AN ANOMALY FREE LITTLE HIGGS MODEL WITH 3-4-1 GAUGE SYMMETRY

MASTER OF SCIENCE THESIS

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A Liliana, mi prometida.

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Resumen

En esta tesis se presentan las contribuciones a nivel árbol a los observables electrodébiles en el contexto de un modelo Little Higgs simple, con una simetría global aproximada $[SU(4)/SU(3)]^4$ y una inmersión libre de anomalías del grupo gauge electrodébil $SU(2)_L \otimes U(1)_Y$ en el grupo gauge $SU(4)_L \otimes U(1)_X$. Mediante la realización de un ajuste global a 22 observables, se restringe el ángulo de mezcla θ de $Z - Z'$, y se obtienen los límites inferiores de la masa correspondiente al nuevo bosón de gauge neutro Z_2, y sobre el parámetro de escala f asociado a el rompimiento $SU(4) \rightarrow SU(2)$. Los datos de precisión electrodébiles producen las restricciones $0 \leq \theta \leq 1.33 \times 10^{-3}$, $M_{Z_2} \geq 1.23$ TeV, y $f \geq 1.56$ TeV.

Keywords: Little Higgs, Fenomenología de Física de Altas Energías, Física más allá del Modelo Estándar, Rompimineto Colectivo de Simetría.

Abstract

We calculate the tree-level contributions to electroweak observables in the context of a simple group little Higgs model with approximate $[SU(4)/SU(3)]^4$ global symmetry and with anomaly-free embedding of the $SU(2)_L \otimes U(1)_Y$ electroweak gauge group into $SU(4)_L \otimes U(1)_X$. By performing a global fit to 22 observables we bound the $Z - Z'$ mixing angle θ, and obtain lower bounds on the mass of the corresponding physical new neutral gauge boson Z_2, and on the parameter scale f associated to the $SU(4) \to SU(2)$ breaking. Electroweak precision data produce the constraints $0 \leq \theta \leq 1.33 \times 10^{-3}$, $M_{Z_2} \geq 1.23$ TeV, and $f \geq 1.56$ TeV.

Keywords: Little Higgs, High Energy Physics Phenomenology, Physics Beyond the Standard Model, Collective Symmetry Breaking.

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Introduction

Two unanswered questions in the framework of the successful standard model (SM) of the electroweak interactions concern the nature of the electroweak symmetry breaking and the number of fermion families. Closely related to the first question is the fact that, if the electroweak symmetry breaking is triggered by the Higgs mechanism, the Higgs boson mass in the SM is quadratically sensitive to the cutoff scale Λ of the SM effective theory via radiative corrections. The quantum-corrected Higgs mass is given at one-loop level by

$$m_H^2 = (m_H^2)_{bare} + \frac{3g^2\Lambda^2}{32\pi^2 m_W^2}\left(m_H^2 + 2m_W^2 + m_Z^2 - \frac{4}{3}N_c\sum_f m_f^2\right). \tag{1}$$

For a high cutoff scale Λ, this cancellation must be fine-tuned; for example, for $\Lambda = 10$ TeV, $(m_H^2)_{bare}$ must be tuned at the 1% level to cancel the radiative corrections. In fact, requiring that the one-loop contributions to the Higgs mass-squared parameter are no more than 10 times the size of the renormalized Higgs mass-squared term (i.e., no more than 10% fine-tuning), leads to the requirement that

$$\Lambda_t \leq 2 \text{ TeV} \qquad \Lambda_{W,Z} \leq 5 \text{ TeV} \qquad \Lambda_H \leq 10 \text{ TeV}. \tag{2}$$

Therefore, the "natural" mass for the Higgs particle would must be Λ unless we fine tune the value of its tree level mass in order to cancel the large loop corrections. However, electroweak precision measurements, which indirectly test the SM predictions at scales in the range $1 - 10$ TeV (including one-loop quantum corrections) without finding significant deviations, favor a light Higgs (200 GeV) and hence suggest that the mechanism responsible for the breakdown of the electroweak symmetry should manifest itself at or below the TeV scale. Due to its quadratic sensitivity to heavy physics, a Higgs mass of order a few hundred GeV implies $\Lambda \sim 1$ TeV, a cutoff clearly experimentally disfavored. Since the Higgs mass $m_H \sim v = 246$ GeV set the electroweak energy scale, it follows that the experimental data impose the hierarchy $v < \Lambda$ with Λ of order of tens of TeV. The need to stabilize the scale v against the large loop corrections is dubbed as "the little hierarchy problem".

The classic solution to this puzzle is Supersymmetry. From the bottom-up point of view, the quadratic divergences in the Higgs mass due to top quark, gauge boson and Higgs loops are canceled by the top squark, gaugino and Higgsino loops, respectively. From the top-down point of view, the Higgs mass is protected by supersymmetry to be one loop factor below the soft supersymmetry breaking scale. Thus weak scale supersymmetry is natural if $M_{SUSY} \sim \mathcal{O}\,(1 \text{ TeV})$.

An alternative proposal to supersymmetry softly broken at ~ 1 TeV to solve the hierarchy problem is provided by the so-called "Little Higgs Models" [1, 2, 3, 4, 5, 6, 7, 8]. In these models the Higgs is a pseudo-Nambu-Goldstone boson whose zero tree-level mass is protected by an approximate global symmetry G which is spontaneously broken down to $H \subset G$ at a scale $f \sim 1$ TeV. The global symmetry is also explicitly broken by gauge and Yukawa couplings to the Higgs, and by Higgs quartic couplings, all of them provided by a gauge symmetry $F \subset G$, larger than the SM one, that breaks to the SM electroweak gauge group $SU(2)_L \otimes U(1)_Y = F \cap G$ at the scale f (the additional $U(1)$ gauge group is required in order to embed quarks in the theory). These couplings generate the Higgs mass at the one-loop level because no single coupling explicitly breaks the global symmetry ("collective symmetry breaking"). This is also the reason for the absence of quadratically divergent one-loop contributions to m_H^2; in fact, the new heavy gauge bosons, fermions and scalars associated to the enlarged symmetry F cancel the quadratic divergences coming from the SM fields. In this way, the cutoff scale at which the theory becomes strongly coupled is parametrically one loop factor above the scale f, that is, the cutoff is pushed up to $\Lambda \sim 4\pi f \sim 10$ TeV, a scale that is safe with respect to electroweak precision measurements.

Summarily, the Little Higgs idea is as follows

(i) The Higgs field is a pseudo-Nambu-Goldstone boson of a global symmetry that is spontaneously broken at a scale $\Lambda \sim 4\pi f \sim 10$ TeV;

(ii) The quadratic divergences in the Higgs mass are canceled at the one-loop level by new particles with masses $M \sim gf \sim 1 - 3$ TeV;

(iii) The Higgs acquires a mass radiatively at the electroweak scale $v \sim \frac{g^2 f}{4\pi} \sim 100 - 300$ TeV.

From the bottom-up point of view, the quadratic divergences in the Higgs mass are canceled by loops of new particles of the same statistics (in contrast to supersymmetry, in which the cancellations are due to particles of opposite statistics). From the top-down point of view, the Higgs mass is protected by the global symmetry. Little Higgs models are constructed so that at least two operators are needed to explicitly break all of the global symmetry that protects the Higgs mass. This forbids quadratic divergences at one-loop; the Higgs mass is then smaller than Λ by not one but two loop factors, leading to the little hierarchy $\Lambda \gg f \gg v$.

Concerning the question on the number of fermion generations in nature, two alternative scenarios, which provide some insight for the solution of this puzzle by relating the number of generations to the cancellation of chiral anomalies, have been proposed in the literature. In one of them anomalies constrain the number of generations provided their cancellation takes place either in a nonsupersymmetric $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ theory that lives in a six-dimensional spacetime [9], or in a sixdimensional $(1,1)$ supersymmetric gauge theory

[10]. In the other one the SM is extended either to the gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$ (the 3-3-1 model) [11, 12] or to the gauge symmetry $SU(3)_c \otimes SU(4)_L \otimes U(1)_Y$ (the 3-4-1 model) [13, 14, 15, 16], with anomalies cancelling among the families (three-family models) and not family by family as in the SM. In the 3-3-1 extension this happens only if we have an equal number of left-handed triplets and antitriplets, taking into account the color degree of freedom. Correspondingly, an equal number of 4-plets and 4^*-plets is required in the 3-4-1 extension. As a consequence, the number of fermion families N_f must be divisible by the number of colors N_c of $SU(3)_c$, being $N_f = N_c = 3$ the simplest solution.

Now, little Higgs models can be separated into two classes [4, 17]: the "product group" models in which the enlarged gauge symmetry F is the direct product of two or more gauge groups whose diagonal breaking leads to the SM $SU(2)_L$ gauge group, and the "simple group" models where the SM $SU(2)_L$ gauge group comes from the breaking of a single larger gauge group and the collective symmetry breaking is achieved from the gauge couplings to two or more nonlinear sigma fields which acquire vacuum expectation values (VEV) at the scale f. In the simplest realization of the simple group class the approximate global symmetry is $[SU(3)/SU(2)]^2$, and the SM electroweak gauge group is enlarged to $SU(3)_L \otimes U(1)_X$ which is basically a kind of 3-3-1 model without exotic electric charges in the fermion sector (that is, without electric charges different from $\pm 2/3$ and $\pm 1/3$ for exotic heavy quarks and different from 0 and ± 1 for exotic leptons) [4]. This model, however, lacks a quartic Higgs coupling that can be generated, without quadratic divergences at one loop, in a $SU(4)_L \otimes U(1)_X$ extension which is, in turn, a kind of 3-4-1 model without exotic electric charges, too. In this case, the global symmetry protecting the Higgs mass is an approximate $[SU(4)/SU(3)]^4$ [4].

The embedding of the SM fermions both into the $SU(3)_L \otimes U(1)_X$ extension and into the $SU(4)_L \otimes U(1)_X$ one can be done in an universal, but anomalous, way or in an anomaly-free way. A detailed study of the experimental signatures of the little Higgs model with universal and anomaly-free $SU(3)_L \otimes U(1)_X$ embeddings has been done in [17], while some aspects of their electroweak constraints has been presented in [18]. Electroweak constraints on the little Higgs model with universal $SU(4)_L \otimes U(1)_X$ embedding have been studied in [19] and the analysis of these constraints on the model with anomaly-free embedding have been initiated in [20].

The systematic study on the way of constructing anomaly-free 3-3-1 fermion spectra without exotic electric charges (which unavoidably also produces $SU(4)$ gauge bosons with electric charges 0 and ± 1 only) was done, for the first time, in Ref. [12] outside the little Higgs context. For the 3-4-1 extension, the same analysis was done in Refs. [13, 14]. An analogous study, but now in the framework of the little Higgs model, was presented in Ref. [21].

In this work we will be involved with anomaly-free little Higgs models with $SU(4)_L \otimes U(1)_X$ gauge symmetry. The analysis of the 3-4-1 gauge theory carried out in Refs. [13, 14] has shown that the restriction to ordinary electric charges in the fermion and gauge boson sectors, allows only for eight different anomaly-free spectra. Four of them correspond to three-family models and can be classified according to the values of the parameters b and c

in the most general expression for the electric charge generator in $SU(4) \otimes U(1)_X$

$$Q = aT_{3L} + \frac{b}{\sqrt{3}}T_{8L} + \frac{c}{\sqrt{6}}T_{15L} + XI_4, \tag{3}$$

where $T_{iL} = \lambda_{iL}/2$ (λ_{iL} are the Gell-Mann matrices for $SU(4)_L$ normalized as $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$), $I_4 = Dg(1,1,1,1)$ is the diagonal 4×4 unit matrix, and $a = 1$ gives the usual isospin of the electroweak interaction.

Under the condition of absence of exotic electric charges, the allowed values for these parameters are: $b = c = 1$ (Model A) and $b = 1, c = -2$ (Model B). Two of the four anomaly-free three-family models belongs to the $b = c = 1$ class, while the other two models belong to the $b = 1, c = -2$ class. Conspicuously, in the little Higgs scenario both the $SU(3)_L \otimes U(1)_X$ embedding and the $SU(4)_L \otimes U(1)_X$ one are three-family models in which all the exotic fermions have only ordinary electric charges, and the complete anomaly-free fermion content obtained in [21] for the $SU(4)_L \otimes U(1)_X$ case exactly coincides with two of the four possible spectra obtained in [13, 14]. Moreover, in the simple group little Higgs models, the anomaly-free embedding only works if the number of fermion generations is a multiple of three, and the fermion spectra remain anomaly-free up to the scale Λ. Then, additional spectator fermions, required at this scale in the universal embedding in order to cancel anomalies, are not necessary [17] and the UV completion of the model can be constructed more easily [22]. Thus, within a single simple group little Higgs model we find a solution to the little hierarchy problem and obtain some insight on the problem of the number of fermion families in nature.

The enlargement of the electroweak symmetry to $SU(4)_L \otimes U(1)_X$ leads to the prediction of two extra neutral gauge bosons Z' and Z'' which, in general, mix up with the known Z boson of the SM. As we shall see below, by imposing not particularly strong conditions on the scales f and v, this mixing can be constrained to occur between Z and Z' only, which leaves $Z'' \equiv Z_3$ as a heavy mass eigenstate. This fact produces an enormous simplification in the study of the low energy deviations of the Z couplings to the SM families [4, 23]. The diagonalization of the $Z - Z'$ mass matrix produces a light mass eigentate Z_1, which can be identified as the neutral gauge boson of the SM, and an additional heavy Z_2. Under these conditions, in this work we examine electroweak precision constraints on an anomaly-free $SU(4)_L \otimes U(1)_X$ little Higgs model which is representative of the $b = 1,\ c = -2$ class. In particular, we do a χ^2 fit to Z-pole observables and atomic parity violation (APV) data in order to constraint three related parameters: the scale f, the mixing angle θ between Z and Z', and the mass scale M_{Z_2} of the corresponding physical new neutral gauge boson.

Chapter 1

Why to construct a Little Higgs Model?

The drawbacks of the SM have led to a strong belief that the model must be regarded as a low-energy effective field theory originating from a more fundamental one. Nearly from the proposal of the SM many scenarios for a most fundamental theory have been advocated in several attempts for solving the various puzzles of the model. All those scenarios introduce theoretically well motivated ideas associated to physics beyond the SM.

One can extend the SM by adding new fermion fields (adding a right-handed neutrino constitutes its simplest extension), by enlarging the local gauge group (introducing new global or local, discrete or continuous symmetries), or by augmenting the scalar sector to more than one Higgs representation. These extensions can occur simultaneously when the SM is embeded in an underlying *simple* group (Grand Unification Theories (GUT)), when Supersymmetry (SUSY) is introduced or with a totally new alternative called Little Higgs Models.

There exist in the literature a plethora of excellent reviews of the Little Higgs Model (LH) [17], so that we start by giving only a brief review pointing out the more relevant aspects of the model and emphasizing on its motivation, successes and deficiencies in order to identify suggestions for new models that can use this idea as an alternative for Physics Beyond the SM.

1.1 Introduction

Given the Standard Model's remarkable success in accurately describing Physics at length scales ranging from atomic scales all the way down to the shortest currently probed scales of about 10^{-18} m it may appear puzzling that the most ambitious, expensive and intricate experiment in all the history of the humanity, the CERN Large Hadron Collider (LHC), is devoted to discovering Physics Beyond the Standard Model (PBSM).

The basic response is "the hierarchy problem" or the sometimes called "fine-tuning problem", which motivates much of PBSM research.

1.2 The SM Fine-Tuning Problem

In SM, the mass of the $W^{\pm}$ and the Higgs boson is given by

$$M_W = \frac{gv}{2} \sim 80 \text{ TeV} \qquad\qquad M_H = v\sqrt{\frac{\lambda}{2}} \tag{1.1}$$

where g is the $SU(2)$ gauge coupling constant, and v is a parameter within the electroweak sector, with dimensions of energy, whose approximate value is

$$v \simeq 246 \text{ GeV}. \tag{1.2}$$

The vacuum expectation value (vev) of the Higgs field $\frac{v}{\sqrt{2}}$ is given in terms of this quantity and, at the same time, v signals the spontaneous breaking of electroweak symmetry. It is the largest importance to note that this parameter also sets the scale, in principle, of all masses in the theory.

The parameter λ is the strength of the Higgs self-interaction in the Higgs potential

$$V = -\mu^2\phi^\dagger\phi + \frac{\lambda}{4}\left(\phi^\dagger\phi\right)^2, \tag{1.3}$$

where $\lambda > 0$ and $\mu^2 > 0$. Here ϕ is the $SU(2)$ doublet field

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{1.4}$$

being ϕ^+ and ϕ^0, the charged and neutral components respectively.

With the μ^2 sign as in (1.3) the minimum of V interpreted as a classical potential is at the non-zero value

$$|\phi| = \frac{\sqrt{2}\mu}{\sqrt{\lambda}} \equiv \frac{v}{\sqrt{2}}, \tag{1.5}$$

where $\mu \equiv \sqrt{\mu^2}$. This classical minimum is conventionally interpreted as the expectation value of the quantum field in the quantum vacuum at tree level, that means, no loops have been considered.

The last analysis assume a classical potential and, quantum corrections only can appear if the field ϕ in the potential V is figured out like a quantum operator. If it be so, the 4-boson self-interaction in (1.3) generates, at one-loop order, a contribution to the $\phi^\dagger\phi$ term, corresponding to a self-energy diagram [24], which is proportional to

$$\lambda\int^{\Lambda} d^4k\frac{1}{k^2 - M_H^2}, \tag{1.6}$$

where Λ is the SM cut-off. This integral clearly diverges quadratically, and it turns out to be positive, producing a correction

$$\sim \lambda\Lambda^2\phi^\dagger\phi \tag{1.7}$$

to the bare $-\mu^2\phi^\dagger\phi$ term in V. The coefficient $-\mu^2$ of $\phi^\dagger\phi$ is then replaced by the one-loop corrected physical value $-\mu^2_{phys}$ where

$$-\mu^2_{phys} = -\mu^2 + \lambda\Lambda^2. \tag{1.8}$$

Re-minimizing V, we obtain (1.5) but with μ replaced by $\mu_{phys} \equiv \sqrt{\mu^2_{phys}}$. Note that if we want to be able to treat the Higgs coupling λ perturbatively, such value cannot hardly be greater than 1. Furthermore, if $\Lambda \sim M_P \sim 10^{19}$ GeV, the one-loop correction in (1.8) is then vastly greater than $\sim (100 \text{ GeV})^2$, so that to arrive at a value $\sim (100 \text{ GeV})^2$ after inclusion of this loop correction [24] would seem to require that we start with an equally huge value of the Lagrangian parameter μ^2, relying on a remarkable cancellation, or *fine-tuning*, to get us from $\sim (10^{19} \text{ GeV})^2$ down to $\sim (10^2 \text{ GeV})^2$.

In the SM, this fine-tuning problem involving the parameter μ_{phys} affects not only the mass of Higgs particle, but also all masses in the SM, which is given in terms of v and hence μ_{phys}.

At the LHC the 1-10 TeV energy scale have been probed directly for the first time on March 30, 2010. Thus an important question to answer is whether it is natural for the SM to be valid up to these scales. To accomplish it, let us see that if we replace $\Lambda = 10$ TeV in equation (1), as well as the appropriate value of the other constants, then the total Higgs mass is approximately

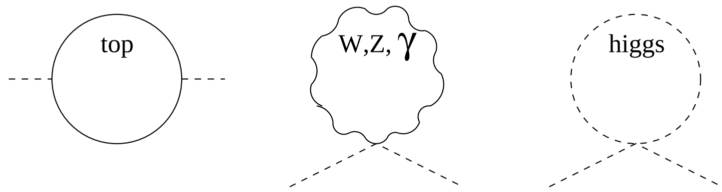
$$m_H^2 = \left(m_H^2\right)_{tree} - [100 - 10 - 5]\,(200 \text{ GeV})^2\,, \tag{1.9}$$

where the three more important contributions come from the top, gauge and Higgs loop Fig.(1.1). In order for this to add up to a Higgs mass of order a few hundred GeV as required in the SM, fine tuning of one part in 100 is required. But, setting $\Lambda = 1$ TeV in the mentioned formula (1) we find that the most dangerous contribution from the top loop is only about $(200 \text{ GeV})^2$. Thus no fine tuning is required, the SM with no new physics up to 1 TeV is perfectly natural, and we should not be surprised that we have not yet seen deviations from it at colliders ranging this scale.

1.3 Phenomenological Restrictions for New Physics at 1-10 TeV

At low energies, new physics can be integrated out and its effects are parametrized in terms of higher dimensional operators involving only Standard Model fields [25]. Precision experimental measurements constrain the sizes of various higher dimensional operators and

Figure 1.1: The most significant quadratically divergent contributions to the Higgs mass in the Standard Model.



consequently the scales of the corresponding new physics [26]. The most stringent bounds are on the operators which break the (approximate) symmetries of the Standard Model, such as those violating baryon number, flavor and CP symmetries. New physics which occurs at the TeV scale should respect these Standard Model symmetries in order not to generate any dangerous operator with a significant size. In the low energy effective theory, however, there are operators, generated by the new physics, which conserve baryon number, flavor and CP symmetries. Precision electroweak measurements put strong constraints on many operators of this kind, and so far suggest no evidence for new physics up to $\geq 5-7$ TeV [27]. This creates some tension with the naturalness requirement, however, which expects new physics at 1 TeV to cut off the quadratic divergence to the Higgs mass-squared. Indeed, many models which address the stabilization of the electroweak scale have new particles in the 1 TeV range in order to cancel the quadratic divergences incurred by the Standard Model particles. As we already stated, the amount of fine-tuning required to reconcile the difference here is not severe, and one may or may not take this *little hierarchy problem* seriously.

1.4 Precision Electroweak Data and the Little Hierarchy Problem

Current experimental data already give some constraints on possible new physics at the TeV scale. Absence of nucleon decays and strong bounds on flavor-changing neutral currents indicate that these effects cannot receive any significant contributions [8] from the TeV scale physics, which implies baryon number conservation and approximate flavor symmetries at the TeV scale. Precision electroweak measurements also put constraints on many operators consistent with baryon, flavor and CP symmetries. The scales which suppress these operators are required to be larger than $2-7$ TeV, depending on the operators and the Higgs mass [8], as was discussed in Ref. [27]. Generally speaking these operators arise by exchanging new heavy particles, and the bound on the sizes of the operators translates into the bound on the masses of the new particles and their couplings to the Standard Model fields. If the new particles are responsible for cancelling the quadratic divergences to the Higgs mass-squared, their masses have to be at ~ 1 TeV by naturalness. One therefore needs to worry about

Table 1.1: Lower bounds on the scale which suppresses higher-dimensional operators that violate approximate symmetries of the Standard Model [8].

Broken Symmetry	Operators	Scale Λ
B, L	$(QQQL)/\Lambda^2$	10^{13} TeV
flavor (1, 2^{nd} family), CP	$\left(\bar{d}s\bar{d}s\right)/\Lambda^2$	1000 TeV
flavor (1, 3^{nd} family)	$m_b\left(\bar{s}\sigma_{\mu\nu}F^{\mu\nu}b\right)/\Lambda^2$	50 TeV
Custodial $SU(2)$	$\left(h^\dagger D_\mu h\right)/\Lambda^2$	5 TeV
None (S-parameter)	$\left(D^2h^\dagger D^2 h\right)/\Lambda^2$	5 TeV

the compatibility of the existence of these particles with the precision electroweak data. Note, however, that the quadratic sensitivity to the high energy physics of the Higgs mass-squared parameter is a result of loop contributions. To cancel the quadratic divergences the new particles at the TeV scale[1] only need to contribute to the Higgs mass at the loop level. It must be clear that is possible to suppress the tree level contributions due to the new physics without modifying the cancellation of the loop contributions. The simplest and most natural way to implement this is to have a new symmetry acting on new TeV particles, while all the Standard Model fields are neutral under the new symmetry. Then there can be no interaction vertex involving the Standard Model particles and a single new TeV particle charged under the symmetry. The interactions containing more than one TeV particles, on the other hand, can still be allowed. Of course, not every TeV scale particle would induce large higher dimensional operators [8] which affect the precision electroweak measurements, so in practice we only need the dangerous particles, for example W' and Z', to be charged under this symmetry. With the new symmetry, higher dimensional operators are generated only at the loop level, and new particles as light as a few hundred GeV can be perfectly consistent with the precision electroweak data.

The higher-dimensional operators can be categorized by the symmetries which they break. More precisely, the relevant symmetries are baryon and lepton number (B and L), CP and flavor symmetries, and custodial $SU(2)$ symmetry. The wealth of indirect experimental data can then be translated into bounds on the scale suppressing the operators. Examples of such operators and the resulting bounds are summarized in Table (1.1). The bounds imply that physics at the TeV scale has to conserve B and L, flavor and CP to a very high accuracy, and that violations of custodial [8] symmetry and contributions to the S-parameter should also be small.

The question then becomes if it is possible to add new physics at the TeV scale to the SM which stabilizes the Higgs mass but does not violate the above bounds.

[1]The new particles can be much lighter than 1 TeV.

1.5 Existing Theoretical Alternatives to be Tested at LHC

There are existing successful approaches to solving the little fine-tuning problem with symmetries acting only on the new particles. The most popular and well-known example is the Minimal Supersymmetric Standard Model (MSSM) with R-parity conservation. In MSSM, all Standard Model particles have positive R-parity and all superpartners have negative R-parity. Superpartner [28] loops cancel the quadratic divergences from the Standard Model particle loops, but in the low energies there is no higher dimensional operator induced by superpartners at the tree level. For a large portion of the parameter space, MSSM is consistent with all the precision data. This is one of the major reasons which make the MSSM the leading candidate for physics beyond the Standard Model. On the other hand [28], without R-parity, there are many strong constraints on the R-parity violating couplings which require them to be unnaturally small. Although supersymmetry is aesthetically appealing, R-parity is the reality check that ensures the consistency of supersymmetric models with precision experiments.

Another closely related example is the KK-parity in the Universal Extra Dimensions (UEDs), where all Standard Model particles propagate in some number of compactified extra dimensions [29]. The compactification breaks the translational invariance in the extra dimensions down to some discrete subgroup corresponding to the geometrical symmetry of the compactified space [28]. As a result, the momentum conservation in extra dimensions is reduced to the KK-parity conservation of the Kaluza-Klein (KK) states of the Standard Model fields. The KK-parity prohibits the lowest KK states from contributing to the higher dimensional operators at the tree level, therefore allowing them to be as light as 300 GeV [44]. The contributions from higher KK states may also be suppressed if the mixing with the zero mode is small. Although the simplest UED scenario, where the KK state loops do not cancel the quadratic divergence of the Higgs mass-squared, does not directly address the little hierarchy problem [28], the KK-parity allows the sizes of the extra dimensions to be large enough to be probed in the near future. This feature makes the UED model very interesting phenomenologically. In contrast, extra-dimensional models without KK parity have much stronger bounds on the masses of the KK states, and hence the sizes of the extra dimensions [28], which makes these models beyond direct probe of near future experiments.

1.6 SUSY is not the only solution to the Hierarchy Problem

It was widely believed that supersymmetry represents the only solution to the hierarchy problem. This belief was based on a lack [30] of known alternatives, bolstered by a “folk theorem”. The “folk theorem” loosely states that supersymmetry is the only theory in which quadratic divergences cancel without tuning. The “folk proof” roughly goes as follows:

(i) Boson and fermion loops have opposite signs due to a minus sign in the Feynman rules for fermion loops.

(ii) Therefore cancellation of divergences only occurs between boson and fermion loops.

(iii) In order for the cancellation to be natural the boson and fermion loops need to be related by a symmetry: supersymmetry.

However, the folk theorem is wrong. Amusingly, a counter example to step (ii) in the above "proof" occurs in the MSSM itself. The MSSM extends the Higgs sector of the SM to a two Higgs doublet model. The tree level quartic couplings for the Higgses arise from integrating out the D-auxiliary fields in the $SU(2) \otimes U(1)$ gauge vector multiplets. Looking for example only at the contribution from hypercharge, the relevant terms in the Lagrangian are

$$\mathcal{L} = \frac{1}{2}D^2 + \frac{1}{2}\left(h_u^* D h_u - h^* D h_d\right), \tag{1.10}$$

giving the usual quartic D-term in the Higgs potential

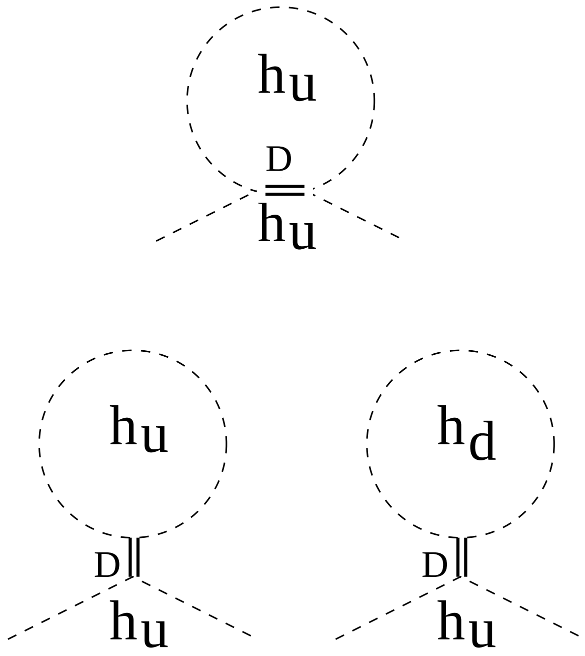
$$V = \frac{1}{2}\left(\frac{g}{2}\right)^2 \left(h_u^* h_u - h_d^* h_d\right)^2. \tag{1.11}$$

The cancellation of quadratic divergences is most easily understood by keeping the D-auxiliary fields in the theory. Then, the quartic coupling contained in (1.10) leads to three distinct diagrams shown in Fig.(1.2).

The first cancels against fermionic loops with gauginos as predicted by folk theorem. However, the other two diagrams cancel between each other. The cancellation occurs between diagrams with only bosons. It requires no fine tuning because the diagrams are proportional to the hypecharges of the Higgses in the loop which are opposite in sign and equal in magnitude (by gauge invariance and supersymmetry). Thus we see even in the MSSM some of the quadratic divergences [30] cancellation between bosons, with the required difference in sign simply arising from a difference in signs between coupling constants.

Elsewhere, although weak-scale (SUSY) is considered the most promising interpretation of the origin of the electroweak symmetry breaking scale, the non-discovery of superpartners and the Higgs at LEP and Tevatron imply that almost all superpartners must be heavier than the W, Z vectors [32], making typical SUSY models fine-tuned. The essential problem is that the mass parameters in the Higgs potential are determined by the soft SUSY breaking terms. It is then hard to understand how the Higgs vev and consequently the W, Z masses could naturally be sufficiently smaller than the soft breaking masses themselves [32]. This problem is exacerbated in the MSSM by the fact that one needs significant one loop corrections to the Higgs quartic self coupling in order to push the Higgs mass above the 115 GeV LEP bound [33]. This can be achieved with heavy and maximally-mixed stops, at the price of more fine-tuning [34].

Figure 1.2: Two loops diagrams [30]



Faced with this problem, alternative interpretations have been sought after. One prominent idea is that the Higgs could be a pseudo-Goldstone boson of a global symmetry broken at a scale f, what is the central point of *Little Higgs Models*.

As we will see, in these theories the Higgs mass is protected from one-loop quadratic divergences by approximate global symmetries under which the Higgs field shifts. New particles must be introduced to ensure that the global symmetries are not broken too severely, and these are the states that cut off the quadratically divergent top, gauge, and Higgs loops.

Chapter 2

Little Higgs Models

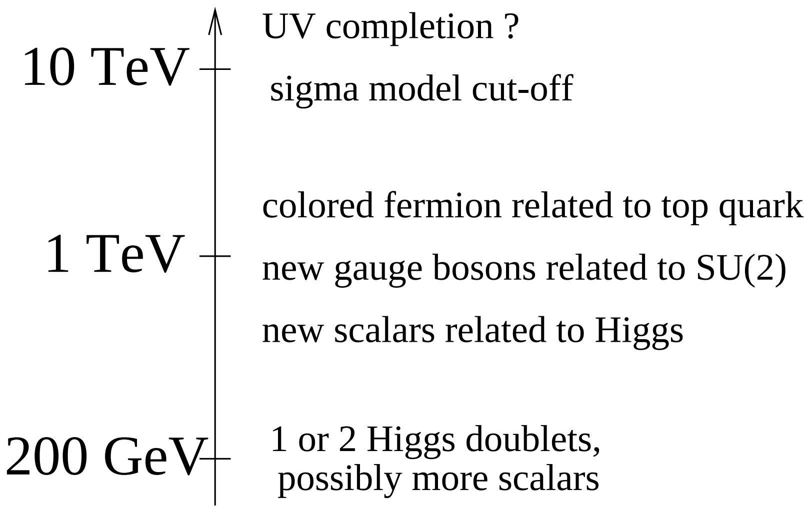
2.1 Introduction

The Little Higgs idea is a new purpose for solving the little hierarchy problem that arises in gauge models for the electroweak interactions, where the symmetry breaking is triggered by the Higgs mechanism. The main idea behind Little Higgs is that the Higgs [35] field is a pseudo-Goldstone boson of an approximate global symmetry. This approximate global symmetry is violated by gauge and Yukawa interactions such that the Higgs field acquires mass from one loop radiative corrections. But now the quadratically divergent contributions are canceled out due to the existence of new particles at TeV scale Fig.(2.1). The low energy degrees of freedom are described by nonlinear sigma models, with a cutoff at an energy scale one loop factor above the spontaneous symmetry breaking scale. Thus the little Higgs models require an ultraviolet (UV) completion at roughly the 10 TeV scale.

From the bottom-up point of view, the quadratic divergences in the Higgs mass are canceled by loops of new particles of the same statistics (in contrast to supersymmetry, in which the cancellations are due to particles of opposite statistics). From the top-down point of view, the Higgs [35] mass is protected by the global symmetry. Little Higgs models are constructed so that at least two operators are needed to explicitly break all of the global symmetry that protects the Higgs mass. This forbids quadratic divergences at one-loop; the Higgs mass is then smaller than Λ by not one but two loop factors, leading to the little hierarchy $\Lambda \gg f \gg v$.

In Little Higgs models, the Higgs boson is embedded among the pseudo-Goldstone boson (PGB) fields arising when a global symmetry G is broken to a subset H at a scale f, assumed to be around a TeV, and the PGBs are described by an G/H non-linear sigma model, as stated. To describe the gauge interactions of the Higgs, a subgroup of G must be weakly gauged. Furthermore, the gauged subgroup in Little Higgs models is not simple, but is a direct product of two (or more) factors, $G_1 \times G_2 \times \ldots$, each of which contains an $SU(2) \otimes U(1)$ subgroup. The gauged subgroup is embedded in such a way that each of the G_i factors commutes with a subgroup of G that acts non-linearly on the Higgs. In other

Figure 2.1: Generic Little Higgs Spectrum [35].



words, if only one of the G_i factors is gauged, the unbroken global symmetry of the theory is sufficient to ensure that the Higgs is an exact pseudo-Goldstone boson, and is therefore massless to all orders in perturbation theory and even non-perturbatively. It is only when the full $G_1 \times G_2 \times \ldots$ group is gauged that the Higgs ceases to be an exact PGB, and acquires non-derivative interactions. This structure is referred to as "collective breaking" of the global [35] symmetries by gauge interactions. It implies that any non-vanishing quantum contribution to the Higgs mass parameter must necessarily be proportional to a product of all the gauge coupling constants corresponding to the different G_i factors: setting any one of the coupling constants to zero must result in a vanishing contribution.

The extended gauge group $G_1 \times G_2 \times \ldots$ of the LH models is typically broken down to the SM $SU(2)_L \otimes U(1)_Y$ at a scale f by the same condensates that break $G \to H$. The models then contain additional gauge bosons at the TeV scale. In the mass eigenbasis, the vanishing of the one-loop quadratic divergence can be understood as a result of a cancellation between the SM bow tie diagrams and their counterparts involving the TeV-scale bosons. The relation between the couplings of these states to the Higgs is not accidental, but is enforced [35] by the collective symmetry breaking mechanism. Quadratic sensitivity of the Higgs mass to the cutoff scale then arises only at the two-loop level, so that a Higgs mass at the 100 GeV scale, two loop factors below the 10 TeV cutoff, is natural. Little Higgs models can thus stabilize the "little hierarchy" between the electroweak scale and the 10 TeV scale at which strongly-coupled new physics is allowed by electroweak precision constraints.

2.2 Successful Classes of Little Higgs Models

The little Higgs models can be categorized into two classes based on the structure of the extended electroweak gauge group [17]: models in which the SM $SU(2)_L$ gauge group arises from the diagonal breaking of two or more gauge groups, called product group models, and models in which the SM $SU(2)_L$ gauge group arises from the breaking of a single larger gauge group down to an SU(2) subgroup, called "simple group" models. These two classes of models also exhibit an important difference in the implementation of the little Higgs mechanism in the fermion sector. As representatives of the two classes, we have the Littlest Higgs model and the SU(4) simple group model, respectively.

Summarily, the so-called Littlest Higgs model consists in a $[SU(2) \otimes U(1)]^2$ gauge symmetry embedded in an $SU(5)$ global symmetry. The gauge symmetry is broken by a single vacuum condensate $f \sim$ TeV down to the SM $SU(2)_L \otimes U(1)_Y$ gauge symmetry. The SM Higgs doublet is contained in the resulting Goldstone bosons, whose interactions are parameterized by a nonlinear sigma model. The gauge and Yukawa couplings radiatively generate a Higgs potential and trigger EWSB.

In contrast, the simple group models share various features that distinguish them from the previous one. First, the simple group models all contain an $SU(N) \otimes U(1)$ gauge symmetry that is broken down to $SU(2)_L \otimes U(1)_Y$, yielding a set of TeV-scale gauge bosons. The two gauge couplings of the $SU(N) \otimes U(1)$ are fixed in terms of the two SM $SU(2)_L \otimes U(1)_Y$ gauge couplings, leaving no free parameters in the gauge sector once the symmetry-breaking scale is fixed. This gauge structure also forbids mixing between the SM $W^{\pm}$ bosons and the TeV-scale gauge bosons, again in contrast to the product group models. Second, in order to implement the collective symmetry breaking, simple group models require at [17] least two sigma-model multiplets. The SM Higgs doublet is embedded as a linear combination of the Goldstone bosons from these multiplets. This introduces at least one additional model parameter, which can be chosen as the ratio of the vevs of the sigma-model multiplets. Moreover, due to the enlarged $SU(N)$ gauge symmetry, all SM fermion representations have to be extended to transform as fundamental (or antifundamental) representations of $SU(N)$, giving rise to additional heavy fermions in all three generations. The existence of multiple sigma-model multiplets generically results in a more complicated structure for the fermion couplings to scalars. On the other hand, the existence of heavy fermion states in all three generations as required by the enlarged gauge symmetry provides extra experimental observables that in principle allow one to test this more complicated structure.

2.3 Pseudo-Goldstone Bosons

The fundamental characteristic of these kind of models is that the Higgs particle is treated as a pseudo-Goldstone boson, which arises whenever a continuous global symmetry is spontaneously broken [4]. If the symmetry is exact, the PGBs are exactly massless and have only derivative couplings. The better example of these kind of particles are the pions which, in fact, are the only light scalar particles that we know in nature.

The strength of their derivative interactions is set by an energy scale f, the decay constant. At energies larger than $\Lambda \sim 4\pi f$, the Goldstone bosons become strongly coupled and some new physics is needed to regulate this behavior. The regulating physics can be strongly coupled at scale Λ, like in QCD, or it can be weakly coupled if the global symmetry is spontaneously broken by an elementary scalar (for example in the SM, the Higgs field regulates WW scattering). Small explicit breaking of the global symmetry can generate a potential for the pseudo-Goldstone bosons. For instance, in QCD, the quark masses explicitly break the flavor symmetry and as a result, the pions are not exactly massless [4]. The gauging of electromagnetism also breaks the global symmetry and a quadratically divergent photon loop is responsible for the $\pi^+ - \pi^0$ mass difference:

$$m^2_{\pi^+} - m^2_{\pi^0} \sim \frac{\alpha_{em}}{4\pi} \Lambda^2_{QCD}, \tag{2.1}$$

which is parametrically of order gf_π (f_π is the pion decay constant).

The precise modern formulation of PGB is due to Weinberg, suggested in the times of the early 1970s. The issue that Weinberg addressed was the puzzle of approximate symmetries. He showed how they could arise in renormalizable quantum field theories (QFTs) as accidental consequences of the constraints of renormalizability. It is known how symmetries could be imposed on a QFT, and how, once imposed, they are (in the absence of explicit anomalies) impervious to quantum corrections [4]. In fact, Weinberg noticed that in some QFTs symmetries of part of the Lagrangian arose automatically from the constraints of renormalizability without being imposed. If these symmetries were not shared by the rest of the Lagrangian, the symmetry breaking quantum corrections to the symmetric part would be non-zero, but they would be protected by renormalizability and they would get a finite and calculable contribution from the Coleman-Weinberg terms.

Furthermore, he noted that as result of an accidental symmetry spontaneously broken, the mass and non-derivative interactions of the PGBs were symmetry breaking effects, finite and calculable quantum corrections. We now know that the approximate chiral $SU(3) \otimes SU(3)$ symmetry of low-energy QCD is an automatic consequence of the fact that the light quark masses are small compared to the QCD scale. No symmetry has been imposed by hand. All that is necessary for a PGB is an automatic degeneracy of the surface of minimum potential.

To earn a better comprehension, let us consider a theory with a single complex scalar field ϕ with potential $V = V(\phi^*\phi)$. The kinetic energy term $\partial_\mu\phi^*\partial^\mu\phi$ and the potential are invariant under the $U(1)$ symmetry transformation

$$\phi \longrightarrow e^{i\alpha}\phi. \tag{2.2}$$

If the minimum of the potential is not at the origin but at some distance f away as in the so called Mexican hat potential, then the $U(1)$ symmetry is [4] spontaneously broken in the vacuum. When we expand the field for small fluctuations around its vev, it takes the form

$$\phi\,(x) = \frac{1}{\sqrt{2}}\,(f + r(x))\,e^{i\frac{\theta(x)}{f}}, \tag{2.3}$$

where again f is the vev, $r(x)$ is the massive radial mode and $\theta(x)$ is the PGB. The factor of $\frac{1}{\sqrt{2}}$ ensures canonical kinetic terms [30] for the real fields r and θ.

The radial field r is invariant under the $U(1)$ symmetry transformation of Eq.(2.2), whereas the PGB field θ shifts,

$$\theta \longrightarrow \theta + \alpha \tag{2.4}$$

under $U(1)$ transformations. We say that the $U(1)$ symmetry is non-linearly realized. Suppose that we now integrate out the massive field r. We can be sure that the resulting effective Lagrangian for the PGB $\theta(x)$ will not include a mass term for θ, or any potential terms for that matter, because the shift symmetry forbids all non-derivative couplings of θ.

2.4 Range of Validity for a Model Using the Little Higgs Mechanism

Now, let us consider a complex scalar field ϕ invariant under the global non-abelian transformation $SU(3)$ and parametrized as $\phi = e^{i\frac{\pi}{f}}\phi_0$, such that $\phi^\dagger\phi = f^2$. The most general effective Lagrangian involving only the massless PGB fields [30], which respects the full $SU(3)$ symmetry, can contain just derivative and constant terms. It can be written at quadratic order as

$$\mathcal{L} = const. + f^2|\partial_\mu\phi|^2 + \mathcal{O}(\partial^4). \tag{2.5}$$

If the symmetry breaking pattern of interest is $SU(3) \longrightarrow SU(2)$ then the number of PGBs is the total number of generators of $SU(3)$ minus the number of generators of $SU(2)$, i.e. $2(3) - 1 = 5$. The unbroken generators are identified with five fields of which π_0 is real and, the other four $\pi_1 \ldots \pi_4$ are complex in general. Neglecting for simplicity the π_0 field, the rest of the PGBs can be treated as two doublet complex fields and conveniently parametrized by writing

$$\phi = exp\left[\frac{i}{f}\begin{pmatrix} 0 & h \\ h^\dagger & 0 \end{pmatrix}\right]\begin{pmatrix} 0 \\ f \end{pmatrix} = e^{i\frac{\pi}{f}}\phi_0. \tag{2.6}$$

To see what interactions we get for h (where clearly this h field can be formally treated as a Higgs doublet), we expand and therefore obtain for the kinetic term

$$f^2|\partial_\mu\phi|^2 = |\partial_\mu h|^2 + \frac{|\partial_\mu h|^2 h^\dagger h}{f^2} + \ldots \tag{2.7}$$

which contains the Higgs kinetic term as well as interactions suppressed by the symmetry-breaking scale f.

Because the Lagrangian contains non-renormalizable interactions, it can only be an effective low-energy description of physics. To determine the cut-off Λ at which the theory becomes strongly coupled, we can compute a loop and ask at what scale it becomes as important [30] as a corresponding tree-level diagram. The simplest example is the quadratically divergent one-loop contribution to the kinetic term that stems from contracting $h^\dagger h$ into a loop in the second term in Eq. (2.7). Cutting the divergence off at Λ we find a renormalization of the kinetic term proportional to

$$\frac{1}{f^2}\frac{\Lambda^2}{16\pi^2}, \tag{2.8}$$

and therefore $\Lambda \sim 4\pi f$.

Hence, the theory produces a Higgs doublet transforming under an exactly preserved global symmetry $SU(2)$. This Higgs is a PGB and therefore exactly massless. It has non-renormalizable interactions suppressed by the scale f, which become strongly coupled at $\Lambda = 4\pi f$. Because of the shift symmetry, no diagrams, divergent or not, can give rise to a mass for h. However, a PGB can only have derivative interactions, which means no gauge interactions, no Yukawa couplings and no quartic potential because any of these interactions explicitly break the shift symmetry $h \to h + const$.

2.5 Collective Symmetry Breaking

In order for us to introduce gauge interactions and Higgs mass, it is necessary to use two copies of PGBs, ϕ_1 and ϕ_2, and add $SU(3)$ invariant covariant derivatives [4] for both (same arguments can be used for a model based in the gauge symmetry $SU(4)$). We expect no quadratic divergence for either of the PGBs, and only one linear combination will be eaten. To see how this works in detail we parametrize

$$\phi_1 = e^{\frac{i\pi_1}{f}} \begin{pmatrix} \\ f \end{pmatrix} \qquad \phi_2 = e^{\frac{i\pi_2}{f}} \begin{pmatrix} \\ f \end{pmatrix}, \tag{2.9}$$

where we have picked up aligned vevs for ϕ_1 and ϕ_2, and, for simplicity, identical symmetry breaking scales $f_1 = f_2 = f$. The Lagrangian is

$$\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2. \tag{2.10}$$

The two interaction terms produce two sets of quadratically divergent one-loop diagrams which give Fig. [2.2]

$$\frac{g^2}{16\pi^2}\Lambda^2\left(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2\right) = \frac{g^2}{16\pi^2}\Lambda^2\left(f^2 + f^2\right) \tag{2.11}$$

i.e. no potential for any of the PGBs. Moreover, only one linear combination of π_1 and π_2 is eaten as there is only one set of hungry massive $SU(3)$ gauge bosons. A simple way to understand this result is to notice that each set of diagrams involves only one of the ϕ fields.

Figure 2.2: a) Quadratically divergent gauge loop contributions which do not contribute to the Higgs potential, b) log-divergent contribution to the Higgs mass [4].

In turn, once we consider diagrams involving both ϕ_1 and ϕ_2 as the diagram in Figure[2.2b], we obtain

$$\frac{g^4}{16\pi^2} log\left(\frac{\Lambda^2}{\mu^2}\right) |\phi_1^\dagger \phi_2|^2, \tag{2.12}$$

which does depend on h but is not quadratically divergent. To calculate the Higgs dependence we choose a convenient parametrization

$$\begin{aligned} \phi_1 &= exp\left[\frac{i}{f}\begin{pmatrix} & k \\ k^\dagger & \end{pmatrix}\right] exp\left[\frac{i}{f}\begin{pmatrix} & h \\ h^\dagger & \end{pmatrix}\right]\begin{pmatrix} 0 \\ f \end{pmatrix} \\ \phi_2 &= exp\left[\frac{i}{f}\begin{pmatrix} & k \\ k^\dagger & \end{pmatrix}\right] exp\left[-\frac{i}{f}\begin{pmatrix} & h \\ h^\dagger & \end{pmatrix}\right]\begin{pmatrix} 0 \\ f \end{pmatrix}. \end{aligned} \tag{2.13}$$

The field k can be removed by an $SU(3)$ gauge transformation [4], and corresponds to the eaten PGBs, while h cannot simultaneously be removed from ϕ_1 and ϕ_2, and is physical. In the following we will work in the unitary gauge for $SU(3)$, where k is rotated away. Then we have

$$\begin{aligned} \phi_1^\dagger \phi_2 &= \begin{pmatrix} 0 & f \end{pmatrix} exp\left[\frac{-2i}{f}\begin{pmatrix} & h \\ h^\dagger & \end{pmatrix}\right]\begin{pmatrix} 0 \\ f \end{pmatrix} \\ \phi_1^\dagger \phi_2 &= f^2 - 2h^\dagger h + \ldots. \end{aligned} \tag{2.14}$$

Which means that, the theory of two complex triplets which both break $SU(3) \to SU(2)$ automatically contains a Higgs doublet PGB which does not receive quadratically divergent contributions to its mass [4]. There are log-divergent and finite contributions, and from these the natural size for the Higgs mass is $\frac{f}{4\pi} \sim M_{weak}$.

2.6 Simple Little Higgs Models

Among the realistic realizations of the mechanism behind the Little Higgs the most economical one is based on the $SU(3)_L \otimes U(1)_X$ gauge group. This model is known as the Simplest Little Higgs model (SLHM). Besides recovering the standard particles spectrum, the SLHM predicts new vector gauge bosons, three of them are neutral and one is charged. In the fermion sector [4], the model has three new quarks, one of them being a heavy top-like quark T . Finally, the scalar sector is composed of two scalar triplets in a non-linear sigma model realization. In this way, after spontaneous symmetry breaking, only the standard Higgs and a new pseudo-scalar survive.

Thus, in fact, the physical scalars are pseudo Goldstone bosons and their masses generated at one loop level are proportional to the gauge and Yukawa coupling constants times the logarithm of the cutoff $\Lambda \simeq 4\pi f$, with f the energy scale related to the global symmetry breakdown present in Little Higgs Models. The new particles in the SLHM have masses related to this scale, so that a lower bound on f reflects as a lower bound on these masses. Once the experimental support to a specific Little Higgs Model does not concern only the Higgs production, but also the identification of a new particle content, such a lower bound for f turns out to be an important issue.

Although, this $SU(3) \otimes U(1)$ model contains a great simplicity and large possibilities to be tested at CERN, one of his theoretical weaknesses is the difficulty of looking at possible terms which one might add to the lagrangian to generate a quartic self coupling for the Higgs in order to stabilize its vev. An alternative way to solve this, is the expansion of the gauge group to $SU(4) \otimes U(1)$, which allow us to use a new mechanism to generate a quartic coupling.

To reproduce the successes of the $SU(3)\otimes U(1)$ LHM and produce uneaten Higgs doublets [4], it is necessary to add two more scalars in the non-linear sigma model $[SU(4)/SU(3)]^2$ with the diagonal $SU(4)$ gauged and, to break the gauged $SU(4) \to SU(2)$ twice. Note that one distinction from the previous model is that the $SU(4)$ breaking is not aligned

$$\Phi_i = \mathrm{e}^{+i\frac{\phi_i}{f}} \begin{pmatrix} 0 \\ 0 \\ f \\ 0 \end{pmatrix}, \quad \Psi_i = \mathrm{e}^{-i\frac{\psi_i}{f}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix}, \tag{2.15}$$

where $i = 1, 2$. Since the product $\Phi_i^\dagger \Psi_j$ contains no constant term it could potentially carry a term quartic.

The complete counting goes as follows: the $[SU(4)/SU(3)]^2$ represents $(15-8) \times 4 = 28$ real components, 12 of which are eaten when the $SU(4)$ gauge group is broken to $SU(2)$. The remaining 16 consist of two complex doublets h_u and h_d, three complex $SU(2)$ singlets σ_1, σ_2 and σ_3, and two real scalars η_u and η_d.

Chapter 3

Anomaly-free Little Higgs Models with 3-4-1 Gauge Symmetry

The hierarchy problem, or the very unnatural fine-tuning required to fix the electroweak scale due to the quadratic divergent quantum corrections to the Higgs boson mass, is a major theoretical shortcoming of the standard model (SM). The fine-tuning problem can be alleviated, only if there is new physics at the TeV scale that guarantees the cancellation of the quadratic divergence to an acceptable level, or totally changes our picture of SM physics. A guaranteed cancellation has to come from some mechanism protected by a symmetry. Candidates of the kind include supersymmetry and the recently proposed little Higgs mechanism. With the little Higgs idea, the SM Higgs boson is identified as the pseudo-Nambu Goldstone boson(s) of some global symmetries. Our background little Higgs model is a model with $SU(3)_L \otimes U(1)_X$ gauge symmetry given in Ref. [8]. That model has a problem with the quartic Higgs coupling, which can be fixed in a $SU(4)_L \otimes U(1)_X$ extension. Our aim is to illustrate the basic features of a $SU(4)$ little Higgs model with a complete and consistent anomaly-free fermionic content.

3.1 Fermions and Scalars

As stated, we shall consider Little Higgs models with $SU(4)_L \otimes U(1)_X$ gauge symmetry characterized by the values $b = 1,\ c = -2$ of the parameters in the electric charge operator in Eq. (3). There exist two anomaly-free models of this type [13, 14]. We select as model case model E in Ref. [14]. It has the anomaly-free fermion content displayed in Table 3.1 where $i = 1, 2$ and $\alpha = 1, 2, 3$ are family indexes and the numbers in parentheses refer to the $[SU(3)_c, SU(4)_L, U(1)_X]$ quantum numbers, respectively. U_i and U'_i are exotic up-type quarks of electric charge 2/3, while D_3 and D'_3 are exotic down-type quarks of electric charge $-1/3$. E^-_α and E'^-_α are exotic electrons. Notice that universality for the know leptons in the three families is present at the tree level in the weak basis.

In the $[SU(4)/SU(3)]^4$ simple group model the gauged $SU(4)_L \otimes U(1)_X$ symmetry is

Table 3.1: Anomaly-free fermion content.

$Q_{iL} = \begin{pmatrix} u_i \\ d_i \\ iD_i \\ iU_i \end{pmatrix}_L$	iu^c_{iL}	id^c_{iL}	iD^c_{iL}	iU^c_{iL}
$[3, 4, \frac{1}{6}]$	$[3^*, 1, -\frac{2}{3}]$	$[3^*, 1, \frac{1}{3}]$	$[3^*, 1, \frac{1}{3}]$	$[3^*, 1, -\frac{2}{3}]$
$Q_{3L} = \begin{pmatrix} d_3 \\ u_3 \\ iU_3 \\ iD_3 \end{pmatrix}_L$	id^c_{3L}	iu^c_{3L}	iU^c_{3L}	iD'^c_{3L}
$[3, 4^*, \frac{1}{6}]$	$[3^*, 1, \frac{1}{3}]$	$[3^*, 1, -\frac{2}{3}]$	$[3^*, 1, -\frac{2}{3}]$	$[3^*, 1, \frac{1}{3}]$
$L_{\alpha L} = \begin{pmatrix} e^-_\alpha \\ \nu^0_{e\alpha} \\ iN^0_\alpha \\ iE^-_\alpha \end{pmatrix}_L$	$ie^+_{\alpha L}$	$iE^+_{\alpha L}$		
$[1, 4^*, -\frac{1}{2}]$	$[1, 1, 1]$	$[1, 1, 1]$		

broken down to the SM electroweak gauge group by four sets of nonlinear sigma model fields Φ_i and Ψ_i, $i = 1, 2$, which are quadruplets under $SU(4)$ with misaligned VEV f_i, $i = 1, 2, 3, 4$, all of them of order 1 TeV. With $b = 1,\ c = -2$ in Eq. (3), the sigma model fields, which contain two Higgs doublets h_1 and h_2, transform under $SU(4)_L \otimes U(1)_X$ as: $\Phi_1 \sim [1, 4, 1/2]$, $\Phi_2 \sim [1, 4^*, -1/2]$, $\Psi_1 \sim [1, 4, -1/2]$, and $\Psi_2 \sim [1, 4^*, 1/2]$, and they can be parameterized as

$$\Phi_1 = \mathrm{e}^{+i\mathcal{H}_d \frac{f_2}{f_1}} \begin{pmatrix} 0 \\ 0 \\ f_1 \\ 0 \end{pmatrix}, \quad \Phi_2 = \mathrm{e}^{-i\mathcal{H}_d \frac{f_1}{f_2}} \begin{pmatrix} 0 \\ 0 \\ f_2 \\ 0 \end{pmatrix},$$
$$\Psi_1 = \mathrm{e}^{+i\mathcal{H}_u \frac{f_4}{f_3}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_3 \end{pmatrix}, \quad \Psi_2 = \mathrm{e}^{-i\mathcal{H}_u \frac{f_3}{f_4}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ f_4 \end{pmatrix}, \tag{3.1}$$

where

$$\mathcal{H}_d = \frac{1}{f_{12}} \begin{pmatrix} \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & h_d & \begin{matrix} 0 \\ 0 \end{matrix} \\ h_d^\dagger & 0 & 0 \\ \begin{matrix} 0 & 0 \end{matrix} & 0 & 0 \end{pmatrix}, \tag{3.2}$$

$$\mathcal{H}_u = \frac{1}{f_{34}} \begin{pmatrix} 0 & 0 & 0 & \\ 0 & 0 & 0 & h_u \\ 0 & 0 & 0 & 0 \\ & h_u^\dagger & 0 & 0 \end{pmatrix}, \tag{3.3}$$

and $f_{ij}^2 = f_i^2 + f_j^2$. The Higgs fields, which accomplish the electroweak symmetry breaking, acquire VEV of the form

$$\langle h_d \rangle = \begin{pmatrix} 0 \\ \frac{v_d}{\sqrt{2}} \end{pmatrix} \qquad \text{and} \qquad \langle h_u \rangle = \begin{pmatrix} \frac{v_u}{\sqrt{2}} \\ 0 \end{pmatrix}. \tag{3.4}$$

3.2 Gauge Bosons

In this model the gauge bosons in $SU(4)_L$ are obtained from [13]

$$\frac{1}{2}\lambda_{Lk} A_\mu^k = \frac{1}{\sqrt{2}} \begin{pmatrix} D_{1\mu}^0 & W_\mu^+ & K_\mu^+ & X_\mu^0 \\ W_\mu^- & D_{2\mu}^0 & K_\mu^0 & V_\mu^- \\ K_\mu^- & K_\mu'^0 & D_{3\mu}^0 & Y_\mu^- \\ X_\mu'^0 & V_\mu^+ & Y_\mu^+ & D_{4\mu}^0 \end{pmatrix} \tag{3.5}$$

where $D_{1\mu}^0 = A_{3\mu}/\sqrt{2} + A_{8\mu}/\sqrt{6} + A_{15\mu}/\sqrt{12}$, $D_{2\mu}^0 = -A_{3\mu}/\sqrt{2} + A_{8\mu}/\sqrt{6} + A_{15\mu}/\sqrt{12}$, $D_{3\mu}^0 = -2A_{8\mu}/\sqrt{6} + A_{15\mu}/\sqrt{12}$, $D_{4\mu}^0 = -3A_{15\mu}/\sqrt{12}$.

The gauge couplings g and g_X, associated with the groups $SU(4)_L$ and $U(1)_X$, respectively, are defined through the covariant derivative for quadruplets as $iD^\mu = i\partial^\mu - g\lambda_{Lk}A_k^\mu/2 - g_X X B^\mu$. After the $SU(4)_L \otimes U(1)_X$ symmetry is broken to $U(1)_Q$, we obtain the gauge boson masses from the kinetic terms for the Φ_i and Ψ_i fields, $i = 1, 2$, as

$$tr\left\{\frac{1}{2}g^2 \left(\lambda_{Lk}A_\mu^k - \left(\frac{\sqrt{2}g_X}{g}\right) X B_\mu^x\right)^2 \left(\Phi_i\Phi_i^\dagger + \Psi_i\Psi_i^\dagger\right)\right\}. \tag{3.6}$$

To compute the gauge boson masses at relevant order, it is convenient to start by computing the matrix

$$\left\langle \Phi_i\Phi_i^\dagger + \Psi_i\Psi_i^\dagger \right\rangle = \begin{pmatrix} \frac{1}{2}v_1^2 & & & \\ & \frac{1}{2}v_2^2 & & \\ & & 2f_{12}^2 - \frac{1}{2}v_1^2 & \\ & & & 2f_{34}^2 - \frac{1}{2}v_2^2 \end{pmatrix} \tag{3.7}$$

where

$$\begin{aligned} v_1^2 &= v_d^2 - \frac{v_d^4}{3f_{12}^2}\left[\frac{f_2^2}{f_1^2} + \frac{f_1^2}{f_2^2} - 1\right], \\ v_2^2 &= v_u^2 - \frac{v_u^4}{3f_{34}^2}\left[\frac{f_4^2}{f_3^2} + \frac{f_3^2}{f_4^2} - 1\right]. \end{aligned} \tag{3.8}$$

The charged gauge bosons do not mix with each other and get the following squared masses

$$\begin{aligned} M^2_{W^\pm} &= \frac{1}{4}g^2v^2, & M^2_{Y^\pm} &= \tfrac{1}{4}g^2\left(4f^2-v^2\right), \\ M^2_{X^0} &= \frac{1}{4}g^2\left(4f^2_{34}+v_1^2-v_2^2\right), & M^2_{V^\pm} &= g^2f^2_{34}, \\ M^2_{K^0} &= \frac{1}{4}g^2\left(4f^2_{12}+v_2^2-v_1^2\right), & M^2_{K^\pm} &= g^2f^2_{12}, \end{aligned} \tag{3.9}$$

where

$$f^2 = f^2_{12}+f^2_{34}, \qquad \text{and} \quad v^2 = v_1^2+v_2^2, \tag{3.10}$$

and the hypercharge gauge coupling is

$$\frac{1}{g'^2} = \frac{1}{g_X^2}+\frac{1}{g^2}, \tag{3.11}$$

where g and g' are the gauge coupling constants of the $SU(2)_L$ and $U(1)_Y$ groups of the SM, respectively.

Notice that $W^\pm$ does not mix with the other charged bosons. Instead, for the neutral gauge bosons we get a 4×4 mass matrix with a zero eigenvalue corresponding to the photon. Once the photon field has been identified, we remain with a 3×3 mass matrix for three neutral gauge bosons Z^μ, Z'^μ, and Z''^μ. The mixing between the three neutral gauge bosons can be further simplified [20] by using the approximation $f = f_{12} = f_{34}$. In this case the field Z''^μ decouples from the other two and acquires a squared mass

$$M^2_{Z''} = \frac{1}{2}g^2f^2. \tag{3.12}$$

The remaining 2×2 mass matrix, in the basis (Z^μ, Z'^μ), is

$$g^2\begin{pmatrix} \frac{v^2}{4C_W^2} & \frac{v^2T_W^2}{4C_W\sqrt{1-T_W^2}} \\ \frac{v^2T_W^2}{4C_W\sqrt{1-T_W^2}} & \frac{f^2}{(1-T_W^2)}-\frac{v^2}{4C_W^2} \end{pmatrix} \tag{3.13}$$

where C_W is the cosine of the electroweak mixing angle and is given by $C_W = \sqrt{(1+\epsilon^2)/(1+2\epsilon^2)}$, whith $\epsilon = g_x/g$.

By diagonalizing this mass matrix we get the following mass terms:

$$\begin{aligned} M_Z^2 &= \frac{g^2v^2}{4C_W^2}\left[1-\frac{v^2T_W^4}{4f^2}\right], \\ M^2_{Z'} &= g^2f^2\left(1+\epsilon^2\right)-M_Z^2, \end{aligned} \tag{3.14}$$

In terms of the electroweak basis, the massless photon A_μ and the massive gauge bosons Z_μ, Z'_μ and Z''_μ are given by

$$\begin{aligned} A^\mu &= S_W A_3^\mu \\ &\quad + C_W \left[\frac{T_W}{\sqrt{3}} \left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^\mu \right] , \\ Z^\mu &= C_W A_3^\mu \\ &\quad - S_W \left[\frac{T_W}{\sqrt{3}} \left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^\mu \right] , \\ Z'^\mu &= \frac{1}{\sqrt{3}} (1 - T_W^2)^{1/2} \left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}} \right) - T_W B^\mu, \\ Z''^\mu &= 2A_8^\mu/\sqrt{6} + A_{15}^\mu/\sqrt{3}, \end{aligned} \tag{3.15}$$

from which we identify the Y hypercharge associated with the SM $U(1)_Y$ gauge boson as

$$Y^\mu = \frac{T_W}{\sqrt{3}} \left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^\mu. \tag{3.16}$$

In the case for which the neutral current $Z''^\mu \equiv Z_3^\mu$ decouples from the other two, the remaining mixing between Z_μ and Z'_μ is parametrized by the mixing angle θ as

$$\begin{aligned} Z_1^\mu &= Z^\mu \cos\theta + Z'^\mu \sin\theta \ , \\ Z_2^\mu &= -Z^\mu \sin\theta + Z'^\mu \cos\theta, \end{aligned} \tag{3.17}$$

where Z_1^μ and Z_2^μ are the mass eigenstates and

$$\tan(2\theta) = \frac{v^2 S_W C_W \sqrt{1 - T_W^2}}{[2f^2 C_W^2 - v^2 (1 - T_W^2)]}. \tag{3.18}$$

Concerning the fermion masses, electroweak symmetry breaking also induces mixing between the heavy left-handed fermions U_{iL}, D_{iL}, E_{iL} and the SM fermions. In order for us to reduce the sources of FCNC in the model and to soften the fermion mixing, we can introduce a hierarchy in the Yukawa couplings [17] such that, for instance, in the third generation of the up-quark sector, only the terms involving U_3^c and u_3 lead the mixing. It means that, $\lambda_u^{U_3 u_3}$ is larger than $\lambda_u^{U_3 u_1}$ and $\lambda_u^{U_3 u_2}$. Hence, the last two couplings can be chosen to be small and then suppressed.

The lepton sector is closely similar in both the universal and anomaly-free embedding. The appropriate implementation of masses and mixings for the charged leptons in this model require the following Yukawa terms

$$\mathcal{L}_Y^L = \sum_{\alpha=1}^{3} \sum_{\beta=1}^{3} L_{\beta L}^T C \left\{ \Psi_1 \left(i\lambda_{\alpha\beta}^e e_{\beta L}^+ + i\lambda_{\alpha\beta}^E E_{\beta L}^+ \right) + \Psi_2^* \left(i\lambda_{\alpha\beta}^e e_{\beta L}^+ + i\lambda_{\alpha\beta}^E E_{\beta L}^+ \right) \right\} + H.c, \tag{3.19}$$

where the λs are Yukawa couplings and C is the charge operation conjugation. From this equation the 6×6 mass matrix for the charged leptons decouples into three 2×2 matrices, each describing the mixing between light and heavy partners. By diagonalizing the latest mass matrices we find that the electron-type leptons fields in $L_{\beta L}$ mix with their $SU(2)$ singlet partners by an amount

$$\delta\theta_e = \frac{1}{\sqrt{2}}\frac{v}{f}. \tag{3.20}$$

The model also contains three heavy neutral states N^0_α that can get masses at order $f \sim 1$ TeV by introducing the Lagrangian

$$\mathcal{L}_Y^{N^0} = i\sum_{\beta=1}^{3} L^T_{\beta L} C \Phi_1 \lambda^N_\beta N^0_{\beta L} + i\sum_{\beta=1}^{3} L^T_{\beta L} C \Phi_2^* \lambda^N_\beta N^0_{\beta L}, \tag{3.21}$$

because these scalar interactions terms and the $SU(4)$ anomaly-free structure, N^0_α mixes only with the neutrino states $\nu^0_{e\alpha}$ with a mixing angle that coincide with the already provided for charged Leptons. Such mixing angle will modify the well-measured couplings of neutrinos to W and Z at order $\frac{v^2}{f^2}$.

For the quark sector, the relevant Lagrangian terms for the third generation and for the first two generations are

$$\begin{aligned}
\mathcal{L}_Y^Q &= \sum_{\alpha=1}^{3}\sum_{i=1}^{2} Q^T_{iL} C \left\{ \Psi_1^* \left(i\lambda^u_{\alpha i} u^c_{\alpha L} + i\lambda^U_{\alpha i} U^c_{\alpha L}\right) + \Psi_2 \left(i\lambda^u_{\alpha i} u^c_{\alpha L} + i\lambda^U_{\alpha i} U^c_{iL}\right)\right.\\
&+ \left. \Phi_1^* \left(i\lambda^d_{\alpha i} d^c_{\alpha L} + i\lambda^D_{\alpha i} D^c_{\alpha L}\right) + \Phi_2 \left(i\lambda^d_{\alpha i} d^c_{\alpha L} + i\lambda^D_{\alpha i} D^c_{\alpha L}\right)\right\}\\
&+ Q^T_{3L} C \sum_{\alpha=1}^{3} \left\{ \Phi_1 \left(i\lambda^u_{\alpha 3} u^c_{\alpha L} + i\lambda^U_{\alpha 3} U^c_{\alpha L}\right) + \Phi_2^* \left(i\lambda^u_{\alpha 3} u^c_{\alpha L} + i\lambda^U_{\alpha 3} U^c_{\alpha L}\right)\right.\\
&+ \left. \Psi_1 \left(i\lambda^d_{\alpha 3} d^c_{\alpha L} + i\lambda^D_{\alpha 3} D^c_{\alpha L}\right) + \Psi_2^* \left(i\lambda^d_{\alpha 3} d^c_{\alpha L} + i\lambda^D_{\alpha 3} D^c_{\alpha L}\right)\right\},
\end{aligned} \tag{3.22}$$

where again the λs are Yukawa couplings. From this Lagrangian we get, for the up- and down-type quarks in the basis $(u_1, U_1, u_2, U_2, u_3, U_3)$ and $(d_1, D_1, d_2, D_2, d_3, D_3)$, respectively, 6×6 mass matrices which decouple into three 2×2 matrices each one by side. The mixing is also determined by diagonalizing the (u_i, U_i) and (d_i, D_i) mass matrices and, again, we find that $\delta\theta_{u,d} = \frac{1}{\sqrt{2}}\frac{v}{f}$ is the typical mixing angle for both sectors.

These mass matrices show that all the charged fermions in the model acquire masses at the three level, and that all the ordinary charged fermions get masses at the low scale v, while all the exotic charged fermions acquire masses at the high scale f.

Small neutrino masses can be generated by introducing higher dimensional lepton number violation operators.

3.3 Currents

The Lagrangian for neutral currents can be written as $-\mathcal{L}^{NC} = eA^\mu J_\mu(EM) + (g/C_W)Z^\mu J_\mu(Z) + (g_X/\sqrt{2})Z'^\mu\ J_\mu(Z') + (g/2)Z''^\mu J_\mu(Z'')$, with

$$\begin{aligned} J_\mu(EM) &= \frac{2}{3}[\sum_{i=1}^{2}(\bar{u}_i\gamma_\mu u_i + \bar{U}_i\gamma_\mu U_i) + \bar{u}_3\gamma_\mu u_3 + \bar{U}_3\gamma_\mu U_3] - \frac{1}{3}[\sum_{i=1}^{2}(\bar{d}_i\gamma_\mu d_i + \bar{D}_i\gamma_\mu D_i) \\ &\quad + \bar{d}_3\gamma_\mu d_3 + \bar{D}_3\gamma_\mu D_3] - \sum_{\alpha=1}^{3}\bar{e}_\alpha^-\gamma_\mu e_\alpha^- - \sum_{\alpha=1}^{3}\bar{E}_\alpha^-\gamma_\mu E_\alpha^- \\ &= \sum_f q_f\bar{f}\gamma_\mu f, \end{aligned} \tag{3.23}$$

$$J_\mu(Z) = J_{\mu,L}(Z) - S_W^2 J_\mu(EM), \tag{3.24}$$

$$J_\mu(Z') = J_{\mu,L}(Z') - T_W J_\mu(EM), \tag{3.25}$$

$$\begin{aligned} J_\mu(Z'') &= \sum_{i=1}^{2}(\bar{u}_{iL}\gamma_\mu u_{iL} + \bar{d}_{iL}\gamma_\mu d_{iL} - \bar{D}_{iL}\gamma_\mu D_{iL} - \bar{U}_{iL}\gamma_\mu U_{iL}) - \bar{d}_{3L}\gamma_\mu d_{3L} \\ &- \bar{u}_{3L}\gamma_\mu u_{3L} + \bar{U}_{3L}\gamma_\mu U_{3L} + \bar{D}_{3L}\gamma_\mu D_{3L} + \sum_{\alpha=1}^{3}(-\bar{e}_{\alpha L}^-\gamma_\mu e_{\alpha L}^- - \bar{\nu}_{e\alpha L}\gamma_\mu \nu_{e\alpha L} \\ &+ \bar{N}_{\alpha L}^0\gamma_\mu N_{\alpha L}^0 + \bar{E}_{\alpha L}^-\gamma_\mu E_{\alpha L}^-), \end{aligned} \tag{3.26}$$

where $e = gS_W$ is the electric charge, q_f is the electric charge of the fermion f in units of e and $J_\mu(EM)$ is the electromagnetic current. It is important to remark that $J_\mu(Z'')$ is a pure left-handed current and that, notwithstanding the extra neutral gauge boson Z''_μ does not mix neither with Z_μ nor with Z'_μ (for the particular case $f = f_{12} = f_{34}$), it still couples non-diagonally to ordinary fermions. Thus, at low energy, we have tree-level FCNC transmitted by Z''_μ.

The left-handed currents in (3.24) and (3.25) are

$$\begin{aligned} J_{\mu,L}(Z) &= \frac{1}{2}[\sum_{i=1}^{2}(\bar{u}_{iL}\gamma_\mu u_{iL} - \bar{d}_{iL}\gamma_\mu d_{iL}) - (\bar{d}_{3L}\gamma_\mu d_{3L} - \bar{u}_{3L}\gamma_\mu u_{3L}) \\ &\quad - \sum_{\alpha=1}^{3}(\bar{e}_{\alpha L}^-\gamma_\mu e_{\alpha L}^- - \bar{\nu}_{e\alpha L}\gamma_\mu \nu_{e\alpha L})] \\ &= \sum_f T_{4f}\bar{f}_L\gamma_\mu f_L, \end{aligned} \tag{3.27}$$

$$J_{\mu,L}(Z') = (2T_W)^{-1}\{\sum_{i=1}^{2}[T_W^2(\bar{u}_{iL}\gamma_\mu u_{iL} - \bar{d}_{iL}\gamma_\mu d_{iL}) - \bar{D}_{iL}\gamma_\mu D_{iL} + \bar{U}_{iL}\gamma_\mu U_{iL}]$$

Table 3.2: The $Z_1^\mu \longrightarrow \bar{f}f$ couplings.

f	$g^{(B)}(f)_{1V}$	$g^{(B)}(f)_{1A}$
$u_{1,2,3}$	$\cos\theta(\frac{1}{2}-\frac{4S_W^2}{3})+\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{C_W^2}{4}\frac{v^2}{f^2}-\frac{5S_W^2}{6})$	$\frac{1}{2}\cos\theta+\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{2}+\frac{C_W^2}{4}\frac{v^2}{f^2})$
$d_{1,2,3}$	$\cos\theta(-\frac{1}{2}+\frac{2S_W^2}{3})+\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{6}-\frac{C_W^2}{4}\frac{v^2}{f^2})$	$-\frac{1}{2}\cos\theta-\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{2}+\frac{C_W^2}{4}\frac{v^2}{f^2})$
$D_{1,2,3}$	$\cos\theta(\frac{2S_W^2}{3}-\frac{1}{4}\frac{v^2}{f^2})+\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{7S_W^2}{6}-\frac{S_W^2}{4}\frac{v^2}{f^2}-\frac{1}{2})$	$-\frac{v^2}{4f^2}\cos\theta-\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{C_W^2}{2}+\frac{S_W^2}{4}\frac{v^2}{f^2})$
$U_{1,2,3}$	$\cos\theta(\frac{v^2}{4f^2}-\frac{4S_W^2}{3})+\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{1}{2}+\frac{S_W^2}{2}(\frac{v^2}{2f^2}-\frac{11}{3}))$	$\frac{v^2}{4f^2}\cos\theta+\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{C_W^2}{2}+\frac{S_W^2}{4}\frac{v^2}{f^2})$
$e^-_{1,2,3}$	$\cos\theta(-\frac{1}{2}+2S_W^2)+\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{3S_W^2}{2}-\frac{C_W^2}{4}\frac{v^2}{f^2})$	$-\frac{1}{2}\cos\theta-\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{2}+\frac{C_W^2}{4}\frac{v^2}{f^2})$
$\nu_{1,2,3}$	$\frac{1}{2}\cos\theta+\frac{\sin\theta}{2(C_{2W})^{1/2}}S_W^2$	$\frac{1}{2}\cos\theta+\frac{\sin\theta}{2(C_{2W})^{1/2}}S_W^2$
$N^0_{1,2,3}$	$\frac{\sin\theta}{2(C_{2W})^{1/2}}C_W^2$	$\frac{\sin\theta}{2(C_{2W})^{1/2}}C_W^2$
$E^-_{1,2,3}$	$\cos\theta(2S_W^2-\frac{v^2}{4f^2})+\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{2}(5-\frac{v^2}{2f^2})-\frac{1}{2})$	$-\frac{v^2}{4f^2}\cos\theta-\frac{\sin\theta}{(C_{2W})^{1/2}}(\frac{C_W^2}{2}+\frac{S_W^2}{4}\frac{v^2}{f^2})$

Table 3.3: The $Z_2^\mu \longrightarrow \bar{f}f$ couplings.

f	$g^{(B)}(f)_{2V}$	$g^{(B)}(f)_{2A}$
$u_{1,2,3}$	$-\sin\theta(\frac{1}{2}-\frac{4S_W^2}{3})+\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{C_W^2}{4}\frac{v^2}{f^2}-\frac{5S_W^2}{6})$	$-\frac{1}{2}\sin\theta+\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{2}+\frac{C_W^2}{4}\frac{v^2}{f^2})$
$d_{1,2,3}$	$-\sin\theta(-\frac{1}{2}+\frac{2S_W^2}{3})+\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{6}-\frac{C_W^2}{4}\frac{v^2}{f^2})$	$\frac{1}{2}\sin\theta-\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{2}+\frac{C_W^2}{4}\frac{v^2}{f^2})$
$D_{1,2,3}$	$-\sin\theta(\frac{2S_W^2}{3}-\frac{1}{4}\frac{v^2}{f^2})+\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{7S_W^2}{6}-\frac{S_W^2}{4}\frac{v^2}{f^2}-\frac{1}{2})$	$\frac{v^2}{4f^2}\sin\theta-\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{C_W^2}{2}+\frac{S_W^2}{4}\frac{v^2}{f^2})$
$U_{1,2,3}$	$-\sin\theta(\frac{v^2}{4f^2}-\frac{4S_W^2}{3})+\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{1}{2}+\frac{S_W^2}{2}(\frac{v^2}{2f^2}-\frac{11}{3}))$	$-\frac{v^2}{4f^2}\sin\theta+\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{C_W^2}{2}+\frac{S_W^2}{4}\frac{v^2}{f^2})$
$e^-_{1,2,3}$	$-\sin\theta(-\frac{1}{2}+2S_W^2)+\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{3S_W^2}{2}-\frac{C_W^2}{4}\frac{v^2}{f^2})$	$\frac{1}{2}\sin\theta-\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{2}+\frac{C_W^2}{4}\frac{v^2}{f^2})$
$\nu_{1,2,3}$	$-\frac{1}{2}\sin\theta+\frac{\cos\theta}{2(C_{2W})^{1/2}}S_W^2$	$-\frac{1}{2}\sin\theta+\frac{\cos\theta}{2(C_{2W})^{1/2}}S_W^2$
$N^0_{1,2,3}$	$\frac{\cos\theta}{2(C_{2W})^{1/2}}C_W^2$	$\frac{\cos\theta}{2(C_{2W})^{1/2}}C_W^2$
$E^-_{1,2,3}$	$-\sin\theta(2S_W^2-\frac{v^2}{4f^2})+\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{S_W^2}{2}(5-\frac{v^2}{2f^2})-\frac{1}{2})$	$\frac{v^2}{4f^2}\sin\theta-\frac{\cos\theta}{(C_{2W})^{1/2}}(\frac{C_W^2}{2}+\frac{S_W^2}{4}\frac{v^2}{f^2})$

$$
\begin{aligned}
& -T_W^2(\bar{d}_{3L}\gamma_\mu d_{3L}-\bar{u}_{3L}\gamma_\mu u_{3L})+\bar{U}_{3L}\gamma_\mu U_{3L}-\bar{D}_{3L}\gamma_\mu D_{3L} \\
& +\sum_{\alpha=1}^{3}[-T_W^2(\bar{e}^-_{\alpha L}\gamma_\mu e^-_{\alpha L}-\bar{\nu}_{\alpha L}\gamma_\mu\nu_{\alpha L})+\bar{N}^0_{\alpha L}\gamma_\mu N^0_{\alpha L}-\bar{E}^-_{\alpha L}\gamma_\mu E^-_{\alpha L}]\} \\
= & \sum_f T'_{4f}\bar{f}_L\gamma_\mu f_L,
\end{aligned} \tag{3.28}
$$

where $T_{4f} = Dg(1/2,-1/2,0,0)$ is the third component of the weak isospin and $T'_{4f} = (1/2T_W)Dg(T_W^2, -T_W^2, -1, 1) = T_W\lambda_3/2 + (1/T_W)(\lambda_8/(2\sqrt{3}) - \lambda_{15}/\sqrt{6})$ is a convenient 4×4 diagonal matrix, both of them acting on the representation 4 of $SU(4)_L$. Since $J_\mu(Z)$ is the generalization of the neutral current present in the SM, we can identify Z_μ as the neutral gauge boson of the SM. Notice from Eq. (3.28) that the left-handed couplings of fermions to

Z' are flavor-diagonal so, there are not tree-level FCNC transmitted by the Z' gauge boson in this model. The couplings between the mass eigenstates Z_1^μ, Z_2^μ and the fermion fields are obtained from the Hamiltonian

$$\mathcal{H}^{NC} = \frac{e}{2S_W C_W} \sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu [g^{(B)}(f)_{iV} - g^{(B)}(f)_{iA}\gamma_5] f\}, \tag{3.29}$$

The couplings $g_{iV}^{(B)}$, $g_{iA}^{(B)}$ $(i = 1,2)$ are listed in Tables 3.2 and 3.3, from which we see that these couplings are family-universal. Notice that there is a relevant contribution to these coupling constants coming from the factor $\frac{1}{S_W C_W}$ in Eq.(3.29) that must be taken into account

$$\frac{1}{S_W C_W} = \frac{1}{\bar{S}_W \bar{C}_W}\left[1 - \frac{v^2}{4f^2}\left(1 + \frac{T_W^4}{2}\right)\right], \tag{3.30}$$

where $\bar{S}_W$ is the effective Weinberg mixing angle.

The relevant terms of the Hamiltonian for the charged currents in this model is given by

$$\mathcal{H}^{CC} = \frac{e}{\sqrt{2}S_W} W_\mu^+ \left[\left(\sum_{\alpha=2}^{3} \bar{u}_{\alpha L}\gamma^\mu d_{\alpha L}\right) - \bar{u}_{1L}\gamma^\mu d_{1L} - \left(\sum_{\alpha}^{3} \bar{\nu}_{e\alpha L}\gamma^\mu e_{\alpha L}^-\right)\right], \tag{3.31}$$

from here it is important to note that, because the mass mixing between light neutrinos and heavy neutral states, there are also corrections to the charged current couplings that lead to a shift in G_F, which in fact, also receives corrections from M_W. The shift on G_F provided by the mentioned mass mixing is

$$\delta G_F = \frac{1}{3\sqrt{2}f^2}. \tag{3.32}$$

3.4 Constraints on the parameters of the model

In this section we are going to set bounds on the mass of the new neutral gauge boson Z_2^μ, and its mixing angle θ with the ordinary neutral gauge boson Z_1^μ, as well as on the breaking scale f.

To get bounds on the parameter space $(\theta - M_{Z_2})$ and $(f - M_{Z_2})$ and to test the model by low energy data, we use electroweak observables measured at the Z-pole from the CERN e^+e^- collider (LEP), SLAC Linear Collider (SLC), and atomic parity violation data which are given in Table 3.4 [31]. Let us start by briefly describing each one of the observables in the Table. The expression for the partial decay width for the gauge boson Z_1^μ to decay into massless ordinary SM fermions $f\bar{f}$, including the electroweak and QCD virtual corrections is given, in the on-shell scheme, by [31, 36]

$$\Gamma(Z_1^\mu \to f\bar{f}) = \frac{N_C G_F M_{Z_1}^3}{6\pi\sqrt{2}}\rho_f \Big\{\frac{3\beta - \beta^3}{2}[g(f)_{1V}]^2 + \beta^3[g(f)_{1A}]^2\Big\}(1+\delta_f)R_{\rm EW}R_{\rm QCD}. \tag{3.33}$$

In the modified minimal subtraction ($\overline{\text{MS}}$) scheme, which we use through this section, the normalization is changed according to $G_F M_{Z_1}^2/(2\sqrt{2}\pi) \rightarrow \widehat{\alpha}/[4\sin^2\widehat{\theta}_W(M_{Z_1})\cos^2\widehat{\theta}_W(M_{Z_1})]$. In (3.33), Z_1^μ is the physical gauge boson observed at LEP, $N_C = 1$ for leptons while for quarks $N_C = 3(1+\alpha_s/\pi + 1.405\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3)$, where the 3 is due to colour and the factor in parentheses represents the universal part of the QCD corrections for massless quarks. R_{EW} are electroweak corrections which include the leading order QED corrections given by $R_{\text{QED}} = 1+3\alpha q_f^2/(4\pi)$. R_{QCD} are further QCD corrections, and $\beta = \sqrt{1-4m_f^2/M_{Z_1}^2}$ is a kinematic factor which can be taken equal to 1 for all the SM fermions except for the bottom quark. The parameter ρ_f is written as $\rho_f = 1+\rho_t$ where $\rho_t = 3G_F m_t^2/(8\pi^2\sqrt{2})$ with m_t being the top quark pole mass and, G_F must be written as $\bar{G}_F = G_F + \delta G_F$ in order for us to count adequately the corrections. As we already stated, such parameter receives important contributions from M_W and the mass mixing described in Ec. (3.32). Taken into account this, and by treating it as the same way as we do with S_W, we can write

$$\frac{1}{G_F} = \frac{1}{\bar{G}_F}\left(1+\frac{v^2}{3f^2}\right), \tag{3.34}$$

where the simplification $f_1 = f_2 = f_3 = f_4$ has been used. Universal electroweak corrections are included in ρ_t, and in the coupling constants $g(f)_{1V}$ and $g(f)_{1A}$ of the physical Z_1^μ field with ordinary fermions which are written in terms of the electroweak mixing angle. The last, also embodies radiative corrections and must be replaced in the coupling constants $g(f)_{1V}$ and $g(f)_{1A}$ in Tables 3.2 and 3.3 by the so called effective Weinberg mixing angle

$$\bar{S}_W^2 = S_W^2 + \frac{S_W^2 C_W^2}{C_W^2 - S_W^2}\left(\frac{v^2}{2f^2}\right)\left(1+\frac{T_W^4}{2}\right). \tag{3.35}$$

In the $\overline{\text{MS}}$ scheme, $\widehat{\rho}_f \sim 1$ for $f \neq b$, while $\widehat{\rho}_b \sim 1-(4/3)\rho_t$ and $\widehat{\kappa}_b \sim 1+(2/3)\rho_t$. The factor δ_f contains the one loop vertex contribution which is negligible for all fermion fields except for the bottom quark for which the contribution coming from the top quark at the one loop vertex radiative correction is parametrized as $\delta_b \approx 10^{-2}[-m_t^2/(2M_{Z_1}^2)+1/5]$. In the $\overline{\text{MS}}$ scheme this correction is included in $\widehat{\rho}_b$. The total hadronic cross-section is

$$\sigma_{\text{had}} = \frac{12\pi}{M_{Z_1}^2}\frac{\Gamma_{(e^+e^-)}\Gamma(\text{had})}{\Gamma_Z^2}, \tag{3.36}$$

where Γ_Z is the total width for $Z_1^\mu \rightarrow f\bar{f}$.

The ratios of partial widths are defined as

$$R_l \equiv \frac{\Gamma(\text{had})}{\Gamma(l^+l^-)} \quad \text{for} \quad l = e, \mu, \tau, \tag{3.37}$$

and

$$R_\eta \equiv \frac{\Gamma_\eta}{\Gamma(\text{had})} \quad \text{for} \quad \eta = b, c. \tag{3.38}$$

Table 3.4: Experimental data and SM values for the observables used for the χ^2 fit.

	Experimental results	SM value
Γ_Z [GeV]	2.4952 ± 0.0023	2.4968 ± 0.0010
Γ(had) [GeV]	1.7444 ± 0.0020	1.7434 ± 0.0010
$\Gamma_{(l^+l^-)}$ [MeV]	83.984 ± 0.086	83.988 ± 0.016
σ_{had} [nb]	41.541 ± 0.037	41.466 ± 0.009
R_e	20.804 ± 0.050	20.758 ± 0.011
R_μ	20.785 ± 0.033	20.758 ± 0.011
R_τ	20.764 ± 0.045	20.803 ± 0.011
R_b	0.21629 ± 0.00066	0.21584 ± 0.00006
R_c	0.1721 ± 0.0030	0.17228 ± 0.00004
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01627 ± 0.00023
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013	
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017	
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1033 ± 0.0007
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0738 ± 0.0006
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1034 ± 0.0007
A_e	0.15138 ± 0.00216	0.1473 ± 0.0011
A_μ	0.142 ± 0.015	
A_τ	0.136 ± 0.015	
A_b	0.923 ± 0.020	0.9347 ± 0.0001
A_c	0.670 ± 0.027	0.6678 ± 0.0005
A_s	0.895 ± 0.091	0.9536 ± 0.0001
Q_W(Cs)	-72.62 ± 0.46	-73.16 ± 0.03

The forward-backward asymmetries at the Z-pole are given by

$$A_{FB}^{(0,f)} = \frac{3}{4} A_e A_f, \quad \text{where} \quad A_f = \frac{2g(f)_{1V} g(f)_{1A}}{g(f)_{1V}^2 + g(f)_{1A}^2}, \tag{3.39}$$

$(f = e, \mu, \tau, s, c, b)$, which are also written in terms of $\bar{S}_W^2$.

The anomaly-free $SU(4)_L \otimes U(1)_X$ Little Higgs model new physics contributions to the SM observables listed in the first column of Table 3.4 are obtained by noticing that, with the assumed approximation $f = f_{12} = f_{34}$, and from (3.18), the $Z - Z'$ mixing angle is expected to be very small so, $\cos\theta = \sqrt{1 - \sin^2\theta} \simeq 1 - (1/2)\sin^2\theta \simeq 1$, and the coupling constants $g(f)_{1V}$ and $g(f)_{1A}$ of the physical Z_1^μ gauge boson to ordinary fermions can be written as (remember that in the limit $\theta \to 0$ and $f \to \infty$ these couplings are the same as in the SM)

$$g(f)_{1V,A} = g(f)_{1V,A}^{\text{SM}} + \delta g(f)_{1V,A}, \tag{3.40}$$

where the expressions for $\delta g(f)_{1V,A}$ depend linearly on $\sin\theta$. They can be easily read from Table 3.2 taking into account the contribution of the coeficiente in Eq. (3.30.)

To facilitate the numerical analysis we express the changes in the physical observables relative to their SM values as [37]

$$O^{341} = O^{\rm SM}(1+\delta_O), \quad \text{where} \quad \delta_O = \frac{\delta O}{O^{\rm SM}}, \tag{3.41}$$

with $O^{\rm SM}$ being the SM value for the observable O, including the one-loop SM corrections, and with δO representing the corrections due to new physics. Equation (3.41) allows us to quickly assess the percentage changes in the SM observables brought about by the various $SU(4)_L \otimes U(1)_X$ corrections.

For the observables in Table 3.4, and taking into account from Table 3.2 that the couplings $g(f)_{1V}$ and $g(f)_{1A}$ are family universal, the several δ_O are given by

$$\delta_Z = \frac{1}{\Gamma_Z^{\rm SM}}(2\Gamma_u^{\rm SM}\delta_u + 2\Gamma_d^{\rm SM}\delta_d + \Gamma_b^{\rm SM}\delta_b + 3\Gamma_\nu^{\rm SM}\delta_\nu + 3\Gamma_e^{\rm SM}\delta_l), \tag{3.42}$$

$$\delta_{\rm had} = 2R_c^{\rm SM}\delta_u + R_b^{\rm SM}\delta_b + 2\frac{\Gamma_d^{\rm SM}}{\Gamma_{\rm had}^{\rm SM}}\delta_d, \tag{3.43}$$

$$\delta_\sigma = \delta_{\rm had} + \delta_l - 2\delta_Z, \tag{3.44}$$

$$\delta_{A_f} = \frac{\delta g(f)_{1V}}{g(f)_{1V}^{\rm SM}} + \frac{\delta g(f)_{1A}}{g(f)_{1A}^{\rm SM}} - \delta_f, \tag{3.45}$$

where, for $f \neq b$

$$\delta_f = \frac{\delta G_f}{G_f} + \frac{3}{2}\frac{\delta M_Z^2}{M_Z^2} + 2\frac{g(f)_{1V}^{\rm SM}\delta g(f)_{1V} + g(f)_{1A}^{\rm SM}\delta g(f)_{1A}}{(g(f)_{1V}^{\rm SM})^2 + (g(f)_{1A}^{\rm SM})^2}, \tag{3.46}$$

while for the bottom quark

$$\delta_b = \frac{\delta G_f}{G_f} + \frac{3}{2}\frac{\delta M_Z^2}{M_Z^2} + \frac{(3-\beta^2)g(b)_{1V}^{\rm SM}\delta g(b)_{1V} + 2\beta^2 g(b)_{1A}^{\rm SM}\delta g(b)_{1A}}{\frac{3-\beta^2}{2}(g(b)_{1V}^{\rm SM})^2 + \beta^2(g(b)_{1A}^{\rm SM})^2}, \tag{3.47}$$

where

$$\delta M_Z^2 = -\frac{g^2 v^4 T_W^4}{16 C_W^2 f^2}.$$

The tree-level contribution to the Z_1 partial decays due to the $Z - Z'$ mixing is included by multiplying $\Gamma(Z_1^\mu \to f\bar{f})$ in (3.33) by the factor $1+\delta\rho$, where

$$\delta\rho \approx \left(\frac{T_W^4}{4}\right)\frac{v^2}{f^2}. \tag{3.48}$$

The theoretical value for the effective weak charge for the Cesium atom is given by [38]

$$Q_W({\rm Cs}) = Q_W^{\rm SM}({\rm Cs}) + \Delta Q_W = Q_W^{\rm SM}({\rm Cs})\Big[1 + \delta_{Q_W}\Big], \tag{3.49}$$

where [39, 40]

$$\Delta Q_W = \left[Z\left(1+4\frac{S_W^4}{1-2S_W^2}\right)-N\right]\delta\rho + \Delta Q'_W. \tag{3.50}$$

with

$$\begin{aligned}\Delta Q'_W &= 16[(2Z+N)(g(e)_{1A}g(u)_{2V}+g(e)_{2A}g(u)_{1V}) \\ &+ (Z+2N)(g(e)_{1A}g(d)_{2V}+g(e)_{2A}g(d)_{1V})]\sin\theta \\ &- 16[(2Z+N)g(e)_{2A}g(u)_{2V}+(Z+2N)g(e)_{2A}g(d)_{2V}]\frac{M_{Z_1}^2}{M_{Z_2}^2}.\end{aligned} \tag{3.51}$$

Z and N are, respectively, the number of protons and of neutrons in the nucleus of the considered atom. For the Cesium: $Z = 55$ and $N = 78$.

Clearly, ΔQ_W accounts for the contribution of the new physics. Notice that $\Delta Q'_W$ is model dependent; in particular, it is a function of the couplings $g(q)_{2V}$ and $g(q)_{2A}$ $(q = u, d)$ of the first family of quarks to the new neutral gauge boson Z_2. Because of this, the new physics in $\Delta Q'_W$ depends on which family of quarks transforms differently under the gauge group.

For the partial decays in (3.42) we use [31]

$$\begin{aligned}\Gamma_u^{\rm SM} &= 300.10\pm0.09 \ \text{MeV}, \ \ \Gamma_\nu^{\rm SM} = 167.18\pm0.02 \ \text{MeV}, \\ \Gamma_d^{\rm SM} &= 382.89\pm0.08 \ \text{MeV}, \ \ \Gamma_e^{\rm SM} = 83.97\pm0.03 \ \text{MeV}, \\ \Gamma_b^{\rm SM} &= 376.01\pm0.05 \ \text{MeV}.\end{aligned} \tag{3.52}$$

With the anomaly-free $SU(4)_L \otimes U(1)_X$ with Little Higgs predictions written in the form (3.41), we need the following well measured input parameters [31]: $\bar{G}_F = 1.166367(5)\times10^{-5}$ GeV, $M_{Z_1} = 91.1874 \pm 0.0021$ GeV and $m_t = 170.9 \pm 1.9$ GeV. For S_W we use the value $\sin^2\widehat{\theta}_W(M_{Z_1}) \equiv \widehat{s}_Z^2 = 0.23119 \pm 0.00014$ in the $\overline{\rm MS}$ scheme because is less sensitive to m_t than its value in the on-shell scheme, and for the bottom quark mass we use the running mass in the $\overline{\rm MS}$ scheme at the Z_1 scale: $\widehat{m}_b(M_{Z_1}) = 2.67 \pm 0.19$ GeV [41].

By using $g(e)_{iA}$ and $g(q)_{iV}, i = 1, 2$ from Tables 3.2 and 3.3, we obtain the following value for $\Delta Q'_W$

$$\Delta Q'_W = 401.34\sin\theta - 97.84\frac{M_{Z_1}^2}{M_{Z_2}^2}. \tag{3.53}$$

Using the experimental values for the Z-pole observables in Table 3.4 and with ΔQ_W in terms of new physics in (3.50), we do a one parameter χ^2 fit of the theoretical expressions in Table 3.5 to the data and find the best allowed region in the $(\theta - M_{Z_2})$ and $(f - M_{Z_2})$ plane at 95% confidence level (C.L.). We plot χ^2 as a function of the mass of Z_2 boson for 20 d.o.f in Figure (3.1) and, by mean of the same statistical method, we show the allowed variation of f with θ in Figure (3.2). Such a procedure provides us the constraints

$$0 \leq \theta \leq 1.33\times10^{-3} \tag{3.54}$$

Table 3.5: Anomaly-free $SU(4)_L \otimes U(1)_X$ Little Higgs model predictions for the observables in Table 3.4. The third column shows the percentage change in these observables relative to their SM values.

Anomaly-free $SU(4)_L \otimes U(1)_X$ Little Higgs model	Value	Percentage change
$\Gamma_Z^{\text{SM}}[1+\delta_Z(1+\delta\rho)]$ [GeV]	2.4994 ± 0.2491	0.1
$\Gamma^{\text{SM}}(\text{had})[1+\delta_{\text{had}}(1+\delta\rho)]$ [GeV]	1.7441 ± 0.2222	0.04
$\Gamma^{\text{SM}}_{(l^+l^-)}[1+\delta_l(1+\delta\rho)]$ [MeV]	84.031 ± 0.002	0.05
$\sigma^{\text{SM}}_{\text{had}}(1+\delta_\sigma)$ [nb]	41.419 ± 0.015	-0.1
$R_e^{\text{SM}}(1+\delta_{\text{had}}-\delta_e)$	20.756 ± 0.061	-0.009
$R_\mu^{\text{SM}}(1+\delta_{\text{had}}-\delta_\mu)$	20.756 ± 0.061	-0.009
$R_\tau^{\text{SM}}(1+\delta_{\text{had}}-\delta_\tau)$	20.801 ± 0.061	-0.009
$R_b^{\text{SM}}(1+\delta_b-\delta_{\text{had}})$	0.21602 ± 0.06314	0.08
$R_c^{\text{SM}}(1+\delta_c-\delta_{\text{had}})$	0.17227 ± 0.05010	-0.004
$A_{FB}^{(0,e)\text{SM}}(1+2\delta_{A_e})$	0.01608 ± 0.01725	-1.2
$A_{FB}^{(0,\mu)\text{SM}}(1+\delta_{A_\mu}+\delta_{A_e})$	0.0161 ± 0.0173	-1.2
$A_{FB}^{(0,\tau)\text{SM}}(1+\delta_{A_\tau}+\delta_{A_e})$	0.01608 ± 0.01725	-1.2
$A_{FB}^{(0,b)\text{SM}}(1+\delta_{A_b}+\delta_{A_e})$	0.1027 ± 0.0548	-0.06
$A_{FB}^{(0,c)\text{SM}}(1+\delta_{A_c}+\delta_{A_e})$	0.0733 ± 0.0394	-0.7
$A_{FB}^{(0,s)\text{SM}}(1+\delta_{A_s}+\delta_{A_e})$	0.1028 ± 0.0549	-0.6
$A_e^{\text{SM}}(1+\delta_{A_e})$	0.1465 ± 0.0781	-0.6
$A_\mu^{\text{SM}}(1+\delta_{A_\mu})$	0.1465 ± 0.0781	-0.6
$A_\tau^{\text{SM}}(1+\delta_{A_\tau})$	0.1465 ± 0.0781	-0.6
$A_b^{\text{SM}}(1+\delta_{A_b})$	0.9344 ± 0.4955	-0.04
$A_c^{\text{SM}}(1+\delta_{A_c})$	0.6673 ± 0.3540	-0.08
$A_s^{\text{SM}}(1+\delta_{A_s})$	0.9533 ± 0.5055	-0.04
$Q_W^{\text{SM}}(\text{Cs})[1+\delta_{Q_W}]$	-73.15 ± 0.16	-0.01

$$f \geq 1.6 \text{ TeV}, \quad M_{Z_2} \geq 1.2 \text{ TeV}. \tag{3.55}$$

The fit has a χ^2/d.o.f. of 20.40/20, corresponding to a probability of 48%, and the best-fit values are: $f = 2.9$ TeV, $\theta = 4.16 \times 10^{-4}$, $M_{Z_2} = 2.3$ TeV.

Notice that the lower bound on M_{Z_2} is compatible with the bound obtained in $p\bar{p}$ collisions at the Fermilab Tevatron [42].

Using the best-fit values for θ, M_{Z_2} and f, we calculate the anomaly-free $SU(4)_L \otimes U(1)_X$ Little Higgs model predictions for the electroweak observables in Table 3.4 and the percentage changes in these observables relative to their SM values. As a first approximation we neglect correlations between the uncertainties of the input parameters and use standard error propagation. This is partially justified because the errors which enter in the expressions for the anomaly-free $SU(4)_L \otimes U(1)_X$ Little Higgs predictions are of different status and, therefore, there is no clean way of exactly calculating the errors. The results are shown in Table 3.5. Notice that the percentage changes are at the per-cent level and even lower. In any case, no substantial departures from the SM fit is observed.

We remark that, since in this model only $SU(2)_L$ doublets develop VEV, the $Z - Z'$ mixing contribution to the ρ parameter is such that $\delta\rho << 1$. This, together with the fact that all the anomaly-free $SU(4)_L \otimes U(1)_X$ Little Higgs model corrections δO to SM observables go to zero in the limits $\theta \to 0$ ($M_{Z'} \to \infty$) and $f \to \infty$, justifies our fitting procedure in which we treat the new physics effects as small corrections to the well established SM results [43].

Figure 3.1: Contour plot displaying the allowed region for χ^2 vs. M_{Z_2} at 95% C.L.

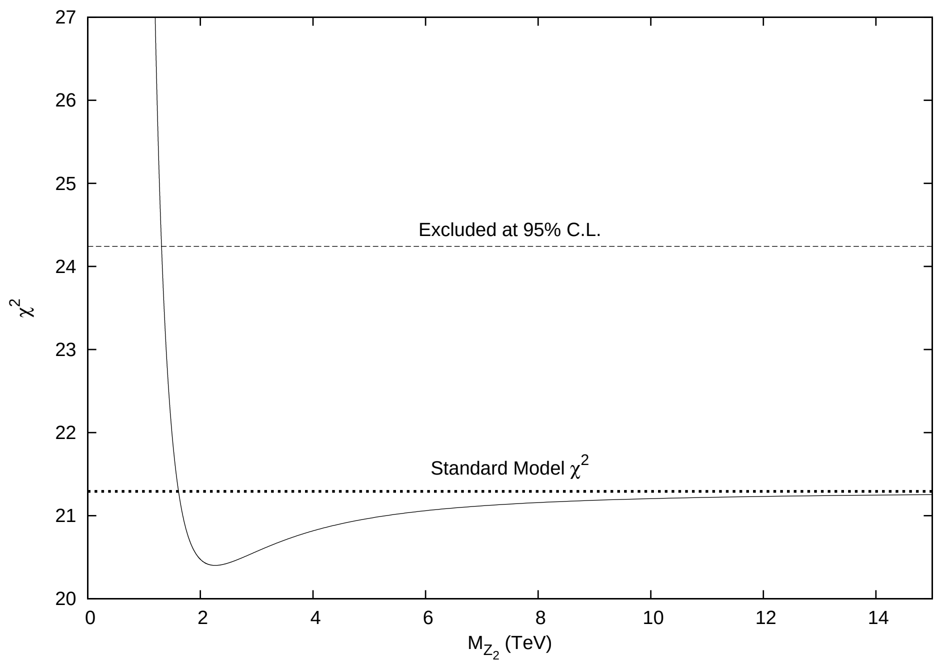
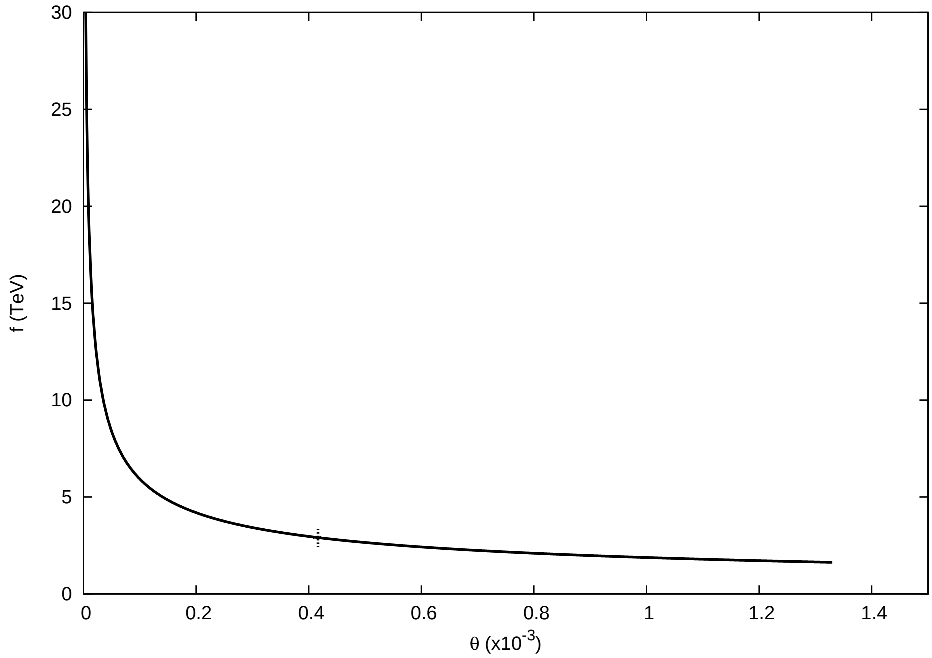


Figure 3.2: Contour plot displaying the allowed region for f vs. θ at 95% C.L.



Conclusions

In this work we have studied an anomaly-free Little Higgs model with $SU(4)_L \otimes U(1)_X$ gauge symmetry which predict the existence of two extra neutral gauge bosons Z' and Z''. We have set bounds on the parameter scale f, on the mixing angle θ between the SM gauge bosons Z and the new Z', and on the masses of the new physical eigenstates, namely: Z_2, which arises from the diagonalization of the $Z - Z'$ mass matrix, and $Z'' \equiv Z_3$ which becomes a mass eigenstate when the relations $f = f_{12} = f_{34}$ are fulfilled.

The very interesting kind of Little Higgs model with $SU(4)_L \otimes U(1)_X$ gauge symmetry that we have analyzed, are characterized by the values $b = 1$, $c = -2$ of the parameters appearing in the electric charge operator in Eq. (3), with fermion content without exotic electric charges and with anomalies cancelling among the fermion families in a non-trivial fashion. This method of cancellation of anomalies leads to a number of fermion families N_f that must be divisible by the number of colours N_c of $SU(3)_c$, being $N_f = N_c = 3$ the simplest solution. In this last case universality in the lepton sector is preserved, but one family of quarks must transform differently than the other two under $SU(4)_L \otimes U(1)_X$.

The limits on the parameters f, θ and M_{Z_2}, have been obtained by doing a χ^2 fit of the theoretical predictions of the $SU(4)_L \otimes U(1)_X$ Little Higgs simple group model, for 22 precision electroweak observables, to the experimental data at the Z-pole from LEP and SLAC Linear Collider and atomic parity violation data. After imposing the simplification $f_1 = f_2 = f_3 = f_4$ on the scale parameters, we have got for this model: $0 \leq \theta \leq 1.33 \times 10^{-3}$, $f \geq 1.6$ TeV, $M_{Z_2} \geq 1.2$ TeV, and their best fit are $f = 2.9$ TeV, $\theta = 4.16 \times 10^{-4}$ and $M_{Z_2} = 2.3$ TeV.

The analysis we presented here, although is close related to the made in Ref. [19], show a large reduction to the bound on f, because the couplings of the the anomaly-free fermions to the Z and Z' is such that, the magnitude of the predicted weak charge of Cesium is strongly reduced. However, even in this model, the largest deviations from standard model derive from tree level $Z - Z'$ mixing.

Despite the fact that almost all Little Higgs models could be criticized from a naturalness point of view, a detailed analysis presented in Ref. [45] shows that while the scale parameter f is closer to 1 TeV, a model with a simple little Higgs structure could be better behaved under the fine-tunning analysis than LH models with stronger restrictions over the scale f and, even, become itself in a more serious alternative to SUSY at low energies. It is important to note that earlier studies on different kinds of Simple Little Higgs models consigned in the

Table 3.6: Best-fit and lower bounds on f reported in the literature

Kind of SLHM	Include fermion mixing	Ref.	Lower bound	Best fit
$SU(3)_L \otimes U(1)_X$ (univer. embedding)	yes	[8]	3.9 TeV	
$SU(3)_L \otimes U(1)_X$ (anomaly-free)	no	[18]	5.6 TeV	
$SU(3)_L \otimes U(1)_X$ (anomaly-free)	no	[8]	1.7 TeV	
$SU(4)_L \otimes U(1)_x$ (univer. embedding)	no	[4]	1.5 TeV	2.2 TeV
$SU(4)_L \otimes U(1)_X$ (univer. embedding)	yes	[19]	4.2 TeV	

literature (but without a detailed study of the size of the electroweak corrections like the one developed here), get constraints on the scale f that are consistent with the constraint we have obtained, as it is shown in Tab. 3.6. However, most of these works ignore the contribution of fermion mixing that is always present in this kind of models.

It is fair to realize that there is no way to elude neither the gauge boson mixing nor fermion mixing in this variation of $SU(4) \otimes U(1)$ Little Higgs model [4]. The first statement can be understood easily from Figure 3.2, and although the second one is particularly strong in our model, it is always possible to introduce a Yukawa hierarchy in the fermion sector that alleviate a lot of technical difficulties.

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