left(\frac{y_{fi}S_{fg}}{M_{fg}}\right)\right]\\
&-\frac{x_{fi}S_{fo}\rho_{fo}\phi_f}{M_{fo}}C_{fo}\frac{\partial P_{cfog}}{\partial t}+\frac{\phi_f x_{fi}S_{fo}\rho_{fo}}{M_{fo}}\left[C_{fo}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}+\frac{\phi_f y_{fi}\rho_{fg}S_{fg}}{M_{fg}}\left[C_{fg}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}\\
&-T
\end{aligned}
\tag{A1.52}
$$

A1.6 Ecuación de Presión

La ecuación de presión se obtiene sumando la ecuación (A1.51) y (A1.52) sobre la totalidad de los componentes de la mezcla, y teniendo en cuenta que:

$$\sum_{i=1}^{N}x_i=\sum_{i=1}^{N}y_i=1$$

Para la matriz:

$$
\begin{aligned}
&\nabla\cdot\left[\left(\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla P_{mg}\right]=\tilde{q}_T-\nabla\cdot\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial\overline{u}}{\partial t}+\nabla\cdot\left(\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{cmog}\right)\\
&-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\frac{\partial\sigma_m}{\partial t}-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pm}\frac{\partial\psi_m}{\partial t}\\
&+\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial P_{fg}}{\partial t}-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial\psi_f}{\partial t}\\
&+\phi_m\left[\rho_{mo}\frac{\partial}{\partial t}\left(\frac{S_{mo}}{M_{mo}}\right)+\rho_{mg}\frac{\partial}{\partial t}\left(\frac{S_{mg}}{M_{mg}}\right)\right]-\frac{S_{mo}\rho_{mo}\phi_m}{M_{mo}}C_{mo}\frac{\partial P_{comg}}{\partial t}+\frac{\phi_m S_{mo}\rho_{mo}}{M_{mo}}\left[C_{mo}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}\\
&+\frac{\phi_m\rho_{mg}S_{mg}}{M_{mg}}\left[C_{mg}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}+T
\end{aligned}
\tag{A1.53}
$$

Donde $\tilde{q}_T = \sum_{i=1}^{N} \tilde{q}_i$

Para la Fractura:

$$\nabla \cdot \left[\left(\frac{\rho_{fo} \vec{k}_f k_{rfo}}{M_{fo} \mu_{fo}} + \frac{\rho_{fg} \vec{k}_f k_{rfg}}{M_{fg} \mu_{fg}} \right) \nabla P_{fg} \right] = \tilde{q}_T - \nabla \cdot \left(\frac{\rho_{fo} \phi_f}{M_{fo}} + \frac{\rho_{fg} \phi_f}{M_{fg}} \right) \frac{\partial \bar{u}}{\partial t} + \nabla \cdot \left(\frac{\rho_{fo} \vec{k}_f k_{rfo}}{M_{fo} \mu_{fo}} \nabla P_{cfog} \right)$$
$$- \phi_f \left(\frac{\rho_{fo}}{M_{fo}} S_{fo} + \frac{\rho_{fg}}{M_{fg}} S_{fg} \right) c_{pcf} \frac{\partial \sigma_m}{\partial t} + \phi_f \left(\frac{\rho_{fo}}{M_{fo}} S_{fo} + \frac{\rho_{fg}}{M_{fg}} S_{fg} \right) c_{pcf} \beta_{pm} \frac{\partial P_{mg}}{\partial t}$$
$$- \phi_f \left(\frac{\rho_{fo}}{M_{fo}} S_{fo} + \frac{\rho_{fg}}{M_{fg}} S_{fg} \right) c_{pcf} \beta_{pm} \frac{\partial \psi_m}{\partial t} - \phi_f \left(\frac{\rho_{fo}}{M_{fo}} S_{fo} + \frac{\rho_{fg}}{M_{fg}} S_{fg} \right) c_{pcf} \beta_{pf} \frac{\partial \psi_f}{\partial t}$$
$$+ \phi_f \left[\rho_{fo} \frac{\partial}{\partial t} \left(\frac{S_{fo}}{M_{fo}} \right) + \rho_{fg} \frac{\partial}{\partial t} \left(\frac{S_{fg}}{M_{fg}} \right) \right] - \frac{S_{fo} \rho_{fo} \phi_f}{M_{fo}} C_{fo} \frac{\partial P_{cfog}}{\partial t} + \frac{\phi_f S_{fo} \rho_{fo}}{M_{fo}} \left[C_{fo} + c_{pcf} \beta_{pf} \right] \frac{\partial P_{fg}}{\partial t}$$
$$+ \frac{\phi_f \rho_{fg} S_{fg}}{M_{fg}} \left[C_{fg} + c_{pcf} \beta_{pf} \right] \frac{\partial P_{fg}}{\partial t} - T$$

(A1.54)

A1.7 Equilibrio de Fases y Ecuación de Composición

Para calcular la composición del fluido en cualquier lugar del yacimiento y en cualquier tiempo deseado se utiliza la ecuación de composición. Esta ecuación se encuentra haciendo un cambio de variables teniendo presente los fundamentos de las relaciones de equilibrio líquido – vapor. Desde estas bases puede demostrarse que:

$$z_{mi} = L_m x_{mi} + V_m y_{mi} \tag{A1.55}$$

Donde z_{mi} es la fracción molar del componente i en la mezcla. Las fracciones de líquido y vapor se definen de la siguiente manera

$$L_m = \frac{\frac{\rho_{mo} S_{mo}}{M_{mo}}}{\frac{\rho_{mo} S_{mo}}{M_{mo}} + \frac{\rho_{mg} S_{mg}}{M_{mg}}} \quad \text{y} \quad V_m = \frac{\frac{\rho_{mg} S_{mg}}{M_{mg}}}{\frac{\rho_{mo} S_{mo}}{M_{mo}} + \frac{\rho_{mg} S_{mg}}{M_{mg}}}$$

Reemplazando estos valores en la ecuación (A1.55) se llega a:

$$z_{mi} \left(\frac{\rho_{mo} S_{mo}}{M_{mo}} + \frac{\rho_{mg} S_{mg}}{M_{mg}} \right) = x_{mi} \frac{\rho_{mo} S_{mo}}{M_{mo}} + y_{mi} \frac{\rho_{mg} S_{mg}}{M_{mg}} \tag{A1.56}$$

Llevando la ecuación (A1.56) a la ecuación (A1.51)

$$
\begin{aligned}
&\nabla\cdot\left[\left(x_{mi}\,\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+y_{mi}\,\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla P_{mg}\right]=\tilde{q}_{mi}-\nabla\cdot\left(x_{mi}\,\frac{\rho_{mo}\phi_m}{M_{mo}}+y_{mi}\,\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial\bar{u}}{\partial t}+\\
&\nabla\cdot\left(x_{mi}\,\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{cmog}\right)-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)z_{mi}c_{pcm}\,\frac{\partial\sigma_m}{\partial t}\\
&-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)z_{mi}c_{pcm}\beta_{pm}\,\frac{\partial\psi_m}{\partial t}+\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)z_{mi}c_{pcm}\beta_{pf}\,\frac{\partial P_{fg}}{\partial t}\\
&-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)z_{mi}c_{pcm}\beta_{pf}\,\frac{\partial\psi_f}{\partial t}+\phi_m\left[\rho_{mo}\,\frac{\partial}{\partial t}\left(\frac{x_{mi}S_{mo}}{M_{mo}}\right)+\rho_{mg}\,\frac{\partial}{\partial t}\left(\frac{y_{mi}S_{mg}}{M_{mg}}\right)\right]\\
&-\frac{x_{mi}S_{mo}\rho_{mo}\phi_m}{M_{mo}}C_{mo}\,\frac{\partial P_{comg}}{\partial t}+\frac{\phi_m x_{mi}S_{mo}\rho_{mo}}{M_{mo}}\left[C_{mo}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}+\frac{\phi_m y_{mi}\rho_{mg}S_{mg}}{M_{mg}}\left[C_{mg}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}\\
&+T
\end{aligned}
$$

(A1.57)

De la misma manera para la Fractura:

$$
\begin{aligned}
&\nabla\cdot\left[\left(x_{fi}\,\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}+y_{fi}\,\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\right)\nabla P_{fg}\right]=\tilde{q}_{fi}-\nabla\cdot\left(x_{fi}\,\frac{\rho_{fo}\phi_f}{M_{fo}}+y_{fi}\,\frac{\rho_{fg}\phi_f}{M_{fg}}\right)\frac{\partial\bar{u}}{\partial t}+\\
&\nabla\cdot\left(x_{fi}\,\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}\nabla P_{cfog}\right)-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)z_{fi}c_{pcf}\,\frac{\partial\sigma_f}{\partial t}\\
&-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)z_{fi}c_{pcf}\beta_{pm}\,\frac{\partial\psi_m}{\partial t}+\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)z_{fi}c_{pcf}\beta_{pm}\,\frac{\partial P_{mg}}{\partial t}\\
&-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)z_{fi}c_{pcf}\beta_{pf}\,\frac{\partial\psi_f}{\partial t}+\phi_f\left[\rho_{fo}\,\frac{\partial}{\partial t}\left(\frac{x_{fi}S_{fo}}{M_{fo}}\right)+\rho_{fg}\,\frac{\partial}{\partial t}\left(\frac{y_{fi}S_{fg}}{M_{fg}}\right)\right]\\
&-\frac{x_{fi}S_{fo}\rho_{fo}\phi_f}{M_{fo}}C_{fo}\,\frac{\partial P_{cfog}}{\partial t}+\frac{\phi_f x_{fi}S_{fo}\rho_{fo}}{M_{fo}}\left[C_{fo}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}+\frac{\phi_f y_{fi}\rho_{fg}S_{fg}}{M_{fg}}\left[C_{fg}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}\\
&-T
\end{aligned}
$$

(A1.58)

A1.8 Modelo de Flujo de Fluidos en Forma Incremental

Para dar solución al problema acoplado de Geomecánica y flujo de fluidos composicionales se requiere la definición de valores iniciales para la presión de poro de la matriz y la fractura, y los desplazamientos. Los desplazamientos iniciales no se pueden obtener al momento inicial, por esta razón se trata a los desplazamientos y la presión de poro en forma incremental a partir de las condiciones iniciales, es decir, se calcula el cambio en las variables principales a medida que el yacimiento esta producido. La relación que tendría en cuenta lo anterior se puede escribir como $P_m\left(t+\Delta t\right)=P_m\left(t\right)+\Delta P_m$ o de la misma manera $P_m=P_m^0+\Delta P_m$. Se llegaría a:

$$
\begin{aligned}
&\nabla\cdot\left[\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+y_{mi}\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla P_{mg}^0\right]+\nabla\cdot\left[\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+y_{mi}\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla\Delta P_{mg}\right]=\tilde{q}_{mi}\\
&-\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\phi_m}{M_{mo}}+y_{mi}\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial\overline{u}}{\partial t}+\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{cmog}^0\right)+\nabla\cdot\left(x_{mi}\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla\Delta P_{cmog}\right)\\
&-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)z_{mi}c_{pcm}\frac{\partial\sigma_m}{\partial t}-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)z_{mi}c_{pcm}\beta_{pm}\frac{\partial\psi_m}{\partial t}\\
&+\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)z_{mi}c_{pcm}\beta_{pf}\frac{\partial P_{fg}}{\partial t}-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)z_{mi}c_{pcm}\beta_{pf}\frac{\partial\psi_f}{\partial t}\\
&+\phi_m\left[\rho_{mo}\frac{\partial}{\partial t}\left(\frac{x_{mi}S_{mo}}{M_{mo}}\right)+\rho_{mg}\frac{\partial}{\partial t}\left(\frac{y_{mi}S_{mg}}{M_{mg}}\right)\right]-\frac{x_{mi}S_{mo}\rho_{mo}\phi_m}{M_{mo}}C_{mo}\frac{\partial P_{comg}}{\partial t}+\frac{\phi_m x_{mi}S_{mo}\rho_{mo}}{M_{mo}}\left[C_{mo}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}\\
&+\frac{\phi_m y_{mi}\rho_{mg}S_{mg}}{M_{mg}}\left[C_{mg}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}+T
\end{aligned}
$$

(A1.60)

Para la fractura:

$$
\begin{aligned}
&\nabla\cdot\left[\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}+y_{fi}\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\right)\nabla P_{fg}^0\right]+\nabla\cdot\left[\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}+y_{fi}\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\right)\nabla\Delta P_{fg}\right]=\tilde{q}_{fi}\\
&-\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\phi_f}{M_{fo}}+y_{fi}\frac{\rho_{fg}\phi_f}{M_{fg}}\right)\frac{\partial\overline{u}}{\partial t}+\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}\nabla P_{cfog}^0\right)+\nabla\cdot\left(x_{fi}\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}\nabla\Delta P_{cfog}\right)\\
&-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)z_{fi}c_{pcf}\frac{\partial\sigma_f}{\partial t}-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)z_{fi}c_{pcf}\beta_{pm}\frac{\partial\psi_m}{\partial t}\\
&+\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)z_{fi}c_{pcf}\beta_{pm}\frac{\partial P_{mg}}{\partial t}-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)z_{fi}c_{pcf}\beta_{pf}\frac{\partial\psi_f}{\partial t}\\
&+\phi_f\left[\rho_{fo}\frac{\partial}{\partial t}\left(\frac{x_{fi}S_{fo}}{M_{fo}}\right)+\rho_{fg}\frac{\partial}{\partial t}\left(\frac{y_{fi}S_{fg}}{M_{fg}}\right)\right]-\frac{x_{fi}S_{fo}\rho_{fo}\phi_f}{M_{fo}}C_{fo}\frac{\partial P_{cfog}}{\partial t}+\frac{\phi_f x_{fi}S_{fo}\rho_{fo}}{M_{fo}}\left[C_{fo}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}\\
&+\frac{\phi_f y_{fi}\rho_{fg}S_{fg}}{M_{fg}}\left[C_{fg}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}-T
\end{aligned}
$$

(A1.61)

ANEXO 2. MODELO DE DEFORMACIÓN GEOMECÀNICA

El modelo Esfuerzo – Deformación que se plantean en este trabajo considera que la fase sólida se comporta como un medio elástico no lineal que sufre pequeñas deformaciones. Las relaciones principales en las que se basa este modelo son:

- Relaciones de Equilibrio de Esfuerzos
- Relaciones Deformación – Desplazamiento
- Relaciones Esfuerzo – Deformación - Presión

A2.1 Relaciones de Equilibrio de Esfuerzos

Para conservar el equilibrio de fuerzas después del cambio en la presión, se deben cumplir las siguientes ecuaciones de equilibrio de esfuerzos:

$$\sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} + F_i \qquad \sigma_{ij} = \sigma_{ji} \tag{A2.1}$$

Donde σ_{ij} es la componente ij del tensor de esfuerzos y F_i es la componente del vector de fuerzas de cuerpo que resultan de la fuerza de gravedad.

De la misma manera que se plantea una forma incremental para la presión se hace para los esfuerzos así:

$$\sigma_{ij}\left(t + \Delta t\right) = \sigma_{ij}\left(t\right) + \Delta \sigma_{ij}$$

o también

$$\sigma_{ij} = \sigma_{ij}^{o} + \Delta \sigma_{ij} \quad \text{o} \quad \Delta \sigma_{ij} = \sigma_{ij} - \sigma_{ij}^{o} \tag{A2.2}$$

A2.2 Relaciones Deformación – Desplazamiento

Las ecuaciones de Deformación – Desplazamiento en forma incremental son:

$$\Delta \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial \Delta u_i}{\partial x_j} + \frac{\partial \Delta u_j}{\partial x_i} \right) \qquad \Delta \varepsilon_{ij} = \Delta \varepsilon_{ji} \tag{A2.3}$$

$\Delta \varepsilon_{ij}$ representa la componente ij del tensor de deformaciones, Δu_i es la componente i del vector de desplazamiento.

$$\Delta \varepsilon_v = \Delta \varepsilon_{xx} + \Delta \varepsilon_{yy} + \Delta \varepsilon_{zz} = \frac{\partial \Delta u_x}{\partial x} + \frac{\partial \Delta u_y}{\partial y} + \frac{\partial \Delta u_z}{\partial z} \tag{A2.4}$$

A2.3 Relaciones Esfuerzo – Deformación – Presión

Para un sistema de doble porosidad, con fluido composicional como el que se tiene en cuenta en este trabajo se plantean las siguientes ecuaciones de esfuerzo – deformación - presión, que relacionan los esfuerzos efectivos con las deformaciones:

$$\Delta\sigma_{ij}^{'} = 2G\Delta\varepsilon_{ij} + \lambda\Delta\varepsilon_v\delta_{ij} \tag{A2.5}$$

Para un sistema de doble porosidad, con flujo composicional el esfuerzo efectivo se escribe como:

$$\Delta\sigma_{ij}^{'} = \Delta\sigma_{ij} - \beta_m\Delta\Omega_m\delta_{ij} - \beta_f\Delta\Omega_f\delta_{ij} \tag{A2.6}$$

Donde Ω_m y Ω_f dependen de la presión del gas y de la presión capilar como se menciona en el Anexo 1 y varían de acuerdo a los caso que se presentaron. Los valores de $\beta_m = \beta_{bm}$, $\beta_f = \beta_{bf}$ se utilizan cuando se pretende calcular un cambio en el volumen total de la matriz y la fractura respectivamente y $\beta_m = \beta_{pm}$, $\beta_f = \beta_{pf}$ cuando el cambio en el volumen se toma en el volumen poroso.

Combinando (A2.2), (A2.5), (A2.6) se llega a:

$$\sigma_{ij} = \sigma_{ij}^0 + 2G\Delta\varepsilon_{ij} + \lambda\Delta\varepsilon_v\delta_{ij} + \beta_m\Delta\Omega_m\delta_{ij} + \beta_f\Delta\Omega_f\delta_{ij} \tag{A2.7}$$

Reemplazando (A2.3) y (A2.4) en (A2.7) se obtiene:

$$\frac{\partial\sigma_{xx}^0}{\partial x} + \frac{\partial\sigma_{xy}^0}{\partial y} + \frac{\partial\sigma_{xz}^0}{\partial z} + 2G\left\{\frac{1}{2}\left(\frac{\partial\Delta u_x}{\partial x} + \frac{\partial\Delta u_x}{\partial x}\right) + \frac{1}{2}\left(\frac{\partial\Delta u_x}{\partial y} + \frac{\partial\Delta u_y}{\partial x}\right) + \frac{1}{2}\left(\frac{\partial\Delta u_x}{\partial z} + \frac{\partial\Delta u_z}{\partial x}\right)\right\}$$
$$+ \frac{\partial\left(\lambda\nabla\left(\Delta u\right)\right)}{\partial x} + \frac{\partial\left(\beta_m\Delta\Omega_m\right)}{\partial x} + \frac{\partial\left(\beta_f\Delta\Omega_f\right)}{\partial x} = 0$$

Agrupando adecuadamente y con las definiciones de divergente y gradiente las ecuaciones se pueden expresar:

En dirección X:

$$\frac{\partial\sigma_{xx}^0}{\partial x} + \frac{\partial\sigma_{xy}^0}{\partial y} + \frac{\partial\sigma_{xz}^0}{\partial z} + \nabla\cdot\left[G\nabla\left(\Delta u_x\right)\right] + \nabla\cdot\left[G\frac{\partial\left(\Delta u\right)}{\partial x}\right] + \frac{\partial\left(\lambda\nabla\left(\Delta u\right)\right)}{\partial x} + \frac{\partial\left(\beta_m\Delta\Omega_m\right)}{\partial x} + \frac{\partial\left(\beta_f\Delta\Omega_f\right)}{\partial x} = 0 \tag{A2.8}$$

En dirección Y:

$$\frac{\partial \sigma_{xy}^{0}}{\partial x}+\frac{\partial \sigma_{yy}^{0}}{\partial y}+\frac{\partial \sigma_{yz}^{0}}{\partial z}+\nabla\cdot\left[G\nabla\left(\Delta u_{y}\right)\right]+\nabla\cdot\left[G\frac{\partial\left(\Delta u\right)}{\partial y}\right]+\frac{\partial\left(\lambda\nabla\left(\Delta u\right)\right)}{\partial y}+\frac{\partial\left(\beta_{m}\Delta\Omega_{m}\right)}{\partial y}+\frac{\partial\left(\beta_{f}\Delta\Omega_{f}\right)}{\partial y}=0 \qquad \text{(A2.9)}$$

En dirección Z:

$$\frac{\partial \sigma_{xz}^{0}}{\partial x}+\frac{\partial \sigma_{yz}^{0}}{\partial y}+\frac{\partial \sigma_{zz}^{0}}{\partial z}+\nabla\cdot\left[G\nabla\left(\Delta u_{z}\right)\right]+\nabla\cdot\left[G\frac{\partial\left(\Delta u\right)}{\partial z}\right]+\frac{\partial\left(\lambda\nabla\left(\Delta u\right)\right)}{\partial z}+\frac{\partial\left(\beta_{m}\Delta\Omega_{m}\right)}{\partial z}+\frac{\partial\left(\beta_{f}\Delta\Omega_{f}\right)}{\partial z}=0 \qquad \text{(A2.10)}$$

ANEXO 3. ESQUEMA DE DISCRETIZACIÓN EN DIFERENCIAS FINITAS PARA UNA ECUACIÓN DIFERENCIAL

A continuación se muestra la discretización de forma generalizada de los términos que se presentan en las ecuaciones diferenciales que se observan en el Anexo 1 y Anexo 2 de este trabajo. La discretización se hace por medio de diferencias finitas en una malla de nodo centrado

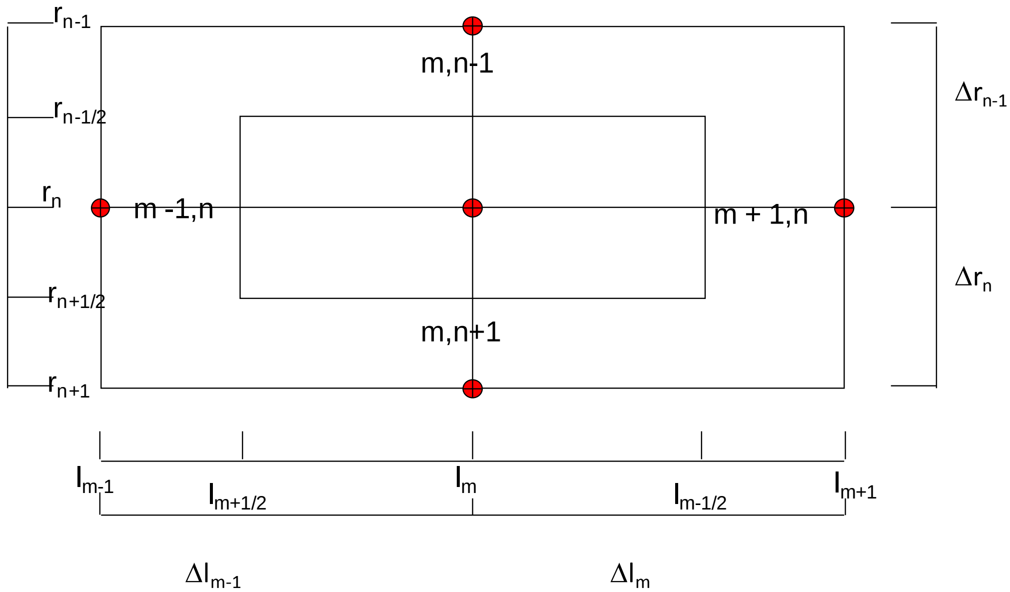

Figura A3. 1 Malla de nodo distribuido (o nodo centrado)

A3.1 Discretización del término: $\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial l}\right)$

Siguiendo la notación que se presenta en la Figura A3.2, el término $\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial l}\right)$ puede ser discretizado de la siguiente forma:

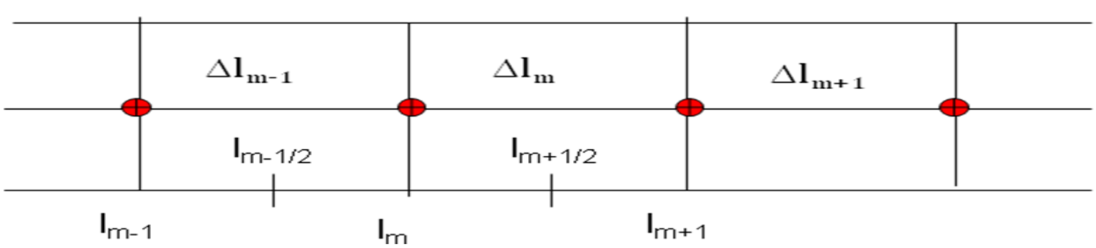

Figura A3. 2 Fila l de una malla de nodo centrado

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial l}\right)_m \cong \frac{\left(T\frac{\partial U}{\partial l}\right)_{lm+1/2} - \left(T\frac{\partial U}{\partial l}\right)_{lm-1/2}}{\frac{\Delta l_{m-1}+\Delta l_m}{2}}$$

Lo cual se puede escribir según las diferencias finitas como

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial l}\right)_m \cong \frac{T_{m+1/2}\left(\frac{U_{m+1}-U_m}{\Delta l_m}\right) - T_{m-1/2}\left(\frac{U_m-U_{m-1}}{\Delta l_{m-1}}\right)}{\frac{\Delta l_{m-1}+\Delta l_m}{2}}$$

O bien,

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial l}\right)_m \cong \frac{2}{\Delta l_{m-1}+\Delta l_m}\left[T_{m+1/2}\left(\frac{U_{m+1}-U_m}{\Delta l_m}\right) - T_{m-1/2}\left(\frac{U_m-U_{m-1}}{\Delta l_{m-1}}\right)\right]$$

Tratando de comprimir más la notación se realiza un cambio de variables como se puede observar a continuación

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial l}\right)_m \cong 2C_m T_{m+1/2}\left(U_{m+1}-U_m\right) - 2C_{m-1}T_{m-1/2}\left(U_m-U_{m-1}\right)$$

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial l}\right)_m \cong \left[2C_{m-1}T_{m-1/2}\right]U_{m-1} - \left[2\left(C_{m-1}T_{m-1/2}+C_m T_{m+1/2}\right)\right]U_m + \left[2C_m T_{m+1/2}\right]U_{m+1} \quad \text{(A3.1)}$$

Donde los términos C_m y C_{m-1} se definen de la siguiente forma:

$$C_m = \frac{1}{\Delta l_m\left(\Delta l_{m-1}+\Delta l_m\right)} \quad \text{(A3.2)}$$

$$C_{m-1} = \frac{1}{\Delta l_{m-1}\left(\Delta l_{m-1}+\Delta l_m\right)} \quad \text{(A3.3)}$$

A3.2 Discretización del término: $\frac{\partial U}{\partial l}$

De la Figura A3.2 se puede escribir:

$$\left(\frac{\partial U}{\partial l}\right)_m \cong \frac{U_{m+1/2}-U_{m-1/2}}{\left(\Delta l_m+\Delta l_{m-1}\right)/2}$$

O bien,

$$\left(\frac{\partial U}{\partial l}\right)_m \cong \frac{\left[\left(U_m - U_{m+1}\right)/2\right] - \left[\left(U_{m-1} - U_m\right)/2\right]}{\left(\Delta l_m + \Delta l_{m-1}\right)/2}$$

Simplificando

$$\left(\frac{\partial U}{\partial l}\right)_m \cong \frac{U_{m+1} - U_{m-1}}{\Delta l_m + \Delta l_{m-1}}$$

Para llevar a una notación más comprimida se define el operador f_m de la siguiente forma:

$$f_m = \frac{1}{\Delta l_m + \Delta l_{m-1}} \qquad \text{(A3.4)}$$

Llegando así a

$$\left(\frac{\partial U}{\partial l}\right)_m \cong f_m\left(U_{m+1} - U_{m-1}\right) \qquad \text{(A3.5)}$$

A3.3 Discretización del término: $\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial r}\right)$

Siguiendo la notación de un plano de una malla de nodo centrado tal como el ilustrado en la Figura A3.1, se tiene:

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial r}\right)_{m,n} \cong \frac{\left(T\frac{\partial U}{\partial r}\right)_{lm+1/2} - \left(T\frac{\partial U}{\partial r}\right)_{lm-1/2}}{\left(\Delta l_{m-1} + \Delta l_m\right)/2}$$

O bien,

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial r}\right)_{m,n} \cong \frac{T_{m+1/2}\left(\frac{\partial U}{\partial r}\right)_{m+1/2,n} - T_{m-1/2}\left(\frac{\partial U}{\partial r}\right)_{m-1/2,n}}{\left(\Delta l_{m-1} + \Delta l_m\right)/2} \qquad \text{(A3.6)}$$

Ahora bien, el término $\left(\dfrac{\partial U}{\partial r}\right)_{m+1/2,n}$ se desarrolla mediante diferencias finitas de la siguiente forma:

$$\left(\frac{\partial U}{\partial r}\right)_{m+1/2,n} \cong \frac{U_{m+1/2,n+1/2} - U_{m+1/2,n+1/2}}{\left(\Delta r_{n-1} + \Delta r_n\right)/2} \qquad \text{(A3.7)}$$

Teniendo en cuenta que:

$$U_{m+1/2,n+1/2} = \frac{U_{m,n} + U_{m+1,n} + U_{m,n+1} + U_{m+1,n+1}}{4}$$

Además,

$$U_{m+1/2,n-1/2} = \frac{U_{m,n-1} + U_{m+1,n-1} + U_{m,n} + U_{m+1,n}}{4}$$

Entonces la ecuación A3.7 puede ser escrita de cómo se puede observar en la ecuación A3.8:

$$\left(\frac{\partial U}{\partial r}\right)_{m+1/2,n} = \frac{\left(U_{m,n+1} + U_{m+1,n+1} - U_{m,n-1} - U_{m+1,n-1}\right)}{2\left(\Delta r_{n-1} + \Delta r_n\right)} \qquad \text{(A3.8)}$$

Similarmente:

$$\left(\frac{\partial U}{\partial r}\right)_{m-1/2,n} = \frac{\left(U_{m-1,n+1} + U_{m,n+1} - U_{m-1,n-1} - U_{m,n-1}\right)}{2\left(\Delta r_{n-1} + \Delta r_n\right)} \qquad \text{(A3.9)}$$

Introduciendo las definiciones encontradas en las Ecuaciones A3.8 y A3.9 a la Ecuación A3.6 se obtiene:

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial r}\right)_{m,n} \cong \frac{\left\{\dfrac{T_{m+1/2}}{2\left(\Delta r_{n-1} + \Delta r_n\right)}\left(U_{m,n+1} + U_{m+1,n+1} - U_{m,n-1} - U_{m+1,n-1}\right) - \dfrac{T_{m-1/2}}{2\left(\Delta r_{n-1} + \Delta r_n\right)}\left(U_{m-1,n+1} + U_{m,n+1} - U_{m-1,n-1} - U_{m,n-1}\right)\right\}}{\left(\Delta l_{m-1} + \Delta l_m\right)/2}$$

O también,

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial r}\right)_{m,n} \cong \frac{\left\{\mathrm{T}_{m+1/2}\left(U_{m,n+1} + U_{m+1,n+1} - U_{m,n-1} - U_{m+1,n-1}\right) - T_{m-1/2}\left(U_{m-1,n+1} + U_{m,n+1} - U_{m-1,n-1} - U_{m,n-1}\right)\right\}}{\left(\Delta r_{n-1} + \Delta r_n\right)\left(\Delta l_{m-1} + \Delta l_m\right)}$$

Considerando el término descrito en la ecuación A3.4, entonces:

$$\frac{\partial}{\partial l}\left(T\frac{\partial U}{\partial r}\right)_{m,n} = f_m \cdot f_n \left\{T_{m+1/2}\left(U_{m,n+1} + U_{m+1,n+1} - U_{m,n-1} - U_{m+1,n-1}\right) - T_{m-1/2}\left(U_{m-1,n+1} + U_{m,n+1} - U_{m-1,n-1} - U_{m,n-1}\right)\right. \quad \text{(A3.10)}$$

A3.4 Discretización del término: $\frac{\partial(TU)}{\partial l}$

El término diferencial $\frac{\partial(TU)}{\partial l}$ puede ser aproximado de la siguiente forma:

$$\frac{\partial}{\partial l}(TU) = \frac{T_{m+1/2}U_{m+1/2} - T_{m-1/2}U_{m-1/2}}{\left(\Delta l_m + \Delta l_{m-1}\right)/2}$$

$$\frac{\partial}{\partial l}(TU) = \frac{\left[T_{m+1/2}\left(U_m + U_{m+1}\right)/2\right] - \left[T_{m-1/2}\left(U_{m-1} + U_m\right)/2\right]}{\left(\Delta l_m + \Delta l_{m-1}\right)/2}$$

Considerando la ecuación A3.4 se llega a:

$$\frac{\partial}{\partial l}(TU) = f_m T_{m+1/2}\left(U_m + U_{m+1}\right) - f_m T_{m-1/2}\left(U_{m-1} + U_m\right)$$

Agrupando términos semejantes se puede tener la ecuación de una manera más simplificada

$$\frac{\partial}{\partial l}(TU) = -\left[f_m T_{m-1/2}\right]U_{m-1} - \left[f_m\left(T_{m-1/2} - T_{m+1/2}\right)\right]U_m + \left[f_m T_{m+1/2}\right]U_{m+1} \quad \text{(A3.11)}$$

A3.5 Discretización del término: $\left(\frac{\partial U_l}{\partial l}\right)_{m,n=1}$

La discretización del término $\left(\frac{\partial U_l}{\partial l}\right)_{m,n=1}$ se realiza siguiendo el procedimiento a continuación:

$$\left(\frac{\partial U_l}{\partial l}\right)_{m,n=1} = \frac{\left(U_{m+1/2} - U_{m-1/2}\right)}{\left(\Delta l_m + \Delta l_{m-1}\right)/2}$$

$$\left(\frac{\partial U_l}{\partial l}\right)_{m,n=1} = \frac{\left[\left(U_{m+1} + U_m\right)/2\right] - \left[\left(U_m + U_{m-1}\right)/2\right]}{\left(\Delta l_m + \Delta l_{m-1}\right)/2}$$

$$\left(\frac{\partial U_l}{\partial l}\right)_{m,n=1} = \frac{\left(U_{m+1,1} - U_{m-1,1}\right)}{\left(\Delta l_m + \Delta l_{m-1}\right)}$$

Recordando la ecuación A3.4 se puede escribir $\left(\frac{\partial U_l}{\partial l}\right)_{m,n=1}$ de la siguiente forma:

$$\left(\frac{\partial U_l}{\partial l}\right)_{m,n=1} = f_m\left(U_{m+1,1} - U_{m-1,1}\right) \tag{A3.12}$$

ANEXO 4. DISCRETIZACIÓN DE LAS ECUACIONES DEL MODELO COMPOSICIONAL DE FLUJO DE FLUIDOS EN LA MATRIZ

A4.1 Discretización de la ecuación de flujo de fluidos para matriz

Para la matriz la ecuación de flujo de fluidos composicionales en forma incremental es:

$$
\begin{aligned}
&\nabla\cdot\left[\left(\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla P_{mg}^0\right]+\nabla\cdot\left[\left(\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right)\nabla\Delta P_{mg}\right]=\tilde{q}_T\\
&-\nabla\cdot\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial\bar{u}}{\partial t}+\nabla\cdot\left(\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla P_{cmog}^0\right)+\nabla\cdot\left(\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}\nabla\Delta P_{cmog}\right)\\
&-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\frac{\partial\sigma_m}{\partial t}-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pm}\frac{\partial\psi_m}{\partial t}\\
&+\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial P_{fg}}{\partial t}-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial\psi_f}{\partial t}\\
&+\phi_m\left[\rho_{mo}\frac{\partial}{\partial t}\left(\frac{S_{mo}}{M_{mo}}\right)+\rho_{mg}\frac{\partial}{\partial t}\left(\frac{S_{mg}}{M_{mg}}\right)\right]-\frac{S_{mo}\rho_{mo}\phi_m}{M_{mo}}C_{mo}\frac{\partial P_{comg}}{\partial t}+\frac{\phi_m S_{mo}\rho_{mo}}{M_{mo}}\left[C_{mo}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}\\
&+\frac{\phi_m\rho_{mg}S_{mg}}{M_{mg}}\left[C_{mg}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}+T
\end{aligned}
$$

(A4.1)

Gradiente ∇, en coordenadas cartesianas:

$$\nabla U=\frac{\partial U}{\partial x}\vec{i}+\frac{\partial U}{\partial y}\vec{j}+\frac{\partial U}{\partial z}\vec{k} \qquad \text{(A4.2)}$$

Divergencia $\nabla\cdot$, en coordenadas cilíndricas:

$$\nabla\cdot F=\frac{\partial F_x}{\partial x}\vec{i}+\frac{\partial F_y}{\partial y}\vec{j}+\frac{\partial F_z}{\partial z}\vec{k} \qquad \text{(A4.3)}$$

Con el fin de manejar más fácilmente las ecuaciones se realizan los siguientes cambios de variables:

$$\lambda=\left(\frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}}+\frac{\rho_{mg}\vec{k}_m k_{rmg}}{M_{mg}\mu_{mg}}\right) \qquad \text{(A4.4a)}$$

De la misma manera:

$$\tau = \frac{\rho_{mo}\vec{k}_m k_{rmo}}{M_{mo}\mu_{mo}} \tag{A4.4b}$$

Aplicando las definiciones (A4.2), (A4.3) y el cambio de variable de las ecuaciones (A4.4a) y (A4.4b) en la ecuación (A4.1) se llega a:

$$\begin{aligned}
&\frac{\partial}{\partial x}\left(\lambda\frac{\partial P_{mg}^0}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial P_{mg}^0}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial P_{mg}^0}{\partial z}\right)+\frac{\partial}{\partial x}\left(\lambda\frac{\partial \Delta P_{mg}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial \Delta P_{mg}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial \Delta P_{mg}}{\partial z}\right)=\tilde{q}_T\\
&-\left(\frac{\partial}{\partial x}\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial \bar{u}}{\partial t}+\frac{\partial}{\partial y}\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial \bar{u}}{\partial t}+\frac{\partial}{\partial z}\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)\frac{\partial \bar{u}}{\partial t}\right)\\
&+\frac{\partial}{\partial x}\left(\tau\frac{\partial P_{cmog}^0}{\partial x}\right)+\frac{\partial}{\partial y}\left(\tau\frac{\partial P_{cmog}^0}{\partial y}\right)+\frac{\partial}{\partial z}\left(\tau\frac{\partial P_{cmog}^0}{\partial z}\right)+\frac{\partial}{\partial x}\left(\tau\frac{\partial \Delta P_{cmog}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\tau\frac{\partial \Delta P_{cmog}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\tau\frac{\partial \Delta P_{cmog}}{\partial z}\right)\\
&-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\frac{\partial \sigma_m}{\partial t}-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pm}\frac{\partial \psi_m}{\partial t}\\
&+\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial P_{fg}}{\partial t}-\phi_m\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)c_{pcm}\beta_{pf}\frac{\partial \psi_f}{\partial t}\\
&+\phi_m\left[\rho_{mo}\frac{\partial}{\partial t}\left(\frac{S_{mo}}{M_{mo}}\right)+\rho_{mg}\frac{\partial}{\partial t}\left(\frac{S_{mg}}{M_{mg}}\right)\right]-\frac{S_{mo}\rho_{mo}\phi_m}{M_{mo}}C_{mo}\frac{\partial P_{comg}}{\partial t}\\
&+\frac{\phi_m S_{mo}\rho_{mo}}{M_{mo}}\left[C_{mo}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}+\frac{\phi_m\rho_{mg}S_{mg}}{M_{mg}}\left[C_{mg}+c_{pcm}\beta_{pm}\right]\frac{\partial P_{mg}}{\partial t}+q_{Tm}
\end{aligned} \tag{A4.5}$$

Teniendo en cuenta lo establecido en el Anexo 3 se realiza la discretización de la ecuación (A4.5).

$$
\begin{aligned}
&2\left(C_{i-1}\lambda_{i-1/2,j,k}\,p^{0}{}_{mg\ i-1,j,k}-\left(C_{i-1}\lambda_{i-1/2,j,k}+C_{i}\lambda_{i+1/2,j,k}\right)p^{0}{}_{mg\ i,j,k}+C_{i}\lambda_{i+1/2,j,k}\,p^{0}{}_{mg\ i+1,j,k}\right)+\\
&2\left(C_{j-1}\lambda_{i,j-1/2,k}\,p^{0}{}_{mg\ i,j-1,k}-\left(C_{j-1}\lambda_{i,j-1/2,k}+C_{j}\lambda_{i,j+1/2,k}\right)p^{0}{}_{mg\ i,j,k}+C_{j}\lambda_{i,j+1/2,k}\,p^{0}{}_{mg\ i,j+1,k}\right)+\\
&2\left(C_{k-1}\lambda_{i,j,k-1/2}\,p^{0}{}_{m\ i,j,k-1}-\left(C_{k-1}\lambda_{i,j,k-1/2}+C_{k}\lambda_{i,j,k+1/2}\right)p^{0}{}_{mg\ i,j,k}+C_{k}\lambda_{i,j,k+1/2}\,p^{0}{}_{mg\ i,j,k+1}\right)+\\
&2\left(C_{i-1}\lambda_{i-1/2,j,k}\Delta p_{mg\ i-1,j,k}-\left(C_{i-1}\lambda_{i-1/2,j,k}+C_{i}\lambda_{i+1/2,j,k}\right)\Delta p_{mg\ i,j,k}+C_{i}\lambda_{i+1/2,j,k}\Delta p_{mg\ i+1,j,k}\right)+\\
&2\left(C_{j-1}\lambda_{i,j-1/2,k}\Delta p_{mg\ i,j-1,k}-\left(C_{j-1}\lambda_{i,j-1/2,k}+C_{j}\lambda_{i,j+1/2,k}\right)\Delta p_{mg\ i,j,k}+C_{j}\lambda_{i,j+1/2,k}\Delta p_{m\ i,j+1,k}\right)+\\
&2\left(C_{k-1}\lambda_{i,j,k-1/2}\Delta p_{mg\ i,j,k-1}-\left(C_{k-1}\lambda_{i,j,k-1/2}+C_{k}\lambda_{i,j,k+1/2}\right)\Delta p_{mg\ i,j,k}+C_{k}\lambda_{i,j,k+1/2}\Delta p_{mg\ i,j,k+1}\right)=\\
&q_{Ti,j,k}-\left\{\begin{array}{l}
\left[Fi\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i+1,j,k}-\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i-1,j,k}\right)*\left(\left(\frac{u^{n+1}_{x\,i,j,k}-u^{n}_{x\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{y\,i,j,k}-u^{n}_{y\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{z\,i,j,k}-u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]+\\
\left[Fj\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j+1,k}-\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j-1,k}\right)*\left(\left(\frac{u^{n+1}_{x\,i,j,k}-u^{n}_{x\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{y\,i,j,k}-u^{n}_{y\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{z\,i,j,k}-u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]+\\
\left[Fk\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k+1}-\left(\frac{\rho_{mo}\phi_m}{M_{mo}}+\frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k-1}\right)*\left(\left(\frac{u^{n+1}_{x\,i,j,k}-u^{n}_{x\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{y\,i,j,k}-u^{n}_{y\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{z\,i,j,k}-u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]
\end{array}\right\}+\\
&2\left(C_{i-1}\tau_{i-1/2,j,k}\,p^{0}{}_{cmog\ i-1,j,k}-\left(C_{i-1}\tau_{i-1/2,j,k}+C_{i}\tau_{i+1/2,j,k}\right)p^{0}{}_{cmog\ i,j,k}+C_{i}\tau_{i+1/2,j,k}\,p^{0}{}_{cmog\ i+1,j,k}\right)\\
&2\left(C_{j-1}\tau_{i,j-1/2,k}\,p^{0}{}_{cmog\ i,j-1,k}-\left(C_{j-1}\tau_{i,j-1/2,k}+C_{j}\tau_{i,j+1/2,k}\right)p^{0}{}_{cmog\ i,j,k}+C_{j}\tau_{i,j+1/2,k}\,p^{0}{}_{cmog\ i,j+1,k}\right)+\\
&2\left(C_{k-1}\tau_{i,j,k-1/2}\,p^{0}{}_{cmog\ i,j,k-1}-\left(C_{k-1}\tau_{i,j,k-1/2}+C_{k}\tau_{i,j,k+1/2}\right)p^{0}{}_{cmog\ i,j,k}+C_{k}\tau_{i,j,k+1/2}\,p^{0}{}_{cmog\ i,j,k+1}\right)+\\
&2\left(C_{i-1}\tau_{i-1/2,j,k}\Delta p_{cmog\ i-1,j,k}-\left(C_{i-1}\tau_{i-1/2,j,k}+C_{i}\tau_{i+1/2,j,k}\right)\Delta p_{cmog\ i,j,k}+C_{i}\tau_{i+1/2,j,k}\Delta p_{cmog\ i+1,j,k}\right)+\\
&2\left(C_{j-1}\tau_{i,j-1/2,k}\Delta p_{cmog\ i,j-1,k}-\left(C_{j-1}\tau_{i,j-1/2,k}+C_{j}\tau_{i,j+1/2,k}\right)\Delta p_{cmog\ i,j,k}+C_{j}\tau_{i,j+1/2,k}\Delta p_{cmog\ i,j+1,k}\right)+\\
&2\left(C_{k-1}\tau_{i,j,k-1/2}\Delta p_{cmog\ i,j,k-1}-\left(C_{k-1}\tau_{i,j,k-1/2}+C_{k}\tau_{i,j,k+1/2}\right)\Delta p_{cmog\ i,j,k}+C_{k}\tau_{i,j,k+1/2}\Delta p_{cmog\ i,j,k+1}\right)-\\
&\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k}c_{pcmi,j,k}\left(\frac{\sigma^{n+1}_{mi,j,k}-\sigma^{n}_{mi,j,k}}{\Delta t}\right)\\
&-\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k}c_{pcmi,j,k}\beta_{pmi,j,k}\left(\frac{\psi^{n+1}_{mi,j,k}-\psi^{n}_{mi,j,k}}{\Delta t}\right)\\
&+\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k}c_{pcmi,j,k}\beta_{pfi,j,k}\frac{\Delta P_{fgi,j,k}}{\Delta t}\\
&-\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo}+\frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k}c_{pcmi,j,k}\beta_{pfi,j,k}\left(\frac{\psi^{n+1}_{fi,j,k}-\psi^{n}_{fi,j,k}}{\Delta t}\right)\\
&+\phi_{mi,j,k}\left[\frac{\rho_{mo}}{M_{mo}}\left(\frac{S^{n+1}_{moi,j,k}-S^{n}_{moi,j,k}}{\Delta t}\right)+\frac{\rho_{mg}}{M_{mg}}\left(\frac{S^{n+1}_{mgi,j,k}-S^{n}_{mgi,j,k}}{\Delta t}\right)\right]-\frac{S_{moi,j,k}\rho_{moi,j,k}\phi_m}{M_{moi,j,k}}C_{moi,j,k}\frac{\Delta P_{comg}}{\Delta t}\\
&+\frac{\phi_{mi,j,k}S_{moi,j,k}\rho_{moi,j,k}}{M_{moi,j,k}}\left[C_{moi,j,k}+c_{pcmi,j,k}\beta_{pmi,j,k}\right]\frac{\Delta P_{mgi,j,k}}{\Delta t}\\
&+\frac{\phi_{mi,j,k}\rho_{mgi,j,k}S_{mgi,j,k}}{M_{mgi,j,k}}\left[C_{mgi,j,k}+c_{pcmi,j,k}\beta_{pmi,j,k}\right]\frac{\Delta P_{mg}}{\Delta t}+q_{Tm}
\end{aligned}
$$

(A4.6)

Esta ecuación puede ser simplificada haciendo un cambio de variable como se puede observar

$$T_{i-1,j,k} = 2C_{i-1}\lambda_{i-1/2,j,k} \quad \text{(A4.7a)}$$

$$T_{i+1,j,k} = 2C_{i+1}\lambda_{i+1/2,j,k} \quad \text{(A4.7b)}$$

$$T_{i,j-1,k} = 2C_{j-1}\lambda_{i,j-1/2,k} \quad \text{(A4.7c)}$$

$$T_{i,j+1,k} = 2C_{j+1}\lambda_{i,j+1/2,k} \quad \text{(A4.7d)}$$

$$T_{i,j,k-1} = 2C_{k-1}\lambda_{i,j,k-1/2} \quad \text{(A4.7e)}$$

$$T_{i,j,k+1} = 2C_{k+1}\lambda_{i,j,k+1/2} \quad \text{(A4.7f)}$$

$$T_{Ci-1,j,k} = 2C_{i-1}\tau_{i-1/2,j,k} \quad \text{(A4.7g)}$$

$$T_{Ci+1,j,k} = 2C_{i+1}\tau_{i+1/2,j,k} \quad \text{(A4.7h)}$$

$$T_{Ci,j-1,k} = 2C_{j-1}\tau_{i,j-1/2,k} \quad \text{(A4.7i)}$$

$$T_{Ci,j+1,k} = 2C_{j+1}\tau_{i,j+1/2,k} \quad \text{(A4.7j)}$$

$$T_{Ci,j,k-1} = 2C_{k-1}\tau_{i,j,k-1/2} \quad \text{(A4.7k)}$$

$$T_{Ci,j,k+1} = 2C_{k+1}\tau_{i,j,k+1/2} \quad \text{(A4.7l)}$$

La ecuación quedaría

$$\left(T_{i-1/2,j,k}\, p^0{}_{mg\ i-1,j,k} - \left(T_{i-1/2,j,k} + T_{i+1/2,j,k}\right) p^0{}_{mg\ i,j,k} + T_{i+1/2,j,k}\, p^0{}_{mg\ i+1,j,k}\right) +$$
$$\left(T_{i,j-1/2,k}\, p^0{}_{mg\ i,j-1,k} - \left(T_{i,j-1/2,k} + T_{i,j+1/2,k}\right) p^0{}_{mg\ i,j,k} + T_{i,j+1/2,k}\, p^0{}_{mg\ i,j+1,k}\right) +$$
$$\left(T_{i,j,k-1/2}\, p^0{}_{m\ i,j,k-1} - \left(T_{i,j,k-1/2} + C_k \lambda_{i,j,k+1/2}\right) p^0{}_{mg\ i,j,k} + T_{i,j,k+1/2}\, p^0{}_{mg\ i,j,k+1}\right) +$$
$$\left(T_{i-1/2,j,k} \Delta p_{mg\ i-1,j,k} - \left(T_{i-1/2,j,k} + T_{i+1/2,j,k}\right) \Delta p_{mg\ i,j,k} + T_{i+1/2,j,k} \Delta p_{mg\ i+1,j,k}\right) +$$
$$\left(T_{i,j-1/2,k} \Delta p_{mg\ i,j-1,k} - \left(T_{i,j-1/2,k} + T_{i,j+1/2,k}\right) \Delta p_{mg\ i,j,k} + T_{i,j+1/2,k} \Delta p_{m\ i,j+1,k}\right) +$$
$$\left(T_{i,j,k-1/2} \Delta p_{mg\ i,j,k-1} - \left(T_{i,j,k-1/2} + T_{i,j,k+1/2}\right) \Delta p_{mg\ i,j,k} + T_{i,j,k+1/2} \Delta p_{mg\ i,j,k+1}\right) =$$
$$q_{Ti,j,k} - \left\{ \begin{array}{l} \left[Fi\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i+1,j,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i-1,j,k} \right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^n_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^n_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^n_{z\,i,j,k}}{\Delta t}\right) \right) \right] + \\ \left[Fj\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j+1,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j-1,k} \right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^n_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^n_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^n_{z\,i,j,k}}{\Delta t}\right) \right) \right] + \\ \left[Fk\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k+1} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k-1} \right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^n_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^n_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^n_{z\,i,j,k}}{\Delta t}\right) \right) \right] \end{array} \right\} +$$
$$\left(T_{Ci-1/2,j,k}\, p^0{}_{cmog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right) p^0{}_{cmog\ i,j,k} + T_{Ci+1/2,j,k}\, p^0{}_{cmog\ i+1,j,k}\right)$$
$$\left(T_{Ci,j-1/2,k}\, p^0{}_{cmog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right) p^0{}_{cmog\ i,j,k} + T_{Ci,j+1/2,k}\, p^0{}_{cmog\ i,j+1,k}\right) +$$
$$\left(T_{Ci,j,k-1/2}\, p^0{}_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right) p^0{}_{cmog\ i,j,k} + T_{Ci,j,k+1/2}\, p^0{}_{cmog\ i,j,k+1}\right) +$$
$$\left(T_{Ci-1/2,j,k} \Delta p_{cmog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right) \Delta p_{cmog\ i,j,k} + T_{Ci+1/2,j,k} \Delta p_{cmog\ i+1,j,k}\right) +$$
$$\left(T_{Ci,j-1/2,k} \Delta p_{cmog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right) \Delta p_{cmog\ i,j,k} + T_{Ci,j+1/2,k} \Delta p_{cmog\ i,j+1,k}\right) +$$
$$\left(T_{Ci,j,k-1/2} \Delta p_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right) \Delta p_{cmog\ i,j,k} + T_{Ci,j,k+1/2} \Delta p_{cmog\ i,j,k+1}\right) -$$
$$\phi_{mi,j,k} \left(\frac{\rho_{mo}}{M_{mo}} S_{mo} + \frac{\rho_{mg}}{M_{mg}} S_{mg}\right)_{i,j,k} c_{pcmi,j,k} \left(\frac{\sigma^{n+1}_{mi,j,k} - \sigma^n_{mi,j,k}}{\Delta t}\right)$$
$$-\phi_{mi,j,k} \left(\frac{\rho_{mo}}{M_{mo}} S_{mo} + \frac{\rho_{mg}}{M_{mg}} S_{mg}\right)_{i,j,k} c_{pcmi,j,k} \beta_{pmi,j,k} \left(\frac{\psi^{n+1}_{mi,j,k} - \psi^n_{mi,j,k}}{\Delta t}\right)$$
$$+\phi_{mi,j,k} \left(\frac{\rho_{mo}}{M_{mo}} S_{mo} + \frac{\rho_{mg}}{M_{mg}} S_{mg}\right)_{i,j,k} c_{pcmi,j,k} \beta_{pfi,j,k} \frac{\Delta P_{fgi,j,k}}{\Delta t}$$
$$-\phi_{mi,j,k} \left(\frac{\rho_{mo}}{M_{mo}} S_{mo} + \frac{\rho_{mg}}{M_{mg}} S_{mg}\right)_{i,j,k} c_{pcmi,j,k} \beta_{pfi,j,k} \left(\frac{\psi^{n+1}_{fi,j,k} - \psi^n_{fi,j,k}}{\Delta t}\right)$$
$$+\phi_{mi,j,k} \left[\frac{\rho_{mo}}{M_{mo}} \left(\frac{S^{n+1}_{moi,j,k} - S^n_{moi,j,k}}{\Delta t}\right) + \frac{\rho_{mg}}{M_{mg}} \left(\frac{S^{n+1}_{mgi,j,k} - S^n_{mgi,j,k}}{\Delta t}\right)\right] - \frac{S_{moi,j,k}\rho_{moi,j,k}\phi_m}{M_{moi,j,k}} C_{moi,j,k} \frac{\Delta P_{comg}}{\Delta t}$$
$$+\frac{\phi_{mi,j,k} S_{moi,j,k} \rho_{moi,j,k}}{M_{moi,j,k}} \left[C_{moi,j,k} + c_{pcmi,j,k} \beta_{pmi,j,k}\right] \frac{\Delta P_{mgi,j,k}}{\Delta t}$$
$$+\frac{\phi_{mi,j,k} \rho_{mgi,j,k} S_{mgi,j,k}}{M_{mgi,j,k}} \left[C_{mgi,j,k} + c_{pcmi,j,k} \beta_{pmi,j,k}\right] \frac{\Delta P_{mg}}{\Delta t} + q_{Tm}$$

La Ecuación puede escribirse de una forma más simplificada como:

$$BCm_{i,j,k}\Delta p_{m\ i,j,k-1}^{n+1}+Sm_{i,j,k}\Delta p_{m\ i,j-1,k}^{n+1}+Wm_{i,j,k}\Delta p_{m\ i-1,j,k}^{n+1}+Cm_{i,j,k}\Delta p_{m\ i,j,k}^{n+1}+Em_{i,j,k}\Delta p_{m\ i+1,j,k}^{n+1}$$
$$+Nm_{i,j,k}\Delta p_{m\ i,j+1,k}^{n+1}+TCm_{i,j,k}\Delta p_{m\ i,j,k+1}^{n+1}=Fm_{i,j,k} \quad \text{(A4.8)}$$

Donde:

$$BCm_{i,j,k}=T_{i,j,k-1/2} \quad \text{(A4.8a)}$$

$$Sm_{i,j,k}=T_{i,j-1/2,k} \quad \text{(A4.8b)}$$

$$Wm_{i,j,k}=T_{i-1/2,j,k} \quad \text{(A4.8c)}$$

$$Em_{i,j,k}=T_{i+1/2,j,k} \quad \text{(A4.8d)}$$

$$Nm_{i,j,k}=T_{i,j+1/2,k} \quad \text{(A4.8e)}$$

$$TCm_{i,j,k}=T_{i,j,k+1/2} \quad \text{(A4.8f)}$$

$$Cm_{i,j,k}=-T_{i-1/2,j,k}-T_{i+1/2,j,k}-T_{i,j-1/2,k}-T_{i,j+1/2,k}-T_{i,j,k-1/2}-T_{i,j,k+1/2}$$
$$-\frac{\phi_{mi,j,k}S_{moi,j,k}\rho_{moi,j,k}}{M_{moi,j,k}*\Delta t}\left(c_{mo\ i,j,k}+c_{pcm\ i,j,k}\beta_{pm\ i,j,k}\right)-\frac{\phi_{mi,j,k}S_{mgi,j,k}\rho_{mgi,j,k}}{M_{mgi,j,k}*\Delta t}\left(c_{mg\ i,j,k}+c_{pcm\ i,j,k}\beta_{pm\ i,j,k}\right)$$

(A4.8g)

$$Fm_{i,j,k} = -\left(T_{i-1/2,j,k}\,p^0{}_{mg\ i-1,j,k} - \left(T_{i-1/2,j,k} + T_{i+1/2,j,k}\right) p^0{}_{mg\ i,j,k} + T_{i+1/2,j,k}\,p^0{}_{mg\ i+1,j,k}\right)$$

$$-\left(T_{i,j-1/2,k}\,p^0{}_{mg\ i,j-1,k} - \left(T_{i,j+1/2,k} + T_{i,j-1/2,k}\right) p^0{}_{mg\ i,j,k} + T_{i,j+1/2,k}\,p^0{}_{mg\ i,j+1,k}\right) -$$

$$\left(T_{i,j,k-1/2}\,p^0{}_{m\ i,j,k-1} - \left(T_{i,j,k+1/2} + T_{i,j,k-1/2}\right) p^0{}_{m\ i,j,k} + T_{i,j,k+1/2}\,p^0{}_{m\ i,j,k+1}\right) + q_{Ti,j,k}$$

$$-\left\{\begin{array}{l}\left[Fi\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i+1,j,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i-1,j,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^n_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^n_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^n_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\ \left[Fj\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j+1,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j-1,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^n_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^n_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^n_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\ \left[Fk\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k+1} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k-1}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^n_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^n_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^n_{z\,i,j,k}}{\Delta t}\right)\right)\right]\end{array}\right\} -$$

$$\left(T_{Ci-1/2,j,k}\,p^0{}_{cmog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right) p^0{}_{cmog\ i,j,k} + T_{Ci+1/2,j,k}\,p^0{}_{cmog\ i+1,j,k}\right)$$

$$\left(T_{Ci,j-1/2,k}\,p^0{}_{cmog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right) p^0{}_{cmog\ i,j,k} + T_{Ci,j+1/2,k}\,p^0{}_{cmog\ i,j+1,k}\right) +$$

$$\left(T_{Ci,j,k-1/2}\,p^0{}_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right) p^0{}_{cmog\ i,j,k} + T_{Ci,j,k+1/2}\,p^0{}_{cmog\ i,j,k+1}\right) +$$

$$\left(T_{Ci-1/2,j,k}\Delta p_{cmog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right)\Delta p_{cmog\ i,j,k} + T_{Ci+1/2,j,k}\Delta p_{cmog\ i+1,j,k}\right) +$$

$$\left(T_{Ci,j-1/2,k}\Delta p_{cmog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right)\Delta p_{cmog\ i,j,k} + T_{Ci,j+1/2,k}\Delta p_{cmog\ i,j+1,k}\right) +$$

$$\left(T_{Ci,j,k-1/2}\Delta p_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right)\Delta p_{cmog\ i,j,k} + T_{Ci,j,k+1/2}\Delta p_{cmog\ i,j,k+1}\right) -$$

$$\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\left(\frac{\sigma^{n+1}_{mi,j,k} - \sigma^n_{mi,j,k}}{\Delta t}\right)$$

$$-\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\beta_{pmi,j,k}\left(\frac{\psi^{n+1}_{mi,j,k} - \psi^n_{mi,j,k}}{\Delta t}\right)$$

$$+\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\beta_{pfi,j,k}\frac{\Delta P_{fgi,j,k}}{\Delta t}$$

$$-\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\beta_{pfi,j,k}\left(\frac{\psi^{n+1}_{fi,j,k} - \psi^n_{fi,j,k}}{\Delta t}\right)$$

$$+\phi_{mi,j,k}\left[\frac{\rho_{mo}}{M_{mo}}\left(\frac{S^{n+1}_{moi,j,k} - S^n_{moi,j,k}}{\Delta t}\right) + \frac{\rho_{mg}}{M_{mg}}\left(\frac{S^{n+1}_{mgi,j,k} - S^n_{mgi,j,k}}{\Delta t}\right)\right] - \frac{S_{moi,j,k}\rho_{moi,j,k}\phi_m}{M_{moi,j,k}}C_{moi,j,k}\frac{\Delta P_{comg}}{\Delta t} + q_{Tmi,j,k} + q_{i,j,k}$$

(A4.8h)

A4.2 Discretización de la ecuación de transferencia de fluidos de matriz a fractura

Sabiendo que el término de transferencia es igual a $q_T = q_{To} + q_{Tg}$ se llega a la ecuación siguiente:

$$q_T = \left(8 \frac{\rho_{om}}{\mu_o M_{om}} \left(\frac{k_{xm} k_{ro\ xm}}{l_x^2} + \frac{k_{ym} k_{ro\ ym}}{l_y^2} + \frac{k_{zm} k_{ro\ zm}}{l_z^2} \right) + 8 \frac{\rho_{gm}}{\mu_g M_{gm}} \left(\frac{k_{xm} k_{rg\ xm}}{l_x^2} + \frac{k_{ym} k_{rg\ ym}}{l_y^2} + \frac{k_{zm} k_{rg\ zm}}{l_z^2} \right) \right) \left(\Omega_m - \Omega_f \right) \tag{A4.9}$$

Discretizando y escribiendo la ecuación (A4.9) en forma incremental se llega a

$$q_{Tm} = \left(8 \frac{\rho_{om}}{\mu_o M_{om}} \left(\frac{k_{xm} k_{ro\ xm}}{l_x^2} + \frac{k_{ym} k_{ro\ ym}}{l_y^2} + \frac{k_{zm} k_{ro\ zm}}{l_z^2} \right) + 8 \frac{\rho_{gm}}{\mu_g M_{gm}} \left(\frac{k_{xm} k_{rg\ xm}}{l_x^2} + \frac{k_{ym} k_{rg\ ym}}{l_y^2} + \frac{k_{zm} k_{rg\ zm}}{l_z^2} \right) \right)_{i,j,k} \left(\left(\Omega_m^0 + \Delta\Omega_m \right) - \left(\Omega_f^0 + \Delta\Omega_f \right) \right) \tag{A4.10}$$

A4.3 Discretización de la ecuación diferencial incluyendo la transferencia matriz – fractura

A continuación se presentan las ecuaciones de flujo de fluidos según el caso con el que se va a trabajar discretizadas y en forma de esténcil

Caso 1:

$\Omega_m = P_{mg}$, $\psi_m = 0$ y $\Omega_f = P_{fg}$, $\psi_f = 0$

La ecuación (A4.10) se escribe como

$$q_{Tm} = \left(8 \frac{\rho_{om}}{\mu_o M_{om}} \left(\frac{k_{xm} k_{ro\ xm}}{l_x^2} + \frac{k_{ym} k_{ro\ ym}}{l_y^2} + \frac{k_{zm} k_{ro\ zm}}{l_z^2} \right) + 8 \frac{\rho_{gm}}{\mu_g M_{gm}} \left(\frac{k_{xm} k_{rg\ xm}}{l_x^2} + \frac{k_{ym} k_{rg\ ym}}{l_y^2} + \frac{k_{zm} k_{rg\ zm}}{l_z^2} \right) \right)_{i,j,k} \left(\left(P_{mg}^0 + \Delta P_{mg} \right) - \left(P_{fg}^0 + \Delta P_{fg} \right) \right) \tag{A4.10a}$$

El esténcil central se puede escribir como:

$$\begin{aligned} Cm_{i,j,k} = & -T_{i-1/2,j,k} - T_{i+1/2,j,k} - T_{i,j-1/2,k} - T_{i,j+1/2,k} - T_{i,j,k-1/2} - T_{i,j,k+1/2} \\ & - \frac{\phi_{mi,j,k} S_{moi,j,k} \rho_{moi,j,k}}{M_{moi,j,k} * \Delta t} \left(c_{mo\ i,j,k} + c_{pcm\ i,j,k} \beta_{pm\ i,j,k} \right) - \frac{\phi_{mi,j,k} S_{mgi,j,k} \rho_{mgi,j,k}}{M_{mgi,j,k} * \Delta t} \left(c_{mg\ i,j,k} + c_{pcm\ i,j,k} \beta_{pm\ i,j,k} \right) \\ & - \left(8 \frac{\rho_{om}}{\mu_o M_{om}} \left(\frac{k_{xm} k_{ro\ xm}}{l_x^2} + \frac{k_{ym} k_{ro\ ym}}{l_y^2} + \frac{k_{zm} k_{ro\ zm}}{l_z^2} \right) + 8 \frac{\rho_{gm}}{\mu_g M_{gm}} \left(\frac{k_{xm} k_{rg\ xm}}{l_x^2} + \frac{k_{ym} k_{rg\ ym}}{l_y^2} + \frac{k_{zm} k_{rg\ zm}}{l_z^2} \right) \right)_{i,j,k} \end{aligned}$$

El esténcil libre se puede escribir como:

$$
\begin{aligned}
&Fm_{i,j,k} = -\left(T_{i-1/2,j,k}\,p^0{}_{mg\ i-1,j,k} - \left(T_{i-1/2,j,k} + T_{i+1/2,j,k}\right)p^0{}_{mg\ i,j,k} + T_{i+1/2,j,k}\,p^0{}_{mg\ i+1,j,k}\right) \\
&-\left(T_{i,j-1/2,k}\,p^0{}_{mg\ i,j-1,k} - \left(T_{i,j+1/2,k} + T_{i,j-1/2,k}\right)p^0{}_{mg\ i,j,k} + T_{i,j+1/2,k}\,p^0{}_{mg\ i,j+1,k}\right) - \\
&\left(T_{i,j,k-1/2}\,p^0{}_{mg\ i,j,k-1} - \left(T_{i,j,k+1/2} + T_{i,j,k-1/2}\right)p^0{}_{mg\ i,j,k} + T_{i,j,k+1/2}\,p^0{}_{mg\ i,j,k+1}\right) + q_{Ti,j,k} \\
&-\left\{
\begin{aligned}
&\left[Fi\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i+1,j,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i-1,j,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\
&\left[Fj\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j+1,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j-1,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\
&\left[Fk\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k+1} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k-1}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]
\end{aligned}
\right\} + T - \\
&\left(T_{Ci-1/2,j,k}\,p^0{}_{cmog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right)p^0{}_{cmog\ i,j,k} + T_{Ci+1/2,j,k}\,p^0{}_{cmog\ i+1,j,k}\right) \\
&\left(T_{Ci,j-1/2,k}\,p^0{}_{cmog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right)p^0{}_{cmog\ i,j,k} + T_{Ci,j+1/2,k}\,p^0{}_{cmog\ i,j+1,k}\right) + \\
&\left(T_{Ci,j,k-1/2}\,p^0{}_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right)p^0{}_{cmog\ i,j,k} + T_{Ci,j,k+1/2}\,p^0{}_{cmog\ i,j,k+1}\right) + \\
&\left(T_{Ci-1/2,j,k}\,\Delta p_{cmog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right)\Delta p_{cmog\ i,j,k} + T_{Ci+1/2,j,k}\,\Delta p_{cmog\ i+1,j,k}\right) + \\
&\left(T_{Ci,j-1/2,k}\,\Delta p_{cmog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right)\Delta p_{cmog\ i,j,k} + T_{Ci,j+1/2,k}\,\Delta p_{cmog\ i,j+1,k}\right) + \\
&\left(T_{Ci,j,k-1/2}\,\Delta p_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right)\Delta p_{cmog\ i,j,k} + T_{Ci,j,k+1/2}\,\Delta p_{cmog\ i,j,k+1}\right) - \\
&\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\left(\frac{\sigma^{n+1}_{mi,j,k} - \sigma^{n}_{mi,j,k}}{\Delta t}\right) \\
&+\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\,\beta_{pfi,j,k}\,\frac{\Delta P_{fgi,j,k}}{\Delta t} \\
&+\phi_{mi,j,k}\left[\frac{\rho_{mo}}{M_{mo}}\left(\frac{S^{n+1}_{moi,j,k} - S^{n}_{moi,j,k}}{\Delta t}\right) + \frac{\rho_{mg}}{M_{mg}}\left(\frac{S^{n+1}_{mgi,j,k} - S^{n}_{mgi,j,k}}{\Delta t}\right)\right] - \frac{S_{moi,j,k}\,\rho_{moi,j,k}\,\phi_m}{M_{moi,j,k}}C_{moi,j,k}\,\frac{\Delta P_{comg}}{\Delta t} + \\
&8*\left(
\begin{aligned}
&\frac{\rho_{om}}{\mu_o M_{om}}\left(\frac{k_{xm}k_{ro\ xm}}{l_x^2} + \frac{k_{ym}k_{ro\ ym}}{l_y^2} + \frac{k_{zm}k_{ro\ zm}}{l_z^2}\right) + \\
&\frac{\rho_{gm}}{\mu_g M_{gm}}\left(\frac{k_{xm}k_{rg\ xm}}{l_x^2} + \frac{k_{ym}k_{rg\ ym}}{l_y^2} + \frac{k_{zm}k_{rg\ zm}}{l_z^2}\right)
\end{aligned}
\right)_{i,j,k}\left(P^0_{mg} - P^0_{fg} - \Delta P_{fg}\right)
\end{aligned}
$$

<u>Caso 2:</u>

$\Omega_m = P_{mo}$, $\psi_m = P_{mcog}$ y $\Omega_f = P_{fo}$, $\psi_f = P_{fcog}$

Entonces la ecuación (A4.10) se escribe como

$$q_{Tm}=\left(8\frac{\rho_{om}}{\mu_o M_{om}}\left(\frac{k_{xm}k_{ro\ xm}}{l_x^2}+\frac{k_{ym}k_{ro\ ym}}{l_y^2}+\frac{k_{zm}k_{ro\ zm}}{l_z^2}\right)+8\frac{\rho_{gm}}{\mu_g M_{gm}}\left(\frac{k_{xm}k_{rg\ xm}}{l_x^2}+\frac{k_{ym}k_{rg\ ym}}{l_y^2}+\frac{k_{zm}k_{rg\ zm}}{l_z^2}\right)\right)_{i,j,k} * \left(\left(P_{mg}^0+\Delta P_{mg}+P_{comg}^0+\Delta P_{comg}\right)-\left(P_{fg}^0+\Delta P_{fg}+P_{cofg}^0+\Delta P_{cofg}\right)\right) \tag{A4.10b}$$

El esténcil central se puede escribir como:

$$Cm_{i,j,k}=-T_{i-1/2,j,k}-T_{i+1/2,j,k}-T_{i,j-1/2,k}-T_{i,j+1/2,k}-T_{i,j,k-1/2}-T_{i,j,k+1/2}$$
$$-\frac{\phi_{mi,j,k}S_{moi,j,k}\rho_{moi,j,k}}{M_{moi,j,k}*\Delta t}\left(c_{mo\ i,j,k}+c_{pcm\ i,j,k}\beta_{pm\ i,j,k}\right)-\frac{\phi_{mi,j,k}S_{mgi,j,k}\rho_{mgi,j,k}}{M_{mgi,j,k}*\Delta t}\left(c_{mg\ i,j,k}+c_{pcm\ i,j,k}\beta_{pm\ i,j,k}\right)$$
$$-8\left(\frac{\rho_{om}}{\mu_o M_{om}}\left(\frac{k_{xm}k_{ro\ xm}}{l_x^2}+\frac{k_{ym}k_{ro\ ym}}{l_y^2}+\frac{k_{zm}k_{ro\ zm}}{l_z^2}\right)+\frac{\rho_{gm}}{\mu_g M_{gm}}\left(\frac{k_{xm}k_{rg\ xm}}{l_x^2}+\frac{k_{ym}k_{rg\ ym}}{l_y^2}+\frac{k_{zm}k_{rg\ zm}}{l_z^2}\right)\right)_{i,j,k}$$

El esténcil libre se puede escribir como:

$$Fm_{i,j,k} = -\left(T_{i-1/2,j,k}\, p^{0}{}_{mg\ i-1,j,k} - \left(T_{i-1/2,j,k} + T_{i+1/2,j,k}\right) p^{0}{}_{mg\ i,j,k} + T_{i+1/2,j,k}\, p^{0}{}_{mg\ i+1,j,k}\right)$$

$$-\left(T_{i,j-1/2,k}\, p^{0}{}_{mg\ i,j-1,k} - \left(T_{i,j+1/2,k} + T_{i,j-1/2,k}\right) p^{0}{}_{mg\ i,j,k} + T_{i,j+1/2,k}\, p^{0}{}_{mg\ i,j+1,k}\right) -$$

$$\left(T_{i,j,k-1/2}\, p^{0}{}_{mg\ i,j,k-1} - \left(T_{i,j,k+1/2} + T_{i,j,k-1/2}\right) p^{0}{}_{mg\ i,j,k} + T_{i,j,k+1/2}\, p^{0}{}_{mg\ i,j,k+1}\right) + q_{Ti,j,k}$$

$$-\left\{\begin{array}{l}
\left[Fi\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i+1,j,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i-1,j,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\
\left[Fj\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j+1,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j-1,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\
\left[Fk\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k+1} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k-1}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]
\end{array}\right\} + T -$$

$$\left(T_{Ci\text{-}1/2,j,k}\, p^{0}{}_{cmog\ i\text{-}1,j,k} - \left(T_{Ci\text{-}1/2,j,k} + T_{Ci+1/2,j,k}\right) p^{0}{}_{cmog\ i,j,k} + T_{Ci+1/2,j,k}\, p^{0}{}_{cmog\ i+1,j,k}\right) +$$

$$\left(T_{Ci,j\text{-}1/2,k}\, p^{0}{}_{cmog\ i,j\text{-}1,k} - \left(T_{Ci,j\text{-}1/2,k} + T_{Ci,j+1/2,k}\right) p^{0}{}_{cmog\ i,j,k} + T_{Ci,j+1/2,k}\, p^{0}{}_{cmog\ i,j+1,k}\right) +$$

$$\left(T_{Ci,j,k-1/2}\, p^{0}{}_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right) p^{0}{}_{cmog\ i,j,k} + T_{Ci,j,k+1/2}\, p^{0}{}_{cmog\ i,j,k+1}\right) +$$

$$\left(T_{Ci-1/2,j,k}\Delta p_{cmog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right)\Delta p_{cmog\ i,j,k} + T_{Ci+1/2,j,k}\Delta p_{cmog\ i+1,j,k}\right) +$$

$$\left(T_{Ci,j-1/2,k}\Delta p_{cmog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right)\Delta p_{cmog\ i,j,k} + T_{Ci,j+1/2,k}\Delta p_{cmog\ i,j+1,k}\right) +$$

$$\left(T_{Ci,j,k-1/2}\Delta p_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right)\Delta p_{cmog\ i,j,k} + T_{Ci,j,k+1/2}\Delta p_{cmog\ i,j,k+1}\right) -$$

$$\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\left(\frac{\sigma^{n+1}_{mi,j,k} - \sigma^{n}_{mi,j,k}}{\Delta t}\right)$$

$$+\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\beta_{pfi,j,k}\frac{\Delta P_{fgi,j,k}}{\Delta t}$$

$$-\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\beta_{pmi,j,k}\left(\frac{P^{n+1}_{comgi,j,k} - P^{n}_{comgi,j,k}}{\Delta t}\right)$$

$$-\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}}S_{mo} + \frac{\rho_{mg}}{M_{mg}}S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\beta_{pfi,j,k}\left(\frac{P^{n+1}_{cofi,j,k} - P^{n}_{cofi,j,k}}{\Delta t}\right)$$

$$+\phi_{mi,j,k}\left[\frac{\rho_{mo}}{M_{mo}}\left(\frac{S^{n+1}_{moi,j,k} - S^{n}_{moi,j,k}}{\Delta t}\right) + \frac{\rho_{mg}}{M_{mg}}\left(\frac{S^{n+1}_{mgi,j,k} - S^{n}_{mgi,j,k}}{\Delta t}\right)\right] - \frac{S_{moi,j,k}\rho_{moi,j,k}\phi_m}{M_{moi,j,k}}C_{moi,j,k}\frac{\Delta P_{comg}}{\Delta t} +$$

$$8*\left(\begin{array}{l}\frac{\rho_{om}}{\mu_o M_{om}}\left(\frac{k_{xm}k_{ro\ xm}}{l_x^2} + \frac{k_{ym}k_{ro\ ym}}{l_y^2} + \frac{k_{zm}k_{ro\ zm}}{l_z^2}\right) + \\ \frac{\rho_{gm}}{\mu_g M_{gm}}\left(\frac{k_{xm}k_{rg\ xm}}{l_x^2} + \frac{k_{ym}k_{rg\ ym}}{l_y^2} + \frac{k_{zm}k_{rg\ zm}}{l_z^2}\right)\end{array}\right)_{i,j,k}\left(\left(P^{0}_{mg} + P^{0}_{comg} + \Delta P_{comg}\right) - \left(P^{0}_{fg} + \Delta P_{fg} + P^{0}_{cofg} + \Delta P_{cofg}\right)\right)$$

Caso 3:

$$\Omega_m = S_{mo}P_{mo} + S_{mg}P_{mg}$$
$$\Omega_m = \left(S_{mo} + S_{mg}\right)P_{mg} - S_{mo}P_{mcog}$$

$$q_{Tm} = \left(8\frac{\rho_{om}}{\mu_o M_{om}}\left(\frac{k_{xm}k_{ro\ xm}}{l_x^2} + \frac{k_{ym}k_{ro\ ym}}{l_y^2} + \frac{k_{zm}k_{ro\ zm}}{l_z^2}\right) + 8\frac{\rho_{gm}}{\mu_g M_{gm}}\left(\frac{k_{xm}k_{rg\ xm}}{l_x^2} + \frac{k_{ym}k_{rg\ ym}}{l_y^2} + \frac{k_{zm}k_{rg\ zm}}{l_z^2}\right)\right)_{i,j,k} * \left(\left(P_{mg}^0 + \Delta P_{mg} + P_{comg}^0 + \Delta P_{comg}\right) - \left(P_{fg}^0 + \Delta P_{fg} + P_{cofg}^0 + \Delta P_{cofg}\right)\right) \quad \text{(A4.10c)}$$

Caso 4:

$$\Omega_m = P_{mg} - \frac{1}{2}P_{mcog}\,,\ \psi_m = \frac{1}{2}P_{mcog}$$

$$\Omega_f = P_{fg} - \frac{1}{2}P_{fcog}\,,\ \psi_f = \frac{1}{2}P_{fcog}$$

Entonces la ecuación (A4.10) se escribe como

$$q_{Tm} = \left(8\frac{\rho_{om}}{\mu_o M_{om}}\left(\frac{k_{xm}k_{ro\ xm}}{l_x^2} + \frac{k_{ym}k_{ro\ ym}}{l_y^2} + \frac{k_{zm}k_{ro\ zm}}{l_z^2}\right) + 8\frac{\rho_{gm}}{\mu_g M_{gm}}\left(\frac{k_{xm}k_{rg\ xm}}{l_x^2} + \frac{k_{ym}k_{rg\ ym}}{l_y^2} + \frac{k_{zm}k_{rg\ zm}}{l_z^2}\right)\right)_{i,j,k} * \left(\left(P_{mg}^0 + \Delta P_{mg} - \frac{1}{2}\left(P_{comg}^0 + \Delta P_{comg}\right)\right) - \left(P_{fg}^0 + \Delta P_{fg} - \frac{1}{2}\left(P_{cofg}^0 + \Delta P_{cofg}\right)\right)\right) \quad \text{(A4.10d)}$$

El esténcil central se puede escribir como:

$$Cm_{i,j,k} = -T_{i-1/2,j,k} - T_{i+1/2,j,k} - T_{i,j-1/2,k} - T_{i,j+1/2,k} - T_{i,j,k-1/2} - T_{i,j,k+1/2}$$
$$-\frac{\phi_{mi,j,k}S_{moi,j,k}\rho_{moi,j,k}}{M_{moi,j,k} * \Delta t}\left(c_{mo\ i,j,k} + c_{pcm\ i,j,k}\beta_{pm\ i,j,k}\right) - \frac{\phi_{mi,j,k}S_{mgi,j,k}\rho_{mgi,j,k}}{M_{mgi,j,k} * \Delta t}\left(c_{mg\ i,j,k} + c_{pcm\ i,j,k}\beta_{pm\ i,j,k}\right)$$
$$-8\left(\frac{\rho_{om}}{\mu_o M_{om}}\left(\frac{k_{xm}k_{ro\ xm}}{l_x^2} + \frac{k_{ym}k_{ro\ ym}}{l_y^2} + \frac{k_{zm}k_{ro\ zm}}{l_z^2}\right) + \frac{\rho_{gm}}{\mu_g M_{gm}}\left(\frac{k_{xm}k_{rg\ xm}}{l_x^2} + \frac{k_{ym}k_{rg\ ym}}{l_y^2} + \frac{k_{zm}k_{rg\ zm}}{l_z^2}\right)\right)_{i,j,k}$$

El esténcil libre se puede escribir como:

$$Fm_{i,j,k} = -\left(T_{i-1/2,j,k}\, p^0{}_{mg\ i-1,j,k} - \left(T_{i-1/2,j,k} + T_{i+1/2,j,k}\right) p^0{}_{mg\ i,j,k} + T_{i+1/2,j,k}\, p^0{}_{mg\ i+1,j,k}\right)$$

$$-\left(T_{i,j-1/2,k}\, p^0{}_{mg\ i,j-1,k} - \left(T_{i,j+1/2,k} + T_{i,j-1/2,k}\right) p^0{}_{mg\ i,j,k} + T_{i,j+1/2,k}\, p^0{}_{mg\ i,j+1,k}\right) -$$

$$\left(T_{i,j,k-1/2}\, p^0{}_{mg\ i,j,k-1} - \left(T_{i,j,k+1/2} + T_{i,j,k-1/2}\right) p^0{}_{mg\ i,j,k} + T_{i,j,k+1/2}\, p^0{}_{mg\ i,j,k+1}\right) + q_{Ti,j,k}$$

$$-\left\{\begin{array}{l}\left[Fi\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i+1,j,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i-1,j,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\ \left[Fj\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j+1,k} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j-1,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\ \left[Fk\left(\left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k+1} - \left(\frac{\rho_{mo}\phi_m}{M_{mo}} + \frac{\rho_{mg}\phi_m}{M_{mg}}\right)_{i,j,k-1}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]\end{array}\right\} + T -$$

$$\left(T_{Ci-1/2,j,k}\, p^0{}_{cmog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right) p^0{}_{cmog\ i,j,k} + T_{Ci+1/2,j,k}\, p^0{}_{cmog\ i+1,j,k}\right)$$

$$\left(T_{Ci,j-1/2,k}\, p^0{}_{cmog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right) p^0{}_{cmog\ i,j,k} + T_{Ci,j+1/2,k}\, p^0{}_{cmog\ i,j+1,k}\right) +$$

$$\left(T_{Ci,j,k-1/2}\, p^0{}_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right) p^0{}_{cmog\ i,j,k} + T_{Ci,j,k+1/2}\, p^0{}_{cmog\ i,j,k+1}\right) +$$

$$\left(T_{Ci-1/2,j,k}\,\Delta p_{cmog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right)\Delta p_{cmog\ i,j,k} + T_{Ci+1/2,j,k}\,\Delta p_{cmog\ i+1,j,k}\right) +$$

$$\left(T_{Ci,j-1/2,k}\,\Delta p_{cmog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right)\Delta p_{cmog\ i,j,k} + T_{Ci,j+1/2,k}\,\Delta p_{cmog\ i,j+1,k}\right) +$$

$$\left(T_{Ci,j,k-1/2}\,\Delta p_{cmog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right)\Delta p_{cmog\ i,j,k} + T_{Ci,j,k+1/2}\,\Delta p_{cmog\ i,j,k+1}\right) -$$

$$\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}} S_{mo} + \frac{\rho_{mg}}{M_{mg}} S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\left(\frac{\sigma^{n+1}_{mi,j,k} - \sigma^{n}_{mi,j,k}}{\Delta t}\right)$$

$$+\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}} S_{mo} + \frac{\rho_{mg}}{M_{mg}} S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\,\beta_{pfi,j,k}\,\frac{\Delta P_{fgi,j,k}}{\Delta t}$$

$$-\frac{1}{2}\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}} S_{mo} + \frac{\rho_{mg}}{M_{mg}} S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\,\beta_{pmi,j,k}\left(\frac{P^{n+1}_{comgi,j,k} - P^{n}_{comgi,j,k}}{\Delta t}\right)$$

$$-\frac{1}{2}\phi_{mi,j,k}\left(\frac{\rho_{mo}}{M_{mo}} S_{mo} + \frac{\rho_{mg}}{M_{mg}} S_{mg}\right)_{i,j,k} c_{pcmi,j,k}\,\beta_{pfi,j,k}\left(\frac{P^{n+1}_{cofi,j,k} - P^{n}_{cofi,j,k}}{\Delta t}\right)$$

$$+\phi_{mi,j,k}\left[\frac{\rho_{mo}}{M_{mo}}\left(\frac{S^{n+1}_{moi,j,k} - S^{n}_{moi,j,k}}{\Delta t}\right) + \frac{\rho_{mg}}{M_{mg}}\left(\frac{S^{n+1}_{mgi,j,k} - S^{n}_{mgi,j,k}}{\Delta t}\right)\right] - \frac{S_{moi,j,k}\,\rho_{moi,j,k}\,\phi_m}{M_{moi,j,k}} C_{moi,j,k}\,\frac{\Delta P_{comg}}{\Delta t} +$$

$$8*\left(\frac{\rho_{om}}{\mu_o M_{om}}\left(\frac{k_{xm}k_{ro\ xm}}{l_x^2} + \frac{k_{ym}k_{ro\ ym}}{l_y^2} + \frac{k_{zm}k_{ro\ zm}}{l_z^2}\right) + \frac{\rho_{gm}}{\mu_g M_{gm}}\left(\frac{k_{xm}k_{rg\ xm}}{l_x^2} + \frac{k_{ym}k_{rg\ ym}}{l_y^2} + \frac{k_{zm}k_{rg\ zm}}{l_z^2}\right)\right)_{i,j,k} *$$

$$\left(\left(P^0_{mg} - \frac{1}{2}\left(P^0_{comg} + \Delta P_{comg}\right)\right) - \left(P^0_{fg} + \Delta P_{fg} - \frac{1}{2}\left(P^0_{cofg} + \Delta P_{cofg}\right)\right)\right)$$

ANEXO 5. DISCRETIZACIÓN DE LAS ECUACIONES DEL MODELO COMPOSICIONAL DE FLUJO DE FLUIDOS EN LA FRACTURA

A5.1 Discretización de la ecuación de flujo de fluidos para fractura

Para la fractura la ecuación de flujo de fluidos composicionales en forma incremental es:

$$\begin{aligned}
&\nabla\cdot\left[\left(\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}+\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\right)\nabla P_{fg}^{0}\right]+\nabla\cdot\left[\left(\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}+\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\right)\nabla\Delta P_{fg}\right]=\tilde{q}_T\\
&-\nabla\cdot\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)\frac{\partial\bar{u}}{\partial t}+\nabla\cdot\left(\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}\nabla P_{cfog}^{0}\right)+\nabla\cdot\left(\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}\nabla\Delta P_{cfog}\right)\\
&-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\frac{\partial\sigma_f}{\partial t}-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\beta_{pf}\frac{\partial\psi_f}{\partial t}\\
&+\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\beta_{pm}\frac{\partial P_{mg}}{\partial t}-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\beta_{pm}\frac{\partial\psi_m}{\partial t}\\
&+\phi_f\left[\rho_{fo}\frac{\partial}{\partial t}\left(\frac{S_{fo}}{M_{fo}}\right)+\rho_{fg}\frac{\partial}{\partial t}\left(\frac{S_{fg}}{M_{fg}}\right)\right]-\frac{S_{fo}\rho_{fo}\phi_f}{M_{fo}}C_{fo}\frac{\partial P_{cofg}}{\partial t}+\frac{\phi_f S_{fo}\rho_{fo}}{M_{fo}}\left[C_{fo}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}\\
&+\frac{\phi_f\rho_{fg}S_{fg}}{M_{fg}}\left[C_{fg}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}-T
\end{aligned}\tag{A5.1}$$

Gradiente ∇, en coordenadas cartesianas:

$$\nabla U=\frac{\partial U}{\partial x}\vec{i}+\frac{\partial U}{\partial y}\vec{j}+\frac{\partial U}{\partial z}\vec{k}\tag{A5.2}$$

Divergencia $\nabla\cdot$, en coordenadas cartesianas:

$$\nabla\cdot F=\frac{\partial F_x}{\partial x}\vec{i}+\frac{\partial F_y}{\partial y}\vec{j}+\frac{\partial F_z}{\partial z}\vec{k}\tag{A5.3}$$

Con el fin de manejar más fácilmente las ecuaciones se realizan los siguientes cambios de variables:

$$\lambda=\left(\frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}}+\frac{\rho_{fg}\vec{k}_f k_{rfg}}{M_{fg}\mu_{fg}}\right)\tag{A5.4a}$$

De la misma manera:

$$\tau = \frac{\rho_{fo}\vec{k}_f k_{rfo}}{M_{fo}\mu_{fo}} \tag{A5.4b}$$

Aplicando las definiciones (A5.2), (A5.3) y el cambio de variable de las ecuaciones (A5.4a) y (A5.4b) en la ecuación (A5.1) se llega a:

$$\begin{aligned}
&\frac{\partial}{\partial x}\left(\lambda\frac{\partial P_{fg}^{0}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial P_{fg}^{0}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial P_{fg}^{0}}{\partial z}\right)+\frac{\partial}{\partial x}\left(\lambda\frac{\partial \Delta P_{fg}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial \Delta P_{fg}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial \Delta P_{fg}}{\partial z}\right)=\tilde{q}_T\\
&-\left(\frac{\partial}{\partial x}\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)\frac{\partial\bar{u}}{\partial t}+\frac{\partial}{\partial y}\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)\frac{\partial\bar{u}}{\partial t}+\frac{\partial}{\partial z}\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)\frac{\partial\bar{u}}{\partial t}\right)\\
&+\frac{\partial}{\partial x}\left(\tau\frac{\partial P_{cfog}^{0}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\tau\frac{\partial P_{cfog}^{0}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\tau\frac{\partial P_{cfog}^{0}}{\partial z}\right)+\frac{\partial}{\partial x}\left(\tau\frac{\partial \Delta P_{cfog}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\tau\frac{\partial \Delta P_{cfog}}{\partial y}\right)+\frac{\partial}{\partial z}\left(\tau\frac{\partial \Delta P_{cfog}}{\partial z}\right)\\
&-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\frac{\partial\sigma_f}{\partial t}-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\beta_{pf}\frac{\partial\psi_f}{\partial t}\\
&+\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\beta_{pm}\frac{\partial P_{mg}}{\partial t}-\phi_f\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)c_{pcf}\beta_{pm}\frac{\partial\psi_m}{\partial t}\\
&+\phi_f\left[\rho_{fo}\frac{\partial}{\partial t}\left(\frac{S_{fo}}{M_{fo}}\right)+\rho_{fg}\frac{\partial}{\partial t}\left(\frac{S_{fg}}{M_{fg}}\right)\right]-\frac{S_{fo}\rho_{fo}\phi_f}{M_{fo}}C_{fo}\frac{\partial P_{cofg}}{\partial t}\\
&+\frac{\phi_f S_{fo}\rho_{fo}}{M_{fo}}\left[C_{fo}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}+\frac{\phi_f\rho_{fg}S_{fg}}{M_{fg}}\left[C_{fg}+c_{pcf}\beta_{pf}\right]\frac{\partial P_{fg}}{\partial t}+q_{Tf}
\end{aligned} \tag{A5.5}$$

Teniendo en cuenta lo establecido en el Anexo 3 se realiza la discretización de la ecuación (A5.5).

$$
\begin{aligned}
&2\left(C_{i-1}\lambda_{i-1/2,j,k}\,p^{0}{}_{fg\ i-1,j,k}-\left(C_{i-1}\lambda_{i-1/2,j,k}+C_i\lambda_{i+1/2,j,k}\right)p^{0}{}_{fg\ i,j,k}+C_i\lambda_{i+1/2,j,k}\,p^{0}{}_{fg\ i+1,j,k}\right)+\\
&2\left(C_{j-1}\lambda_{i,j-1/2,k}\,p^{0}{}_{fg\ i,j-1,k}-\left(C_{j-1}\lambda_{i,j-1/2,k}+C_j\lambda_{i,j+1/2,k}\right)p^{0}{}_{fg\ i,j,k}+C_j\lambda_{i,j+1/2,k}\,p^{0}{}_{fg\ i,j+1,k}\right)+\\
&2\left(C_{k-1}\lambda_{i,j,k-1/2}\,p^{0}{}_{fgi,j,k-1}-\left(C_{k-1}\lambda_{i,j,k-1/2}+C_k\lambda_{i,j,k+1/2}\right)p^{0}{}_{fg\ i,j,k}+C_k\lambda_{i,j,k+1/2}\,p^{0}{}_{fg\ i,j,k+1}\right)+\\
&2\left(C_{i-1}\lambda_{i-1/2,j,k}\Delta p_{fg\ i-1,j,k}-\left(C_{i-1}\lambda_{i-1/2,j,k}+C_i\lambda_{i+1/2,j,k}\right)\Delta p_{fg\ i,j,k}+C_i\lambda_{i+1/2,j,k}\Delta p_{fg\ i+1,j,k}\right)+\\
&2\left(C_{j-1}\lambda_{i,j-1/2,k}\Delta p_{fg\ i,j-1,k}-\left(C_{j-1}\lambda_{i,j-1/2,k}+C_j\lambda_{i,j+1/2,k}\right)\Delta p_{fg\ i,j,k}+C_j\lambda_{i,j+1/2,k}\Delta p_{fg\ i,j+1,k}\right)+\\
&2\left(C_{k-1}\lambda_{i,j,k-1/2}\Delta p_{fg\ i,j,k-1}-\left(C_{k-1}\lambda_{i,j,k-1/2}+C_k\lambda_{i,j,k+1/2}\right)\Delta p_{fg\ i,j,k}+C_k\lambda_{i,j,k+1/2}\Delta p_{fg\ i,j,k+1}\right)=\\
&q_{Ti,j,k}-\left\{\begin{array}{l}
\left[Fi\left(\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i+1,j,k}-\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i-1,j,k}\right)*\left(\left(\frac{u^{n+1}_{x\,i,j,k}-u^{n}_{x\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{y\,i,j,k}-u^{n}_{y\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{z\,i,j,k}-u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]+\\
\left[Fj\left(\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j+1,k}-\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j-1,k}\right)*\left(\left(\frac{u^{n+1}_{x\,i,j,k}-u^{n}_{x\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{y\,i,j,k}-u^{n}_{y\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{z\,i,j,k}-u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]+\\
\left[Fk\left(\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j,k+1}-\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j,k-1}\right)*\left(\left(\frac{u^{n+1}_{x\,i,j,k}-u^{n}_{x\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{y\,i,j,k}-u^{n}_{y\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{z\,i,j,k}-u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]
\end{array}\right\}+\\
&2\left(C_{i-1}\tau_{i-1/2,j,k}\,p^{0}{}_{cfog\ i-1,j,k}-\left(C_{i-1}\tau_{i-1/2,j,k}+C_i\tau_{i+1/2,j,k}\right)p^{0}{}_{cfog\ i,j,k}+C_i\tau_{i+1/2,j,k}\,p^{0}{}_{cfog\ i+1,j,k}\right)\\
&2\left(C_{j-1}\tau_{i,j-1/2,k}\,p^{0}{}_{cfog\ i,j-1,k}-\left(C_{j-1}\tau_{i,j-1/2,k}+C_j\tau_{i,j+1/2,k}\right)p^{0}{}_{cfog\ i,j,k}+C_j\tau_{i,j+1/2,k}\,p^{0}{}_{cfog\ i,j+1,k}\right)+\\
&2\left(C_{k-1}\tau_{i,j,k-1/2}\,p^{0}{}_{cfog\ i,j,k-1}-\left(C_{k-1}\tau_{i,j,k-1/2}+C_k\tau_{i,j,k+1/2}\right)p^{0}{}_{cfog\ i,j,k}+C_k\tau_{i,j,k+1/2}\,p^{0}{}_{cfog\ i,j,k+1}\right)+\\
&2\left(C_{i-1}\tau_{i-1/2,j,k}\Delta p_{cfog\ i-1,j,k}-\left(C_{i-1}\tau_{i-1/2,j,k}+C_i\tau_{i+1/2,j,k}\right)\Delta p_{cfog\ i,j,k}+C_i\tau_{i+1/2,j,k}\Delta p_{cfog\ i+1,j,k}\right)+\\
&2\left(C_{j-1}\tau_{i,j-1/2,k}\Delta p_{cfog\ i,j-1,k}-\left(C_{j-1}\tau_{i,j-1/2,k}+C_j\tau_{i,j+1/2,k}\right)\Delta p_{cfog\ i,j,k}+C_j\tau_{i,j+1/2,k}\Delta p_{cfog\ i,j+1,k}\right)+\\
&2\left(C_{k-1}\tau_{i,j,k-1/2}\Delta p_{cfog\ i,j,k-1}-\left(C_{k-1}\tau_{i,j,k-1/2}+C_k\tau_{i,j,k+1/2}\right)\Delta p_{cfog\ i,j,k}+C_k\tau_{i,j,k+1/2}\Delta p_{cfog\ i,j,k+1}\right)-\\
&\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k}c_{pcfi,j,k}\left(\frac{\sigma^{n+1}_{fi,j,k}-\sigma^{n}_{fi,j,k}}{\Delta t}\right)\\
&-\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k}c_{pcfi,j,k}\beta_{pfi,j,k}\left(\frac{\psi^{n+1}_{fi,j,k}-\psi^{n}_{fi,j,k}}{\Delta t}\right)\\
&+\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k}c_{pcfi,j,k}\beta_{pmi,j,k}\frac{\Delta P_{mgi,j,k}}{\Delta t}\\
&-\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k}c_{pcfi,j,k}\beta_{pmi,j,k}\left(\frac{\psi^{n+1}_{mi,j,k}-\psi^{n}_{mi,j,k}}{\Delta t}\right)\\
&+\phi_{fi,j,k}\left[\frac{\rho_{fo}}{M_{fo}}\left(\frac{S^{n+1}_{foi,j,k}-S^{n}_{foi,j,k}}{\Delta t}\right)+\frac{\rho_{fg}}{M_{fg}}\left(\frac{S^{n+1}_{fgi,j,k}-S^{n}_{fgi,j,k}}{\Delta t}\right)\right]-\frac{S_{foi,j,k}\rho_{foi,j,k}\phi_f}{M_{foi,j,k}}C_{foi,j,k}\frac{\Delta P_{cofg}}{\Delta t}\\
&+\frac{\phi_{fi,j,k}S_{foi,j,k}\rho_{foi,j,k}}{M_{foi,j,k}}\left[C_{foi,j,k}+c_{pcfi,j,k}\beta_{pfi,j,k}\right]\frac{\Delta P_{fgi,j,k}}{\Delta t}\\
&+\frac{\phi_{fi,j,k}\rho_{fgi,j,k}S_{fgi,j,k}}{M_{fgi,j,k}}\left[C_{fgi,j,k}+c_{pcfi,j,k}\beta_{pfi,j,k}\right]\frac{\Delta P_{fg}}{\Delta t}+q_{Tf}
\end{aligned}
$$

(A5.6)

Esta ecuación puede ser simplificada haciendo un cambio de variable como se puede observar

$$T_{i-1,j,k} = 2C_{i-1}\lambda_{i-1/2,j,k} \quad \text{(A5.7a)}$$

$$T_{i+1,j,k} = 2C_{i+1}\lambda_{i+1/2,j,k} \quad \text{(A5.7b)}$$

$$T_{i,j-1,k} = 2C_{j-1}\lambda_{i,j-1/2,k} \quad \text{(A5.7c)}$$

$$T_{i,j+1,k} = 2C_{j+1}\lambda_{i,j+1/2,k} \quad \text{(A5.7d)}$$

$$T_{i,j,k-1} = 2C_{k-1}\lambda_{i,j,k-1/2} \quad \text{(A5.7e)}$$

$$T_{i,j,k+1} = 2C_{k+1}\lambda_{i,j,k+1/2} \quad \text{(A5.7f)}$$

$$T_{Ci-1,j,k} = 2C_{i-1}\tau_{i-1/2,j,k} \quad \text{(A5.7g)}$$

$$T_{Ci+1,j,k} = 2C_{i+1}\tau_{i+1/2,j,k} \quad \text{(A5.7h)}$$

$$T_{Ci,j-1,k} = 2C_{j-1}\tau_{i,j-1/2,k} \quad \text{(A5.7i)}$$

$$T_{Ci,j+1,k} = 2C_{j+1}\tau_{i,j+1/2,k} \quad \text{(A5.7j)}$$

$$T_{Ci,j,k-1} = 2C_{k-1}\tau_{i,j,k-1/2} \quad \text{(A5.7k)}$$

$$T_{Ci,j,k+1} = 2C_{k+1}\tau_{i,j,k+1/2} \quad \text{(A5.7l)}$$

La ecuación quedaría

$$\left(T_{i-1/2,j,k}\,p^{0}{}_{fg\;i-1,j,k}-\left(T_{i-1/2,j,k}+T_{i+1/2,j,k}\right)p^{0}{}_{fg\;i,j,k}+T_{i+1/2,j,k}\,p^{0}{}_{fg\;i+1,j,k}\right)+$$

$$\left(T_{i,j-1/2,k}\,p^{0}{}_{fg\;i,j-1,k}-\left(T_{i,j-1/2,k}+T_{i,j+1/2,k}\right)p^{0}{}_{fg\;i,j,k}+T_{i,j+1/2,k}\,p^{0}{}_{fg\;i,j+1,k}\right)+$$

$$\left(T_{i,j,k-1/2}\,p^{0}{}_{\mathrm{f}\;i,j,k-1}-\left(T_{i,j,k-1/2}+T_{i,j,k+1/2}\right)p^{0}{}_{fg\;i,j,k}+T_{i,j,k+1/2}\,p^{0}{}_{fg\;i,j,k+1}\right)+$$

$$\left(T_{i-1/2,j,k}\Delta p_{fg\;i-1,j,k}-\left(T_{i-1/2,j,k}+T_{i+1/2,j,k}\right)\Delta p_{fg\;i,j,k}+T_{i+1/2,j,k}\Delta p_{fg\;i+1,j,k}\right)+$$

$$\left(T_{i,j-1/2,k}\Delta p_{fg\;i,j-1,k}-\left(T_{i,j-1/2,k}+T_{i,j+1/2,k}\right)\Delta p_{fg\;i,j,k}+T_{i,j+1/2,k}\Delta p_{fg\;i,j+1,k}\right)+$$

$$\left(T_{i,j,k-1/2}\Delta p_{fg\;i,j,k-1}-\left(T_{i,j,k-1/2}+T_{i,j,k+1/2}\right)\Delta p_{fg\;i,j,k}+T_{i,j,k+1/2}\Delta p_{fg\;i,j,k+1}\right)=$$

$$q_{Ti,j,k}-\left\{\begin{array}{l}\left[Fi\left(\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i+1,j,k}-\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i-1,j,k}\right)*\left(\left(\frac{u^{n+1}_{x\,i,j,k}-u^{n}_{x\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{y\,i,j,k}-u^{n}_{y\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{z\,i,j,k}-u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]+\\ \left[Fj\left(\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j+1,k}-\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j-1,k}\right)*\left(\left(\frac{u^{n+1}_{x\,i,j,k}-u^{n}_{x\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{y\,i,j,k}-u^{n}_{y\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{z\,i,j,k}-u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]+\\ \left[Fk\left(\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j,k+1}-\left(\frac{\rho_{fo}\phi_f}{M_{fo}}+\frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j,k-1}\right)*\left(\left(\frac{u^{n+1}_{x\,i,j,k}-u^{n}_{x\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{y\,i,j,k}-u^{n}_{y\,i,j,k}}{\Delta t}\right)+\left(\frac{u^{n+1}_{z\,i,j,k}-u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]\end{array}\right\}+$$

$$\left(T_{Ci-1/2,j,k}\,p^{0}{}_{cfog\;i-1,j,k}-\left(T_{\mathrm{C}i-1/2,j,k}+T_{\mathrm{C}i+1/2,j,k}\right)p^{0}{}_{cfog\;i,j,k}+T_{Ci+1/2,j,k}\,p^{0}{}_{cfog\;i+1,j,k}\right)$$

$$\left(T_{Ci,j-1/2,k}\,p^{0}{}_{cfog\;i,j-1,k}-\left(T_{\mathrm{C}i,j-1/2,k}+T_{\mathrm{C}i,j+1/2,k}\right)p^{0}{}_{cfog\;i,j,k}+T_{Ci,j+1/2,k}\,p^{0}{}_{cfog\;i,j+1,k}\right)+$$

$$\left(T_{Ci,j,k-1/2}\,p^{0}{}_{cfog\;i,j,k-1}-\left(T_{\mathrm{C}i,j,k-1/2}+T_{\mathrm{C}i,j,k+1/2}\right)p^{0}{}_{cfog\;i,j,k}+T_{\mathrm{C}i,j,k+1/2}\,p^{0}{}_{cfog\;i,j,k+1}\right)+$$

$$\left(T_{Ci-1/2,j,k}\Delta p_{cfog\;i-1,j,k}-\left(T_{\mathrm{C}i-1/2,j,k}+T_{\mathrm{C}i+1/2,j,k}\right)\Delta p_{cfog\;i,j,k}+T_{Ci+1/2,j,k}\Delta p_{cfog\;i+1,j,k}\right)+$$

$$\left(T_{Ci,j-1/2,k}\Delta p_{cfog\;i,j-1,k}-\left(T_{\mathrm{C}i,j-1/2,k}+T_{\mathrm{C}i,j+1/2,k}\right)\Delta p_{cfog\;i,j,k}+T_{Ci,j+1/2,k}\Delta p_{cfog\;i,j+1,k}\right)+$$

$$\left(T_{Ci,j,k-1/2}\Delta p_{cfog\;i,j,k-1}-\left(T_{\mathrm{C}i,j,k-1/2}+T_{\mathrm{C}i,j,k+1/2}\right)\Delta p_{cfog\;i,j,k}+T_{\mathrm{C}i,j,k+1/2}\Delta p_{cfog\;i,j,k+1}\right)-$$

$$\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k}c_{pcfi,j,k}\left(\frac{\sigma^{n+1}_{fi,j,k}-\sigma^{n}_{fi,j,k}}{\Delta t}\right)$$

$$-\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k}c_{pcfi,j,k}\beta_{pfi,j,k}\left(\frac{\psi^{n+1}_{fi,j,k}-\psi^{n}_{fi,j,k}}{\Delta t}\right)$$

$$+\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k}c_{pcfi,j,k}\beta_{pmi,j,k}\frac{\Delta P_{mgi,j,k}}{\Delta t}$$

$$-\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo}+\frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k}c_{pcfi,j,k}\beta_{pmi,j,k}\left(\frac{\psi^{n+1}_{mi,j,k}-\psi^{n}_{mi,j,k}}{\Delta t}\right)$$

$$+\phi_{fi,j,k}\left[\frac{\rho_{fo}}{M_{fo}}\left(\frac{S^{n+1}_{foi,j,k}-S^{n}_{foi,j,k}}{\Delta t}\right)+\frac{\rho_{fg}}{M_{fg}}\left(\frac{S^{n+1}_{fgi,j,k}-S^{n}_{fgi,j,k}}{\Delta t}\right)\right]-\frac{S_{foi,j,k}\rho_{foi,j,k}\phi_m}{M_{foi,j,k}}C_{foi,j,k}\frac{\Delta P_{cofg}}{\Delta t}$$

$$+\frac{\phi_{fi,j,k}S_{foi,j,k}\rho_{foi,j,k}}{M_{foi,j,k}}\left[C_{foi,j,k}+c_{pcfi,j,k}\beta_{pfi,j,k}\right]\frac{\Delta P_{fgi,j,k}}{\Delta t}$$

$$+\frac{\phi_{fi,j,k}\rho_{fgi,j,k}S_{fgi,j,k}}{M_{fgi,j,k}}\left[C_{fgi,j,k}+c_{pcfi,j,k}\beta_{pfi,j,k}\right]\frac{\Delta P_{fg}}{\Delta t}+q_{Tf}$$

La Ecuación puede escribirse de una forma más simplificada como:

$$BCf_{i,j,k}\Delta p_{f\ \ i,j,k-1}^{n+1}+Sf_{i,j,k}\Delta p_{f\ \ i,j-1,k}^{n+1}+Wf_{i,j,k}\Delta p_{f\ \ i-1,j,k}^{n+1}+Cf_{i,j,k}\Delta p_{f\ \ i,j,k}^{n+1}+Ef_{i,j,k}\Delta p_{f\ \ i+1,j,k}^{n+1}$$
$$+Nf_{i,j,k}\Delta p_{f\ \ i,j+1,k}^{n+1}+TCf_{i,j,k}\Delta p_{f\ \ i,j,k+1}^{n+1}=Ff_{i,j,k} \quad \text{(A5.8)}$$

Donde:

$$BCf_{i,j,k}=T_{i,j,k-1/2} \quad \text{(A5.8a)}$$

$$Sf_{i,j,k}=T_{i,j-1/2,k} \quad \text{(A5.8b)}$$

$$Wf_{i,j,k}=T_{i-1/2,j,k} \quad \text{(A5.8c)}$$

$$Ef_{i,j,k}=T_{i+1/2,j,k} \quad \text{(A5.8d)}$$

$$Nf_{i,j,k}=T_{i,j+1/2,k} \quad \text{(A5.8e)}$$

$$TCf_{i,j,k}=T_{i,j,k+1/2} \quad \text{(A5.8f)}$$

$$Cf_{i,j,k}=-T_{i-1/2,j,k}-T_{i+1/2,j,k}-T_{i,j-1/2,k}-T_{i,j+1/2,k}-T_{i,j,k-1/2}-T_{i,j,k+1/2}$$
$$-\frac{\phi_{fi,j,k}S_{foi,j,k}\rho_{foi,j,k}}{M_{foi,j,k}*\Delta t}\left(c_{fo\ i,j,k}+c_{pcf\ i,j,k}\beta_{pf\ i,j,k}\right)-\frac{\phi_{fi,j,k}S_{fgi,j,k}\rho_{fgi,j,k}}{M_{fgi,j,k}*\Delta t}\left(c_{fg\ i,j,k}+c_{pcf\ i,j,k}\beta_{pf\ i,j,k}\right)$$

(A5.8g)

$$
\begin{aligned}
&Ff_{i,j,k} = -\left(T_{i-1/2,j,k}\,p^{0}{}_{fg\ i-1,j,k} - \left(T_{i-1/2,j,k} + T_{i+1/2,j,k}\right) p^{0}{}_{fg\ i,j,k} + T_{i+1/2,j,k}\,p^{0}{}_{fg\ i+1,j,k}\right) \\
&-\left(T_{i,j-1/2,k}\,p^{0}{}_{fg\ i,j-1,k} - \left(T_{i,j+1/2,k} + T_{i,j-1/2,k}\right) p^{0}{}_{fg\ i,j,k} + T_{i,j+1/2,k}\,p^{0}{}_{fg\ i,j+1,k}\right) - \\
&\left(T_{i,j,k-1/2}\,p^{0}{}_{fg\ i,j,k-1} - \left(T_{i,j,k+1/2} + T_{i,j,k-1/2}\right) p^{0}{}_{fgi,j,k} + T_{i,j,k+1/2}\,p^{0}{}_{fg\ i,j,k+1}\right) + q_{Ti,j,k} \\
&-\left\{
\begin{array}{l}
\left[Fi\left(\left(\dfrac{\rho_{fo}\phi_f}{M_{fo}} + \dfrac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i+1,j,k} - \left(\dfrac{\rho_{fo}\phi_f}{M_{fo}} + \dfrac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i-1,j,k}\right) * \left(\left(\dfrac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\dfrac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\dfrac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\
\left[Fj\left(\left(\dfrac{\rho_{fo}\phi_f}{M_{fo}} + \dfrac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j+1,k} - \left(\dfrac{\rho_{fo}\phi_f}{M_{fo}} + \dfrac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j-1,k}\right) * \left(\left(\dfrac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\dfrac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\dfrac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\
\left[Fk\left(\left(\dfrac{\rho_{fo}\phi_f}{M_{fo}} + \dfrac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j,k+1} - \left(\dfrac{\rho_{fo}\phi_f}{M_{fo}} + \dfrac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j,k-1}\right) * \left(\left(\dfrac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\dfrac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\dfrac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]
\end{array}
\right\} + T - \\
&\left(T_{Ci-1/2,j,k}\,p^{0}{}_{cfog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right) p^{0}{}_{cfog\ i,j,k} + T_{Ci+1/2,j,k}\,p^{0}{}_{cfog\ i+1,j,k}\right) \\
&\left(T_{Ci,j-1/2,k}\,p^{0}{}_{cfog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right) p^{0}{}_{cfog\ i,j,k} + T_{Ci,j+1/2,k}\,p^{0}{}_{cfog\ i,j+1,k}\right) + \\
&\left(T_{Ci,j,k-1/2}\,p^{0}{}_{cfog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right) p^{0}{}_{cfog\ i,j,k} + T_{Ci,j,k+1/2}\,p^{0}{}_{cfog\ i,j,k+1}\right) + \\
&\left(T_{Ci-1/2,j,k}\Delta p_{cfog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right)\Delta p_{cfog\ i,j,k} + T_{Ci+1/2,j,k}\Delta p_{cfog\ i+1,j,k}\right) + \\
&\left(T_{Ci,j-1/2,k}\Delta p_{cfog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right)\Delta p_{cfog\ i,j,k} + T_{Ci,j+1/2,k}\Delta p_{cfog\ i,j+1,k}\right) + \\
&\left(T_{Ci,j,k-1/2}\Delta p_{cfog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right)\Delta p_{cfog\ i,j,k} + T_{Ci,j,k+1/2}\Delta p_{cfog\ i,j,k+1}\right) - \\
&\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo} + \frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k} c_{pcfi,j,k}\left(\frac{\sigma^{n+1}_{fi,j,k} - \sigma^{n}_{fi,j,k}}{\Delta t}\right) \\
&-\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo} + \frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k} c_{pcfi,j,k}\beta_{pfi,j,k}\left(\frac{\psi^{n+1}_{fi,j,k} - \psi^{n}_{fi,j,k}}{\Delta t}\right) \\
&+\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo} + \frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k} c_{pcfi,j,k}\beta_{pmi,j,k}\frac{\Delta P_{mgi,j,k}}{\Delta t} \\
&-\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo} + \frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k} c_{pcfi,j,k}\beta_{pmi,j,k}\left(\frac{\psi^{n+1}_{mi,j,k} - \psi^{n}_{mi,j,k}}{\Delta t}\right) \\
&+\phi_{fi,j,k}\left[\frac{\rho_{fo}}{M_{fo}}\left(\frac{S^{n+1}_{foi,j,k} - S^{n}_{foi,j,k}}{\Delta t}\right) + \frac{\rho_{fg}}{M_{fg}}\left(\frac{S^{n+1}_{fgi,j,k} - S^{n}_{fgi,j,k}}{\Delta t}\right)\right] - \frac{S_{foi,j,k}\rho_{foi,j,k}\phi_f}{M_{foi,j,k}}C_{foi,j,k}\frac{\Delta P_{cofg}}{\Delta t} - q_{Tfi,j,k} + q_{i,j,k}
\end{aligned}
$$

(A5.8h)

A5.2 Discretización de la ecuación de transferencia de fluidos de matriz a fractura

Sabiendo que el término de transferencia es igual a $q_T = q_{To} + q_{Tg}$ se llega a la ecuación siguiente:

$$q_T = \left(8\frac{\rho_{om}}{\mu_o M_{om}} \left(\frac{k_{xm}k_{ro\ xm}}{l_x^2} + \frac{k_{ym}k_{ro\ ym}}{l_y^2} + \frac{k_{zm}k_{ro\ zm}}{l_z^2} \right) + 8\frac{\rho_{gm}}{\mu_g M_{gm}} \left(\frac{k_{xm}k_{rg\ xm}}{l_x^2} + \frac{k_{ym}k_{rg\ ym}}{l_y^2} + \frac{k_{zm}k_{rg\ zm}}{l_z^2} \right) \right) \left(\Omega_m - \Omega_f \right)$$

(A5.9)

Discretizando y escribiendo la ecuación (A5.9) en forma incremental se llega a

$$q_{Tf} = \left(8\frac{\rho_{of}}{\mu_o M_{of}} \left(\frac{k_{xf}k_{ro\ xf}}{l_x^2} + \frac{k_{yf}k_{ro\ yf}}{l_y^2} + \frac{k_{zf}k_{ro\ zf}}{l_z^2} \right) + 8\frac{\rho_{gf}}{\mu_g M_{gf}} \left(\frac{k_{xf}k_{rg\ xf}}{l_x^2} + \frac{k_{yf}k_{rg\ yf}}{l_y^2} + \frac{k_{zf}k_{rg\ zf}}{l_z^2} \right) \right)_{i,j,k} \left(\left(\Omega_m^0 + \Delta\Omega_m \right) - \left(\Omega_f^0 + \Delta\Omega_f \right) \right)$$

(A5.10)

A5.3 Discretización de la ecuación diferencial incluyendo la transferencia matriz – fractura

A continuación se presentan las ecuaciones de flujo de fluidos según el caso con el que se va a trabajar discretizadas y en forma de esténcil

Caso 1:

$\Omega_m = P_{mg}$, $\psi_m = 0$ y $\Omega_f = P_{fg}$, $\psi_f = 0$

La ecuación (A5.10) se escribe como

$$q_{Tm} = \left(8\frac{\rho_{om}}{\mu_o M_{om}} \left(\frac{k_{xm}k_{ro\ xm}}{l_x^2} + \frac{k_{ym}k_{ro\ ym}}{l_y^2} + \frac{k_{zm}k_{ro\ zm}}{l_z^2} \right) + 8\frac{\rho_{gm}}{\mu_g M_{gm}} \left(\frac{k_{xm}k_{rg\ xm}}{l_x^2} + \frac{k_{ym}k_{rg\ ym}}{l_y^2} + \frac{k_{zm}k_{rg\ zm}}{l_z^2} \right) \right)_{i,j,k} \left(\left(P_{mg}^0 + \Delta P_{mg} \right) - \left(P_{fg}^0 + \Delta P_{fg} \right) \right)$$

(A5.10a)

El esténcil central se puede escribir como:

$$Cf_{i,j,k} = -T_{i-1/2,j,k} - T_{i+1/2,j,k} - T_{i,j-1/2,k} - T_{i,j+1/2,k} - T_{i,j,k-1/2} - T_{i,j,k+1/2}$$
$$-\frac{\phi_{fi,j,k} S_{foi,j,k} \rho_{foi,j,k}}{M_{foi,j,k} * \Delta t} \left(c_{fo\ i,j,k} + c_{pcf\ i,j,k} \beta_{pf\ i,j,k} \right) - \frac{\phi_{fi,j,k} S_{fgi,j,k} \rho_{fgi,j,k}}{M_{fgi,j,k} * \Delta t} \left(c_{fg\ i,j,k} + c_{pcf\ i,j,k} \beta_{pf\ i,j,k} \right)$$
$$-\left(8\frac{\rho_{of}}{\mu_o M_{of}} \left(\frac{k_{xf}k_{ro\ xf}}{l_x^2} + \frac{k_{yf}k_{ro\ yf}}{l_y^2} + \frac{k_{zf}k_{ro\ zf}}{l_z^2} \right) + 8\frac{\rho_{gf}}{\mu_g M_{gf}} \left(\frac{k_{xf}k_{rg\ xf}}{l_x^2} + \frac{k_{yf}k_{rg\ yf}}{l_y^2} + \frac{k_{zf}k_{rg\ zf}}{l_z^2} \right) \right)_{i,j,k}$$

El esténcil libre se puede escribir como:

$$Ff_{i,j,k} = -\left(T_{i-1/2,j,k}\,p^{0}{}_{fg\ i-1,j,k} - \left(T_{i-1/2,j,k} + T_{i+1/2,j,k}\right)p^{0}{}_{fg\ i,j,k} + T_{i+1/2,j,k}\,p^{0}{}_{fg\ i+1,j,k}\right)$$

$$-\left(T_{i,j-1/2,k}\,p^{0}{}_{fg\ i,j-1,k} - \left(T_{i,j+1/2,k} + T_{i,j-1/2,k}\right)p^{0}{}_{fg\ i,j,k} + T_{i,j+1/2,k}\,p^{0}{}_{fg\ i,j+1,k}\right) -$$

$$\left(T_{i,j,k-1/2}\,p^{0}{}_{fg\ i,j,k-1} - \left(T_{i,j,k+1/2} + T_{i,j,k-1/2}\right)p^{0}{}_{fg\ i,j,k} + T_{i,j,k+1/2}\,p^{0}{}_{fg\ i,j,k+1}\right) + q_{Ti,j,k}$$

$$-\left\{\begin{array}{l}
\left[Fi\left(\left(\frac{\rho_{fo}\phi_f}{M_{fo}} + \frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i+1,j,k} - \left(\frac{\rho_{fo}\phi_f}{M_{fo}} + \frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i-1,j,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\
\left[Fj\left(\left(\frac{\rho_{fo}\phi_f}{M_{fo}} + \frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j+1,k} - \left(\frac{\rho_{fo}\phi_f}{M_{fo}} + \frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j-1,k}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right] + \\
\left[Fk\left(\left(\frac{\rho_{fo}\phi_f}{M_{fo}} + \frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j,k+1} - \left(\frac{\rho_{fo}\phi_f}{M_{fo}} + \frac{\rho_{fg}\phi_f}{M_{fg}}\right)_{i,j,k-1}\right) * \left(\left(\frac{u^{n+1}_{x\,i,j,k} - u^{n}_{x\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{y\,i,j,k} - u^{n}_{y\,i,j,k}}{\Delta t}\right) + \left(\frac{u^{n+1}_{z\,i,j,k} - u^{n}_{z\,i,j,k}}{\Delta t}\right)\right)\right]
\end{array}\right\} + T -$$

$$\left(T_{Ci-1/2,j,k}\,p^{0}{}_{cfog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right)p^{0}{}_{cfog\ i,j,k} + T_{Ci+1/2,j,k}\,p^{0}{}_{cfog\ i+1,j,k}\right)$$

$$\left(T_{Ci,j-1/2,k}\,p^{0}{}_{cfog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right)p^{0}{}_{cfog\ i,j,k} + T_{Ci,j+1/2,k}\,p^{0}{}_{cfog\ i,j+1,k}\right) +$$

$$\left(T_{Ci,j,k-1/2}\,p^{0}{}_{cfog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right)p^{0}{}_{cfog\ i,j,k} + T_{Ci,j,k+1/2}\,p^{0}{}_{cfog\ i,j,k+1}\right) +$$

$$\left(T_{Ci-1/2,j,k}\Delta p_{cfog\ i-1,j,k} - \left(T_{Ci-1/2,j,k} + T_{Ci+1/2,j,k}\right)\Delta p_{cfog\ i,j,k} + T_{Ci+1/2,j,k}\Delta p_{cfog\ i+1,j,k}\right) +$$

$$\left(T_{Ci,j-1/2,k}\Delta p_{cfog\ i,j-1,k} - \left(T_{Ci,j-1/2,k} + T_{Ci,j+1/2,k}\right)\Delta p_{cfog\ i,j,k} + T_{Ci,j+1/2,k}\Delta p_{cfog\ i,j+1,k}\right) +$$

$$\left(T_{Ci,j,k-1/2}\Delta p_{cfog\ i,j,k-1} - \left(T_{Ci,j,k-1/2} + T_{Ci,j,k+1/2}\right)\Delta p_{cfog\ i,j,k} + T_{Ci,j,k+1/2}\Delta p_{cfog\ i,j,k+1}\right) -$$

$$\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo} + \frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k} c_{pcfi,j,k}\left(\frac{\sigma^{n+1}_{fi,j,k} - \sigma^{n}_{fi,j,k}}{\Delta t}\right)$$

$$+\phi_{fi,j,k}\left(\frac{\rho_{fo}}{M_{fo}}S_{fo} + \frac{\rho_{fg}}{M_{fg}}S_{fg}\right)_{i,j,k} c_{pcfi,j,k}\beta_{pmi,j,k}\frac{\Delta P_{mgi,j,k}}{\Delta t}$$

$$+\phi_{fi,j,k}\left[\frac{\rho_{fo}}{M_{fo}}\left(\frac{S^{n+1}_{foi,j,k} - S^{n}_{foi,j,k}}{\Delta t}\right) + \frac{\rho_{fg}}{M_{fg}}\left(\frac{S^{n+1}_{fgi,j,k} - S^{n}_{fgi,j,k}}{\Delta t}\right)\right] - \frac{S_{foi,j,k}\rho_{foi,j,k}\phi_f}{M_{foi,j,k}}C_{foi,j,k}\frac{\Delta P_{cofg}}{\Delta t} +$$

$$8*\left(\begin{array}{l}
\frac{\rho_{of}}{\mu_o M_{of}}\left(\frac{k_{xf}k_{ro\ xf}}{l_x^2} + \frac{k_{yf}k_{ro\ yf}}{l_y^2} + \frac{k_{zf}k_{ro\ zf}}{l_z^2}\right) + \\
\frac{\rho_{gf}}{\mu_g M_{gf}}\left(\frac{k_{xf}k_{rg\ xf}}{l_x^2} + \frac{k_{yf}k_{rg\ yf}}{l_y^2} + \frac{k_{zf}k_{rg\ zf}}{l_z^2}\right)
\end{array}\right)_{i,j,k}\left(P^{0}_{fg} - P^{0}_{mg} - \Delta P_{mg}\right)$$

En forma similar para los otros casos que se plantearon en el Anexo 4.

ANEXO 6. DISCRETIZACIÓN DE LAS ECUACIONES DEL MODELO DE DEFORMACIÓN GEOMECÀNICA

A6.1 Discretización de la ecuación geomecánica en dirección X

La primera ecuación geomecánica planteada en el Anexo 2, ecuación A2.8, puede escribirse de la siguiente forma siguiendo el análisis que se realizó en el Anexo 4 y Anexo 5.

$$BC_{Uxi,j,k}\Delta U_{x\,i,j,k-1}^{n+1} + S_{Uxi,j,k}\Delta U_{x\,i,j-1,k}^{n+1} + W_{Uxi,j,k}\Delta U_{x\,i-1,j,k}^{n+1} + C_{Uxi,j,k}\Delta U_{x\,i,j,k}^{n+1} + E_{Uxi,j,k}\Delta U_{x\,i+1,j,k}^{n+1} + N_{Uxi,j,k}\Delta U_{x\,i,j+1,k}^{n+1} + TC_{Uxi,j,k}\Delta U_{x\,i,j,k+1}^{n+1} = F_{Uxi,j,k} \quad \text{(A6.1)}$$

Donde:

$$BC_{Ux\;i,j,k} = 2C_{k-1}G_{i,j,k-1/2} \quad \text{(A6.1a)}$$

$$S_{Uxi,j,k} = 2C_{j-1}G_{i,j-1/2,k} \quad \text{(A6.1b)}$$

$$W_{Uxi,j,k} = 2C_{i-1}\left(2G_{i-1/2,j,k} + \lambda_{i-1/2,j,k}\right) \quad \text{(A6.1c)}$$

$$E_{Uxi,j,k} = 2C_{i}\left(2G_{i+1/2,j,k} + \lambda_{i+1/2,j,k}\right) \quad \text{(A6.1d)}$$

$$N_{Uxi,j,k} = 2C_{j}G_{i,j+1/2,k} \quad \text{(A6.1e)}$$

$$TC_{Uxi,j,k} = 2C_{k}G_{i,j,k+1/2} \quad \text{(A6.1f)}$$

$$C_{Ux\;i,j,k} = -\left\{BC_{Uxi,j,k} + S_{Uxi,j,k} + W_{Uxi,j,k} + E_{Uxi,j,k} + N_{Uxi,j,k} + TC_{Uxi,j,k}\right\} \quad \text{(A6.1g)}$$

$$\begin{aligned}
F_{Uxi,j,k} = {} & f_i f_j\left(G_{i,j-1/2,k} + \lambda_{i-1/2,j,k}\right)\Delta U_{y\,i-1,j-1,k}^{n+1} - f_i f_j\left(\lambda_{i+1/2,j,k} - \lambda_{i-1/2,j,k}\right)\Delta U_{y\,i,j-1,k}^{n+1} \\
& - f_i f_j\left(G_{i,j-1/2,k} + \lambda_{i+1/2,j,k}\right)\Delta U_{y\,i+1,j-1,k}^{n+1} + f_i f_j\left(G_{i,j-1/2,k} - G_{i,j+1/2,k}\right)\Delta U_{y\,i-1,j,k}^{n+1} \\
& - f_i f_j\left(G_{i,j-1/2,k} - G_{i,j+1/2,k}\right)\Delta U_{y\,i+1,j,k}^{n+1} - f_i f_j\left(G_{i,j+1/2,k} + \lambda_{i-1/2,j,k}\right)\Delta U_{y\,i-1,j+1,k}^{n+1} \\
& - f_i f_j\left(\lambda_{i-1/2,j,k} - \lambda_{i+1/2,j,k}\right)\Delta U_{y\,i,j+1,k}^{n+1} + f_i f_j\left(G_{i,j+1/2,k} + \lambda_{i+1/2,j,k}\right)\Delta U_{y\,i+1,j+1,k}^{n+1} \\
& + f_i f_k\left(G_{i,j,k-1/2} + \lambda_{i-1/2,j,k}\right)\Delta U_{z\,i-1,j,k-1}^{n+1} + f_i f_k\left(\lambda_{i-1/2,j,k} - \lambda_{i+1/2,j,k}\right)\Delta U_{z\,i,j,k-1}^{n+1} \\
& - f_i f_k\left(G_{i,j,k-1/2} + \lambda_{i+1/2,j,k}\right)\Delta U_{z\,i+1,j,k-1}^{n+1} - f_i f_k\left(G_{i,j,k+1/2} - G_{i,j,k-1/2}\right)\Delta U_{z\,i-1,j,k}^{n+1} \\
& - f_i f_k\left(G_{i,j,k-1/2} - G_{i,j,k+1/2}\right)\Delta U_{z\,i+1,j,k}^{n+1} - f_i f_k\left(G_{i,j,k+1/2} + \lambda_{i-1/2,j,k}\right)\Delta U_{z\,i-1,j,k+1}^{n+1}
\end{aligned}$$

$$- f_i f_k \left(\lambda_{i-1/2,j,k} - \lambda_{i+1/2,j,k}\right)\Delta U^{n+1}_{z\,i,j,k+1} + f_i f_k \left(G_{i,j,k+1/2} + \lambda_{i+1/2,j,k}\right)\Delta U^{n+1}_{z\,i+1,j,k+1}$$

$$+ f_i \beta_{pm\,i+1/2,j,k} \Delta P^{n+1}_{m\,i+1,j,k} + f_i \left(\beta_{pm\,i+1/2,j,k} - \beta_{pm\,i-1/2,j,k}\right)\Delta P^{n+1}_{m\,i,j,k}$$

$$- f_i \beta_{pm\,i-1/2,j,k} \Delta P^{n+1}_{m\,i-1,j,k} + f_i \alpha_{bf\,i+1/2,j,k} \Delta P^{n+1}_{f\,i+1,j,k} - f_i \left(\alpha_{bf\,i-1/2,j,k} - \alpha_{bf\,i+1/2,j,k}\right)\Delta P^{n+1}_{f\,i,j,k}$$

$$- f_i \beta_{bf\,i-1/2,j,k} \Delta P^{n+1}_{f\,i-1,j,k} + f_i \left(\sigma^0_{xx\,i+1,j,k} - \sigma^0_{xx\,i-1,j,k}\right) + f_j \left(\sigma^0_{xy\,i,j+1,k} - \sigma^0_{xy\,i,j-1,k}\right)$$

$$+ f_k \left(\sigma^0_{xz\,i,j,k+1} - \sigma^0_{xz\,i,j,k-1}\right) \qquad \text{(A6.1h)}$$

$$F_{Ux\,i,j,k} = -F_{Ux\,i,j,k}$$

A6.2 Discretización de la ecuación geomecánica en dirección Y

La segunda ecuación geomecánica, puede escribirse de la siguiente forma,

$$\begin{aligned} & BC_{Uy\,i,j,k}\Delta U^{n+1}_{y\,i,j,k-1} + S_{Uy\,i,j,k}\Delta U^{n+1}_{y\,i,j-1,k} + W_{Uy\,i,j,k}\Delta U^{n+1}_{y\,i-1,j,k} + C_{Uy\,i,j,k}\Delta U^{n+1}_{y\,i,j,k} + E_{Uy\,i,j,k}\Delta U^{n+1}_{y\,i+1,j,k} \\ & + N_{Uy\,i,j,k}\Delta U^{n+1}_{y\,i,j+1,k} + TC_{Uy\,i,j,k}\Delta U^{n+1}_{y\,i,j,k+1} = F_{Uy\,i,j,k} \end{aligned} \qquad \text{(A6.2)}$$

Donde:

$$BC_{Uy\,i,j,k} = 2C_{k-1} G_{i,j,k-1/2} \qquad \text{(A6.2a)}$$

$$S_{Uy\,i,j,k} = 2C_{j-1}\left(2G_{i,j-1/2,k} + \lambda_{i,j-1/2,k}\right) \qquad \text{(A6.2b)}$$

$$W_{Uy\,i,j,k} = 2C_{i-1} G_{i-1/2,j,k} \qquad \text{(A6.2c)}$$

$$E_{UY\,i,j,k} = 2C_I G_{i+1/2,j,k} \qquad \text{(A6.2d)}$$

$$N_{Uy\,i,j,k} = 2C_j\left(2G_{i,j+1/2,k} + G_{i,j+1/2,k}\right) \qquad \text{(A6.2e)}$$

$$TC_{Uy\,i,j,k} = 2C_k G_{i,j,k+1/2} \qquad \text{(A6.2f)}$$

$$C_{Uy\,i,j,k} = -\left\{BC_{Uy\,i,j,k} + S_{Uy\,i,j,k} + W_{Uy\,i,j,k} + E_{Uy\,i,j,k} + N_{Uy\,i,j,k} + TC_{Uy\,i,j,k}\right\} \qquad \text{(A6.2g)}$$

$$\begin{aligned} F_{Uy\,i,j,k} = {} & f_i f_j \left(G_{i,j,k+1/2} + \lambda_{i,j+1/2,k}\right)\Delta U^{n+1}_{x\,i-1,j-1,k} + f_i f_j \left(G_{i-1/2,j,k} - G_{i+1/2,j,k}\right)\Delta U^{n+1}_{x\,i,j-1,k} \\ & - f_i f_j \left(G_{i+1/2,j,k} + \lambda_{i,j-1/2,k}\right)\Delta U^{n+1}_{x\,i+1,j-1,k} + f_i f_j \left(\lambda_{i,j-1/2,k} - \lambda_{i,j+1/2,k}\right)\Delta U^{n+1}_{x\,i-1,j,k} \end{aligned}$$

$$
\begin{aligned}
& - f_i f_j \left(\lambda_{i,j-1/2,k} - \lambda_{i,j+1/2,k}\right)\Delta U^{n+1}_{x\ i+1,j,k} - f_i f_j \left(G_{i-1/2,j,k} + \lambda_{i,j+1/2,k}\right)\Delta U^{n+1}_{x\ i-1,j+1,k} \\
& - f_i f_j \left(G_{i-1/2,j,k} - G_{i+1/2,j,k}\right)\Delta U^{n+1}_{x\ i,j+1,k} + f_i f_j \left(G_{i+1/2,j,k} + \lambda_{i,j+1/2,k}\right)\Delta U^{n+1}_{x\ i+1,j+1,k} \\
& + f_j f_k \left(G_{i,j,k-1/2} + \lambda_{i,j-1/2,k}\right)\Delta U^{n+1}_{z\ i,j-1,k-1} + f_j f_k \left(\lambda_{i,j-1/2,k} - \lambda_{i,j+1/2,k}\right)\Delta U^{n+1}_{z\ i,j,k-1} \\
& - f_j f_k \left(G_{i,j,k-1/2} + \lambda_{i,j+1/2,k}\right)\Delta U^{n+1}_{z\ i,j+1,k-1} + f_j f_k \left(G_{i,j,k-1/2} - G_{i,j,k+1/2}\right)\Delta U^{n+1}_{z\ i,j-1,k} \\
& - f_j f_k \left(G_{i,j,k-1/2} - G_{i,j,k+1/2}\right)\Delta U^{n+1}_{z\ i,j+1,k} - f_j f_k \left(G_{i,j,k+1/2} + \lambda_{i,j-1/2,k}\right)\Delta U^{n+1}_{z\ i,j-1,k+1} \\
& + f_j f_k \left(G_{i,j,k+1/2} + \lambda_{i,j+1/2,k}\right)\Delta U^{n+1}_{z\ i,j+1,k+1} - f_j f_k \left(\lambda_{i,j-1/2,k} + \lambda_{i,j+1/2,k}\right)\Delta U^{n+1}_{z\ i,j,k+1} \\
& - f_j \beta_{bf\ i,j-1/2,k} \Delta P^{n+1}_{m\ i,j-1,k} - f_j \left(\beta_{bm\ i,j-1/2,k} - \beta_{bm\ i,j+1/2,k}\right)\Delta P^{n+1}_{m\ i,j,k} \\
& + f_j \beta_{bm\ i,j+1/2,k} \Delta P^{n+1}_{m\ i,j+1,k} - f_{jJ} \beta_{bf\mathrm{F}\ i,j-1/2,k} \Delta P^{n+1}_{f\ i,j-1,k} \\
& - f_j \left(\beta_{bf\ i,j-1/2,k} - \beta_{bf\ i,j+1/2,k}\right)\Delta P^{n+1}_{f\ i,j,k} + f_j \beta_{bf\ i,j+1/2,k} \Delta P^{n+1}_{f\ i,j+1,k} \\
& + f_i \left(\sigma^0_{xy\ i+1,j,k} - \sigma^0_{xy\ i-1,j,k}\right) + f_j \left(\sigma_{yy\ i,j+1,k} - \sigma^0_{yy\ i,j-1,k}\right) + f_k \left(\sigma^0_{yz\ i,j,k+1} - \sigma^0_{yz\ i,j,k-1}\right)
\end{aligned}
\tag{A6.2h}
$$

$$
F_{Uyi,j,k} = -F_{Uyi,j,k}
$$

A6.3 Discretización de la ecuación geomecánica en dirección Z

La tercera ecuación geomecánica, puede escribirse de la siguiente manera

$$
\begin{aligned}
& BC_{Uzi,j,k}\Delta U^{n+1}_{z\ i,j,k-1} + S_{Uzi,j,k}\Delta U^{n+1}_{z\ i,j-1,k} + W_{Uzi,j,k}\Delta U^{n+1}_{z\ i-1,j,k} + C_{Uzi,j,k}\Delta U^{n+1}_{z\ i,j,k} + E_{Uzi,j,k}\Delta U^{n+1}_{z\ i+1,j,k} \\
& + N_{Uzi,j,k}\Delta U^{n+1}_{z\ i,j+1,k} + TC_{Uzi,j,k}\Delta U^{n+1}_{z\ i,j,k+1} = F_{Uzi,j,k}
\end{aligned}
\tag{A6.3}
$$

Donde:

$$
BC_{Uzi,j,k} = 2C_{k-1}\left(2G_{i,j,k-1/2} + \lambda_{i,j,k-1/2}\right) \tag{A6.3a}
$$

$$
S_{Uzi,j,k} = 2C_{j-1}G_{i,j-1/2,k} \tag{A6.3b}
$$

$$
W_{Uzi,j,k} = 2C_{i-1}G_{i-1/2,j,k} \tag{A6.3c}
$$

$$
E_{Uzi,j,k} = 2C_i G_{i+1/2,j,k} \tag{A6.3d}
$$

$$
N_{Uzi,j,k} = 2C_j G_{i,j+1/2,k} \tag{A6.3e}
$$

$$
TC_{Uzi,j,k} = 2C_k\left(2G_{i,j,k+1/2} + \lambda_{i,j,k+1/2}\right) \tag{A6.3f}
$$

$$
C_{Uzi,j,k} = -\left\{BC_{Uzi,j,k} + S_{Uzi,j,k} + W_{Uzi,j,k} + E_{Uzi,j,k} + N_{Uzi,j,k} + TC_{Uzi,j,k}\right\} \tag{A6.3g}
$$

$$
\begin{aligned}
F_{Uz i,j,k} = {} & f_i f_k \left(G_{i-1/2,j,k} + \lambda_{i,j,k-1/2}\right)\Delta U^{n+1}_{x\ i-1,j,k-1} + f_i f_k \left(G_{i-1/2,j,k} - G_{i+1/2,j,k}\right)\Delta U^{n+1}_{x\ i,j,k-1} \\
& - f_i f_k \left(G_{i+1/2,j,k} + \lambda_{i,j,k-1/2}\right)\Delta U^{n+1}_{x\ i+1,j,k-1} + f_i f_k \left(\lambda_{i,j,k-1/2} - \lambda_{i,j,k+1/2}\right)\Delta U^{n+1}_{x\ i-1,j,k} \\
& - f_i f_k \left(\lambda_{i,j,k-1/2} - \lambda_{i,j,k+1/2}\right)\Delta U^{n+1}_{x\ i+1,j,k} - f_i f_k \left(\lambda_{i,j,k+1/2} + G_{i-1/2,j,k}\right)\Delta U^{n+1}_{x\ i-1,j,k+1} \\
& - f_i f_k \left(G_{-1/2i,j,k} - G_{i+1/2,j,k}\right)\Delta U^{n+1}_{x\ i,j,k+1} + f_i f_k \left(\lambda_{i,j,k+1/2} + G_{i+1/2,j,k}\right)\Delta U^{n+1}_{x\ i+1,j,k+1} \\
& + f_j f_k \left(\lambda_{i,j,k-1/2} + G_{i,j-1/2,k}\right)\Delta U^{n+1}_{y\ i,j-1,k-1} + f_j f_k \left(G_{i,j-1/2,k} - G_{i,j+1/2,k}\right)\Delta U^{n+1}_{y\ i,j,k-1} \\
& - f_j f_k \left(\lambda_{i,j,k-1/2} + G_{i,j+1/2,k}\right)\Delta U^{n+1}_{y\ i,j+1,k-1} + f_j f_k \left(\lambda_{i,j,k-1/2} - \lambda_{i,j,k+1/2}\right)\Delta U^{n+1}_{y\ i,j-1,k} \\
& - f_j f_k \left(\lambda_{i,j,k-1/2} - \lambda_{i,j,k+1/2}\right)\Delta U^{n+1}_{y\ i,j+1,k} - f_j f_k \left(\lambda_{i,j,k+1/2} + G_{i,j-1/2,k}\right)\Delta U^{n+1}_{y\ i,j-1,k+1} \\
& - f_j f_k \left(G_{i,j-1/2,k} - G_{i,j+1/2,k}\right)\Delta U^{n+1}_{y\ i,j,k+1} + f_j f_k \left(\lambda_{i,j,k+1/2} + G_{i,j+1/2,k}\right)\Delta U^{n+1}_{y\ i,j+1,k+1} \\
& - f_k \beta_{bm\ i,j,k-1/2}\Delta P^{n+1}_{m\ i,j,k-1} - f_k \left(\beta_{bm\ i,j,k-1/2} - \beta_{bm\ i,j,k+1/2}\right)\Delta P^{n+1}_{m\ i,j,k} + f_k \beta_{bm\ i,j,k+1/2}\Delta P^{n+1}_{m\ i,j,k+1} \\
& - f_k \beta_{bf\ i,j,k-1/2}\Delta P^{n+1}_{\mathrm{f}\ i,j,k-1} + f_k \left(\beta_{bf\ i,j,k-1/2} - \beta_{bf\ i,j,k+1/2}\right)\Delta P^{n+1}_{f\ i,j,k} + f_k \beta_{bf\ i,j,k+1/2}\Delta P^{n+1}_{m\ i,j,k+1} \\
& + f_i \left(\sigma^0_{xz\ i+1,j,k} - \sigma^0_{xz\ i-1,j,k}\right) + f_j \left(\sigma^O_{yz\ i,j+1,k} - \sigma^O_{yz\ i,j-1,k}\right) + f_k \left(\sigma^0_{zz\ i,j,k+1} - \sigma^0_{zz\ i,j,k-1}\right)
\end{aligned}
\tag{A6.3h}
$$

$$F_{Uz i,j,k} = -F_{Uz i,j,k}$$