Soil moisture dynamics in water- and energy-limited ecosystems. Application to slope stability

Dinámica de la humedad del suelo en ecosistemas limitados por agua y energía. Aplicación a la estabilidad de laderas

Estefanía Muñoz Hoyos
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Estefanía Muñoz Hoyos

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Advisor: Andrés Ochoa
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Abstract

I investigate the daily dynamics of the soil-water-climate system in water- and energy-limited environments. I analyze the stochastic component of radiation at the Earth’s surface in the photosynthetically active spectral band in 28 sites located around the world and propose a clear-sky based methodology to model it. In order to study the effect of stochastic radiation on soil moisture and evapotranspiration, we use the proposed radiation model and the ecohydrological model of Rodríguez-Iturbe et al. I introduce a semi-analytical equation to analyze the dynamics of evapotranspiration as a function of stochastic rainfall and radiation. With climatic and physiological measurements from the FLUXNET dataset, and taking into account the coupling of water and CO$_2$ fluxes through the leaf plant, I parameterize the link of transpiration and available energy, which is used to extend the ecohydrological model to energy-limited ecosystems. Finally, I carry out an application to landslides triggered by rainfall, relating the factor of safety of a slope to soil moisture, and examine the widely used threshold rainfall approach to estimate the landslides occurrence.

Keywords: Soil-climate-vegetation system, PAR, clear-sky index, transpiration, ecohydrology, stochastic processes, stationarity, slope stability.

Resumen

En esta tesis se investiga la dinámica diaria del agua en el sistema clima-suelo-vegetación en ambientes limitados por agua y energía. Se analiza la componente estocástica de la radiación fotosintéticamente activa en superficie en 28 sitios localizados alrededor del mundo, y se propone una metodología basada en el índice clear-sky para modelar su comportamiento. Con el modelo de radiación se estudia el efecto de la estocasticidad de la energía disponible en la dinámica de la humedad del suelo y de la evapotranspiración, proponiendo una ecuación semianalítica para modelar la transpiración en función de la lluvia y la radiación estocásticas. Usando mediciones de diferentes variables climáticas y fisiológicas, y considerando el acomplamiento de los flujos de agua y CO$_2$ a través de la planta, se parametriza la relación entre la transpiración y la energía disponible, y se extiende el modelo ecohidrológico de Rodríguez-Iturbe et al. a ecosistemas limitados por energía. Por último, se hace una aplicación a deslizamientos detonados por lluvias, relacionando el factor de seguridad de una ladera con la humedad del suelo, y se examina el enfoque de umbrales de lluvia en la determinación de la ocurrencia de deslizamientos.

Palabras clave: Sistema clima-suelo-vegetación, PAR, índice clear-sky, transpiración, ecohidrología, procesos estocásticos, estacionariedad, estabilidad de taludes.
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1. Introduction

The climate-soil-vegetation system involves many variables and processes with high spatiotemporal variability, feedbacks, and nonlinear relationships (Klemes, 1983; Entekhabi and Rodríguez-Iturbe, 1994; Blöschl and Sivapalan, 1995; Rodríguez-Iturbe et al., 2001; Porporato and Rodríguez-Iturbe, 2002; Rodríguez-Iturbe et al., 2006; Manzoni et al., 2011; Bras, 2015). Climate limits the amount of water and energy available and influences diffusion and evapotranspiration processes (Leuning, 1995; De Pury and Farquhar, 1997; Stoy et al., 2009; Manzoni et al., 2011). Soil texture, its mineralogical composition, and particle size distribution restrict the amount of water stored by the soil, and therefore, the stomata closing. Vegetation controls the energy and water fluxes, coupling soil and atmosphere (Feddes et al., 2001; Rodríguez-Iturbe et al., 2001). Soil water content ($s$) is the key variable synthesizing the complex dynamics of the climate-soil-vegetation system (Entekhabi and Brubaker, 1995; Porporato and Rodríguez-Iturbe, 2002; Rodríguez-Iturbe and Porporato, 2004) and markedly depends on the plant physiology, soil and climate type (Rodríguez-Iturbe et al., 1991a; Entekhabi and Rodríguez-Iturbe, 1994; Rodríguez-Iturbe and Porporato, 2004). $s$ controls the meteorological processes (evapotranspiration is governed by $s$, and water that returns to the atmosphere in the form of vapor can become rainfall), plant conditions, soil biochemistry, and the exchange of nutrients and contaminants in the climate-soil-vegetation system (Eagleson, 1978a; Larcher, 1995; Brubaker and Entekhabi, 1996; Dickinson et al., 2002; Daly and Porporato, 2005). Despite the high complexity of the climate-soil-vegetation system, there are several approaches to modeling soil moisture, among which are biophysical process-based, physical-based and statistical models (Wang et al., 2019). Most of these approaches are a function of in-situ (e.g. Korres et al., 2015; Noh et al., 2015; Pirone et al., 2015; Gevaert et al., 2018) and remote sensing (e.g. Wagner et al., 1999; Kim and Barros, 2002; Fang and Lakshmi, 2014; Zehe et al., 2018) data or involve numerical simulations (e.g. Mtundu and Koch, 1987; Albertson and Montaldo, 2003; Rigon et al., 2006; Sela et al., 2012; Chen et al., 2017; de Assunção Montenegro et al., 2018). In-situ data are not easy to extrapolate to spatial scales that allow hydrological applications, remote sensing methods measure continuous spatiotemporal information but only comprise the most superficial centimeters of the soil (Niemann, 2004), and numerical simulations do not permit to generalize the results (Ogren, 1993). Some complete reviews of the state of the art of soil moisture modeling are presented by Daly and Porporato (2005), Seneviratne et al. (2010), Asbjørnsen et al. (2011), Legates et al. (2011) and Wang et al. (2019).

Among the biophysical based approaches is the one initiated by Eagleson (1978a), Cordova and Bras (1981), Hosking and Clarke (1990), and Milly (1993), and extended by Rodríguez-
Iturbe et al. (1999) and others (e.g. D’Odorico et al., 2000; Laio et al., 2001; Milly, 2001; Laio et al., 2002; Porporato et al., 2003b; D’Odorico and Porporato, 2004; De Michele et al., 2008). This approach comprises simplified but realistic conceptual models characterized by analytically describing the phenomena among the climate-soil-vegetation system, the inherent variability of soil and vegetation properties and by involving stochastic components that take into account the unpredictability of precipitation. Precipitation is modeled as a stochastic process with known properties, and the analytical expressions of soil moisture are derived from measurable parameters of soil, climate, and vegetation (Vereecken et al., 2015). The model of Milly (1993) does not consider the dependence of evapotranspiration on \( s \), but Rodríguez-Iturbe et al. (1999) and Laio et al. (2001) solved this problem through a nonlinear function relating both variables. D’Odorico et al. (2000) and Porporato et al. (2003b) regarded the role of the interannual climate variability, Laio et al. (2002) the seasonal water balance, and Laio et al. (2009) the dynamics of the water table and the presence of plant roots interacting with saturated and unsaturated zones of the soil. Brubaker (1995), Brubaker and Entekhabi (1996), Margulis and Entekhabi (2001) and Ridolfi et al. (2003) also used a similar approach, but having to appeal to numerical solutions. Ridolfi et al. (2003) evaluated the dynamics of soil moisture along a hillslope, while Brubaker (1995) and Brubaker and Entekhabi (1996) proposed a conceptual coupling of water and energy balances in 1-D, improved then by Margulis and Entekhabi (2001). The biophysical based approaches initiated by Eagleson capture the essential features of the water cycle and the vegetation response (Ogren, 1993). Nevertheless, all the models mentioned, except the one of Brubaker (1995) and Brubaker and Entekhabi (1996), are developed for arid and semi-arid environments, characterized by the scarcity of rainfall, low soil moisture, frequent water stress, and deep water table (Laio et al., 2009). As the available energy is not directly considered, they are not suitable to be applied in energy-limited environments.

The regimes limiting evapotranspiration \((ET)\) are governed by its relationship with \( s \). In energy-limited ecosystems, \( ET \) is independent of \( s \), while in water-limited ecosystems \( ET \) is very sensitive to it. The first regime has been associated with humid regions, where solar radiation is the limiting variable and the second one with arid and semi-arid regions, where water is scarce (Eagleson, 1982; Seneviratne et al., 2010). Besides, there are seasonal ecosystems in which the availability of water fluctuates sharply, and plants present unique adaptations and effects on the hydrological cycle that differ from water- and energy-limited ecosystems (Asbjornsen et al., 2011). In water-limited ecosystems, water restricts \( ET \) by its scarcity, intermittency, and unpredictability (Porporato and Rodríguez-Iturbe, 2002), and photosynthesis is controlled by soil moisture (Porporato and Rodríguez-Iturbe, 2002; Daly et al., 2004a). In energy-limited ecosystem, light limits by its high spatiotemporal variability. This variability is associated with structural and environmental heterogeneity (gapping and clumping of foliage, gaps in the canopy, leaf orientation, type and distribution of clouds, topography, seasonal trends in plant phenology, and seasonality movements of the sun) (Baldocchi and Collineau, 1994). Solar radiation is essential in modeling soil moisture in energy-limited systems since it is the energy source of biophysical processes, such as photosynthesis, stomatal conductance,
transpiration, evaporation, leaf temperature, plant growth, seedling generation, biochemical cycling, and atmospheric chemistry (Thorpe et al., 1978; Baldocchi and Meyers, 1991; Baldocchi and Collineau, 1994; Ballaré, 1994; Hansen, 1999; Yu et al., 2004; Daly et al., 2004a; Ge et al., 2011), that are directly or indirectly related to \( s \). Stomata movement regulates simultaneously the water and CO\(_2\) fluxes during transpiration and photosynthesis (Collatz et al., 1991; Yu et al., 2004; Medlyn et al., 2017; Shan et al., 2019), tending to maximize photosynthetic carbon gain while minimizes water loss (Farquhar, 1973; Cowan and Farquhar, 1977; Leuning, 1995; Medlyn et al., 2017). As a result, photosynthesis and transpiration should be modeled coupled with the stomatal conductance.

Stomatal conductance (\( g_s \)) can be calculated by physiological and biochemical based models (e.g. Jarvis, 1976; Farquhar et al., 1980; Ball et al., 1987; Farquhar, 1989; Collatz et al., 1991; Leuning, 1995; Gao et al., 2002; Dewar, 2002; Tuzet et al., 2003; Yu et al., 2004). The types of models most widely used are those based on Jarvis (1976) (e.g. Baldocchi and Meyers, 1991; Peters-Lidard et al., 1997; Daly et al., 2004a; Yu et al., 2004) and Ball et al. (1987) (e.g. Leuning, 1990, 1995; Leuning et al., 1995; Daly et al., 2004a) approaches. Jarvis (1976) approach assumes that environmental factors act separately in \( g_s \) response and that \( g_s \) is independent of photosynthesis and transpiration. Ball et al. (1987) model \( g_s \) as a function of relative humidity at the leaf surface, assimilation rate, and surface CO\(_2\) concentration. Leuning (1995) improved this last model taking into account the effect of plant water stress. As mentioned before, in order to correctly model the stomatal conductance it is necessary to treat it as an identity dependent on transpiration and photosynthesis, and Jarvis’s approach does not do so, being this an important limitation (Jones and Rawson, 1979; Leuning, 1995; Daly et al., 2004a). Otherwise, Ball’s model requires the calculation of the CO\(_2\) assimilation rate (\( A_n \)). Bartlett et al. (2014), Daly et al. (2004a) and Leuning et al. (1995) present methodologies to solve simultaneously stomatal conductance, CO\(_2\) assimilation, and energy balance.

In spite of all the important advances that have been carried out in the understanding of soil moisture dynamics, soil and atmosphere interaction through plants remains still an important challenge due to the complex physical, chemical and biological processes occurring in the soil-climate-vegetation system, and that this is forced by external hydroclimatic factors (Katul et al., 2007; Seneviratne et al., 2010; Asbjornsen et al., 2011; Bartlett et al., 2015; Zhang et al., 2016b; Dymond et al., 2017; Haberreiter et al., 2017; Shan et al., 2019; Wang et al., 2019). Moreover, most of the efforts have been made for the sake of describing the phenomena in water-limited ecosystems. The models applicable in humid regions do not usually have analytical solutions and depend on many parameters.

The main objective of this thesis is to advance in the understanding of climate-soil-vegetation system dynamics, both in water- and energy-limited ecosystems, since it is greatly useful in many study areas, such as those related to landslides, agricultural production, plant health, prediction of floods and droughts, water availability, the concentration of greenhouse gases, erosion, transport of solutes, biochemical cycles, among others (Pelletier et al., 1997; Rodríguez-Iturbe and Porporato, 2004; Daly and Porporato, 2005; Ray and Jacobs, 2007; Talebi, 2008;
Gebremichael et al., 2009; Seneviratne et al., 2010; Feng et al., 2012; Ponziani et al., 2012; Stokes et al., 2014; Posner and Georgakakos, 2015; Zhao et al., 2019). To reach this objective I study and model the stochastic behavior of the photosynthetically active radiation (PAR) and analyze its effects on evapotranspiration and soil water dynamics. Then, I develop an application to landslides triggered by rainfall, looking to understand the relationship between the factor of safety, soil water content and precipitation. I select this application since it is one of the most frequent natural phenomena, and results in large economic losses and human casualties (Schuster, 1996; Crosta, 1998; Kjekstad and Highland, 2009; Klimeš and Escobar, 2010; Arnone et al., 2011; Robbins, 2016; Petley, 2012; Froude and Petley, 2018; Wang et al., 2018).

The analyses are initially based on the ecohydrological model of Rodríguez-Iturbe et al. (1999) and Laio et al. (2001), and then I propose an extension of this model towards the representation of the stochastic dynamics of soil moisture in both water- and energy-limited environments. I study the dynamics of the soil-climate-vegetation at the daily time scale since it allows us to model soil moisture in a physical, simple and parsimonious way, avoiding the sub-daily variation of evapotranspiration, photosynthesis, and rainfall storms (Rodríguez-Iturbe and Porporato, 2004; Daly and Porporato, 2005). Furthermore, this time scale permits to easily consider a canopy spatial scale, since it admits to assume some canopy properties constant, such as the leaf area index (Leuning et al., 1995).

Among the most notable contributions of this thesis are: i) the analysis and modeling of the stochastic component of the PAR in different sites around the world, associating its spatial patterns with the climate type through the Holdridge life zones and the Köppen classification, ii) the study of the effect of radiation and its stochasticity in the dynamics of soil moisture and evapotranspiration, iii) a semi-analytic solution of the probability density function of evapotranspiration dynamics forced by stochastic rainfall and radiation, iv) the extension of the ecohydrological model by Rodríguez-Iturbe et al. (1991a) and Laio et al. (2001) to energy-limited environments and its application under various scenarios, and v) the analysis of the effect of the soil moisture dynamics on the factor of safety (FS) of a theoretical slope, concluding that a certain value of FS can be reached as a result of rainfalls with very different characteristics. The above shows a possible limitation of the widely used landslide threshold approach.

This thesis is structured as follows: the second chapter describes and models the stochastic component of radiation, the third chapter compares the soil moisture and evapotranspiration dynamics when soil-climate-vegetation is forced by deterministic and stochastic radiation and proposes a semi-analytic equation based on the Laio et al. (2001) model to describe evapotranspiration forced by stochastic radiation. The fourth chapter describes the relationship between evapotranspiration and available energy, and its parametrization, taking into account the photosynthesis and transpiration coupling. Also, this chapter shows an extension of the Rodríguez-Iturbe et al. (1999) and Laio et al. (2001) model for energy-limited ecosystems. The fifth chapter applies the ecohydrological model to the occurrence probability of landslides.
triggered by rainfall. The last chapter presents general conclusions, recommendations, and potential future work.

The third and fourth chapters derived from the visiting scholars accomplished at the Texas A&M University and the Politecnico di Torino, with professors Ignacio Rodríguez-Iturbe, Luca Ridolfi, Stefania Tamea and Francesco Laio. Professor Germán Poveda also contributed to the fourth chapter.
2. Stochastic daily solar irradiation at Earth’s surface

Abstract: I analyze and model the stochastic component of daily solar radiation at the Earth’s surface in the photosynthetically active spectral band. The stochastic component is analyzed through the clearness and the clear-sky indices, and the empirical probability densities, conditioned on rainfall occurrence and clearness and clear-sky seasons, are fitted to Beta distributions. The results of 28 sites located around the world indicate spatial patterns in the stochastic component related to geographical location and climate type. Northern extratropics show bimodal distributions, humid tropical regions have unimodal distributions with low dispersion and arid to sub-humid climates show unimodal distributions with high dispersion. When data are not discriminated by the seasons of the indices, the information given by the clearness and the clear-sky indices is very similar, but when the seasons are defined, their duration change with the occurrence of rainfall.

2.1. Introduction

Solar irradiation drives physical, chemical and biological processes at the Earth’s surface, being a fundamental variable in the development of human activities, and the energy source of processes such as photosynthesis and evapotranspiration (Hansen, 1999; Remesan et al., 2008; Bojanowski, 2013). Solar irradiance is commonly studied at the daily time scale (e.g., Liu and Jordan, 1960; Bendt et al., 1981; Elizondo et al., 1994; Meek, 1997; Hansen, 1999; Ibañez et al., 2002; Hassan et al., 2018) since it is an important input for agronomic, ecological, and hydrological models.

Earth’s surface irradiation has been commonly analyzed through physical and statistical models (e.g., Bendt et al., 1981; Forero et al., 2007; Podestá et al., 2004). Physical models study the physical processes occurring in the atmosphere, while statistical approaches study the observed frequency distribution of solar irradiance and its relationships with other meteorological or local variables, such as topography, air temperature, relative humidity, and precipitation (e.g., Richardson, 1981; Larsen and Pense, 1981; Wilks, 1999; Hansen, 1999; Wang et al., 2002; Kambezidis and Psiloglou, 2008; Şahin and Şen, 2008). Irradiation at the Earth’s surface comprises stochastic and deterministic components. The deterministic component has an astronomical origin, being an inherent function of the day (Ianetz and Kudish, 2008), while the stochastic component is strongly affected by atmospheric constituents (Goh and Tan, 1977;
such as molecular gases, aerosols, water vapor and clouds; that absorb, reflect and diffuse (scattering) the extraterrestrial irradiation (Ehnberg and Bollen, 2005; Tovar-Pescador, 2008; Bojanowski, 2013). Atmospheric constituents and their concentration vary with geometric location, elevation and season. However, widely used standard atmospheres (e.g., ICAO Standard Atmosphere, ARDC Model Atmosphere, and U.S. Standard Atmosphere) greatly simplify the problem.

The stochastic component of irradiation is commonly analyzed using the clearness index ($k$) (e.g., Liu and Jordan, 1960; Gordon and Hochman, 1984; Bendt et al., 1981; Skartveit and Olseth, 1992; Hansen, 1999; Ianetz and Kudish, 2008; Harrouni, 2008; Tran, 2013; Allen et al., 2006) and the clear-sky index ($c$) (also named relative clearness index, clear day index, and normalized clearness index) (e.g., Olseth and Skartveit, 1984; Polo et al., 2008; Ianetz and Kudish, 2008; Hollands and Suehrcke, 2013; Engerer and Mills, 2014). The clearness index relates the measured irradiation at Earth’s surface and the irradiation when there is no atmosphere, while the clear-sky index relates the measured irradiation and the cloudless-sky irradiation at Earth’s surface. These indices can be used at different time scales, and generally taking into account the irradiance on specific spectral bands, such as shortwave and ultraviolet (e.g., Martinez-Lozano et al., 1999; Cañada et al., 2003; Utrillas et al., 2018), or dividing global irradiance into diffuse and direct (e.g., Gordon and Hochman, 1984; Ibañez et al., 2002; Assunção et al., 2003; Tovar-Pescador, 2008; Ineichen, 2016). The clearness index has been fitted to different probabilistic density functions (pdf) among which are gamma (e.g., Larsen and Pense, 1981), beta (e.g., Larsen and Pense, 1981; Assunção et al., 2003), and Gaussian (e.g., Goh and Tan, 1977). Hansen (1999) emphasizes the non-normality of daily distributions in the United States.

In this chapter is presented a methodology based on the clearness and clear-sky indices to analyze and model the stochastic behavior of daily irradiation at the Earth’s surface in the photosynthetically active radiation (PAR) spectral band. The analysis of this spectral band is very useful in the health of plants, photosynthesis, and transpiration (Lhomme, 2001; Gueymard and Myers, 2008). $k$ and $c$ are calculated using the Solar Spectral Irradiance (SSI) time series, the U.S Standard Atmosphere, and measured radiation at the Earth surface data on 28 sites around the world. The probability density functions of $k$ and $c$ are analyzed through the Holdridge life zones and Köppen climate classification. The seasons of $k$ and $c$ are defined conditioned on the occurrence of rainfall (as a proxy of cloudiness and water vapor). Finally, $c$ and $k$ are fitted to beta distributions discriminated by rainfall occurrence and the seasons of the indices.

The objective of this chapter is to advance in the understanding of the radiation dynamics in the PAR spectral band, looking for spatial patterns in the statistical behavior of the stochastic component and modeling it in a simple way allowing mathematical treatability.
2.2. Data

I use daily observations of incoming Photosynthetic Photon Flux Density (PPFD) and rainfall from 28 sites around the world (see map in Fig. 2-1) from the FLUXNET dataset (Baldocchi et al., 2001; Olson et al., 2004). I select these sites for their location and the availability of data. However, the sites located in the tropics have much less information than extratropical northern sites (see https://fluxnet.fluxdata.org/sites/site-summary/), and extratropical southern sites often have no PAR measurements. PPFD is transformed into PAR through the relation 4570 nmol m$^{-2}$ s$^{-1}$=1 W m$^{-2}$ (Sager and McFarlane, 1997). Information about selected sites is in Table 2-1.

I also use the Solar Spectral Irradiance (SSI) at the top of the atmosphere from the “First European Comprehensive Solar Irradiance Data Exploitation project” (SOLID) (Schöll et al., 2016; Haberreiter et al., 2017). This historical time series has a daily time resolution and covers the wavelengths range between 0.5 and 1991.5 nm (the spectral irradiance is integrated on the spectral domain between 400 and 700 nm). The SOLID data-set spans from 1978/7/11 to 2014/12/31 (13204 days). Data from SOLID can be accessed at http://projects.pmodwrc.ch/solid. Information of Köppen classification and Holdridge life zones is taken from Rubel and Kottek (2010) and NEP-WCMC (https://www.unep-wcmc.org/resources-and-data/holdridges-life-zones), respectively.
Table 2-1.: FLUXNET Sites. The record period refers to complete calendar years, i.e. data for all sites start on January 1st of the initial year and end on December 31st of the last year.

<table>
<thead>
<tr>
<th>Site</th>
<th>Country</th>
<th>Latitude [°]</th>
<th>Longitude [°]</th>
<th>Elevation [m]</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT-Neu</td>
<td>Austria</td>
<td>47.117</td>
<td>11.318</td>
<td>970</td>
<td>2002 - 2012</td>
</tr>
<tr>
<td>BE-Lon</td>
<td>Belgium</td>
<td>50.552</td>
<td>4.746</td>
<td>167</td>
<td>2004 - 2014</td>
</tr>
<tr>
<td>BE-Vie</td>
<td>Belgium</td>
<td>50.305</td>
<td>5.998</td>
<td>493</td>
<td>1996 - 2014</td>
</tr>
<tr>
<td>BR-Sa3</td>
<td>Brazil</td>
<td>-3.018</td>
<td>-54.971</td>
<td>100</td>
<td>2000 - 2004</td>
</tr>
<tr>
<td>CA-Oas</td>
<td>Canada</td>
<td>53.629</td>
<td>-106.198</td>
<td>530</td>
<td>1996 - 2010</td>
</tr>
<tr>
<td>CG-Tch</td>
<td>Congo</td>
<td>-4.289</td>
<td>11.656</td>
<td>82</td>
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2.3. Methods

Daily radiation amount at a site on the Earth’s surface is the integral of instantaneous irradiance over the day length. Instantaneous irradiance comprises solar irradiance at the top of the atmosphere, solar declination, and physical properties of the atmosphere and locality. Astronomical geometrical relations are well-known and easy to calculate. Beer-Lambert law governs light attenuation through the atmosphere. Yet, some atmospheric components are highly variable in space and time. The clear-day index \(c\) and the clearness index \(k\) are often used to account for the effect of the variable atmospheric components. I use the definitions of \(k\) and \(c\) expressed as:

\[
k = \frac{H}{H_0}, \quad c = \frac{H}{H_{cda}},
\]

where \(H\) and \(H_0\) are the surface and extraterrestrial daily global radiations on a horizontal surface, and \(H_{cda}\) is the daily global radiation on the ground for a cloudless-sky, clean and dry atmosphere. The clearness and the clear-sky indices are calculated in the PAR wavelength domain \((c_{PAR}\) and \(k_{PAR}\)\) as:

\[
c_{PAR} = \frac{PAR_{obs}}{PAR_{cda}}, \quad k_{PAR} = \frac{PAR_{obs}}{PAR_0}.
\]

where \(PAR_{obs}\) is the daily observed values from the FLUXNET data-set, \(PAR_0\) is calculated from the SOLID data and \(PAR_{cda}\) as explained in section 2.3.1. The wavelength domains in FLUXNET for PAR is 400-700 nm (Olson et al., 2004).

2.3.1. Daily surface radiation for a cloudless-sky, clean dry atmosphere

To calculate the daily direct and diffuse PAR on a horizontal surface \((H_b\) and \(H_d\)) for a cloudless-sky, clean and dry atmosphere, I model the direct and diffuse instantaneous irradiances, and integrate them along the day length and the PAR spectral domain. Then, I calculate the global daily radiation on a horizontal surface as \(H = H_b + H_d\) (see Fig. 2-2).

Following Iqbal (1983), I assume the clean dry atmosphere to be composed of uniformly mixed gases \((g)\) and ozone \((o)\). \(H_b\) is calculated by

\[
H_b = \int_{\gamma_{sr}}^{\gamma_{ss}} \int_{\lambda_0}^{\lambda_1} I_{0,n,\lambda} \sin(\gamma) \tau_{ma,\lambda} \, d\lambda \, d\gamma,
\]

where \(I_{0,n,\lambda}\) is the extraterrestrial spectral irradiance normal to the rays from the sun (obtained from SOLID), \(\gamma\) is the solar altitude varying from sunrise \((sr)\) to sunset \((ss)\) and \(\tau_{ma,\lambda}\) is
the transmittance due to absorption of the atmosphere. For the assumed atmosphere composition \( \tau_{ma,\lambda} = \tau_o \cdot \tau_g \), where \( \tau_o \) is the ozone transmittance, \( \tau_g \) is the mixed gases transmittance, \( \lambda \) is the wavelength and \( ma \) the air mass (see details in Iqbal (1983, Sec.6.14). \( H_d \) is calculated by

\[
H_d = \int_{\gamma_{ss}}^{\gamma_{sr}} \int_{\lambda_0}^{\lambda_1} I_{0,n,\lambda} \sin(\gamma) \tau_{ma,\lambda} \left[ 0.5(1 - \tau_{r,\lambda}) \right] \, d\lambda \, d\gamma,
\]

(2-4)

where \( \tau_{r,\lambda} \) is the transmittance due to Rayleigh molecular scattering (see details in Iqbal (1983, Sec.6.16)).

Several atmospheric parameters are required by Eqs. 2-3 and 2-4. I assume the 1976 U.S. standard atmosphere (NASA, 1976) (sea level pressure of 101.325 kPa, sea level temperature of 288 K and sea level density of 1.225 kg/m\(^3\)), and the Kasten and Young (1989, Table II) optical air mass function of solar altitude. The Kasten and Young (1989) table for optical air mass has 336 values of solar altitudes between 0° and 90°, corresponding approximately to half a day. Hence, the numerical solution of the integrals in Eqs. 2-3 and 2-4 have approximately \( 2 \times 366 \) data points. Transmittance for ozone (\( \tau_o \)), mixed gases (\( \tau_g \)) and due to Rayleigh molecular scattering (\( \tau_r \)) are calculated for each wavelength (\( \lambda \)) by:

\[
\tau_{o,\lambda} = \exp(-k_o \lambda m_r),
\]

(2-5)

\[
\tau_{g,\lambda} = \exp \left[ \frac{-1.41k_g\lambda m_a}{(1 + 118.93k_g\lambda m_a)^{0.45}} \right],
\]

(2-6)

\[
\tau_{r,\lambda} = \exp(0.008735\lambda^{-4.08} m_a),
\]

(2-7)

where \( m_r \) is relative air mass at standard pressure, \( m_a \) is relative air mass at actual pressure,
$k_o$ and $k_g$ are the absorption attenuation coefficient for oxygen and mixed gases and $l_o$ is the amount of ozone in cm (at normal temperature and pressure, NTP). I calculate $k_{o,\lambda}$ for any $\lambda$ value using the Leckner (1978) interpolation of the classic Vigroux (1953) data. For calculating $l_o$, I interpolate the Table 5.3.2 from Iqbal (1983), that is a reproduction of Robinson (1966, p.114). $k_{o,\lambda}$ is calculated by interpolating Table 6.13.1, which is a reproduction of Table 4 in Leckner (1978, p.146).

### 2.3.2. Statistics of the clearness and clear-sky indices

Daily time series, annual cycles, autocorrelograms and empirical pdfs of $k_{PAR}$ and $c_{PAR}$ are calculated from the FLUXNET data and the global daily radiation calculated by the physical model described in section 2.3.1. Clearness and clear-sky indices data are separated by rainy and dry days (days with and without precipitation according to the FLUXNET data), using precipitation as a proxy of cloudiness and water vapor in the atmosphere. Then, I define $c$ and $k$ seasons by comparing the cumulative distribution function (CDF) of each month with the CDFs of the other months. This comparison is carried out visually and tested by using Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) Goodness of Fit Tests (Pearson, 1900; Dodge, 2008). Finally, I fit data discriminated by rainy and dry days and by $k$ and $c$ seasons to a Beta distributions, and test the fittings by using Chi-square Goodness of Fit Tests (Dodge, 2008; Pearson, 1900) with a significance level of 0.05 and by Q-Q plots. The beta distribution is selected because of the doubly limited domain that makes it good for modeling these indices. Furthermore, it is roughly analytically treatable due to its mathematical form (exponential).

### 2.4. Results and Discussion

Clearness and clear-sky indices relate measured and potential radiation at the Earth’s surface when there is no atmosphere, and when there is a cloudless-sky and clean dry atmosphere, respectively. Although their values are expected in the range $[0,1]$, $c$ and $k$ time series and their annual cycles show values greater than 1 (see Appendix B). These values are found mostly in sites with seasonal snow, during the periods in which it occurs. This can be explained by the multiple refractions of light of the snow, and also by measurement errors, albedo changes, and the cloudless-sky and clean dry atmosphere model.

Fig. 2-3 shows the autocorrelogram functions (ACF) of PAR, $c_{PAR}$ and $k_{PAR}$ of DE-Hai, MY-PSO and SN-Dhr. The photosynthetically active radiation (PAR) series have a more marked annual cycle than $c_{PAR}$ and $k_{PAR}$ in sites located in the extratropics (see Fig. 2-3(a-c)), while in the most tropical sites, the indices and PAR seasonalities are very similar (see Fig. 2-3(d-f)). ACF of the sites located in or near tropical latitudes indicates very low autocorrelation. As clearness and clear-sky indices remove the astronomical seasonality, sites in which it plays an important role, such as in extratropical latitudes, have indices with autocorrelations less strong than PAR (as in DE-Hai). Indeed, by removing the astronomical seasonality, the
2.4 Results and Discussion

Figure 2-3: Autocorrelograms of PAR, $k_{PAR}$ and $c_{PAR}$ of (a-c) DE-Hai, (d-f) MY-PSO and (g-i) SN-Dhr. Dotted lines indicated the confidence intervals.

The clearness index ($k$) has higher autocorrelations than the clear-sky index ($c$) in most sites (see Fig. 2-3(a-c)). The atmosphere is not taken into account to calculate $k$, but to obtain $c$ it is assumed a cloudiness-sky and clean dry atmosphere. Given the above, high autocorrelations of $k$ indicate that the atmosphere (specifically the air mass) has seasonality, and this index does not manage to remove it (Ianetz and Kudish, 2008). Autocorrelogram functions of PAR, $k$ and $c$ show a period of approximately 180 days for all sites studied, except for BR-Sa3 that it is 120 days (for both PAR and indices) and for the indices in AT-Neu where it is 80 days. As seasonality depends largely on the movement of the sun, the periods in sites nearest to the geographical Equator should be shorter than those of the extratropics, as in BR-Sa3. However, seasonality is also a function of local factors, that are not treated in this analysis. The differences in the periods of PAR and its indices in AT-Neu can be explained because climatic and astronomical seasonalities are out of phase.

Fig. 2-4 shows the relationship between $c$ and $k$ of DE-Hai, MY-PSO, and SD-Dem. From Eqs. 2-1, the slope of this relationship ($m$) is given by $H_0/H_{cda}$. $m$ is always greater than 1 since $H_0$ does not consider the absorption by atmosphere components, having a higher value than $H_{cda}$ that does it. Sites in the extratropical northern hemisphere show a notable variability of $m$ throughout the year (see Fig. 2-4(a)), while in tropics the dispersion is very low (Fig. 2-4(b,c)).
mass (purple lines) for different latitudes. $E_0$ denotes the sun-earth distance for any day of the year, indicating the potential energy arriving at the earth’s surface when there is no atmosphere. In the extratropical latitudes, the relative air mass ($m_a$) has a great variation, while in the tropics it is almost constant (dotted line). The $m_a$ changes explain the high dispersion of $m$ in sites located in high latitudes and the low dispersion in sites near the equator. Each color dots in Fig. 2-4 indicates a month of the year, black dots represent the days in January, green dots the days in July and red dots the days in December. In the extratropical northern, the greatest differences between $c$ and $k$ occur at the beginning and the end of the year, since the path length that light rays must pass through is longer than in the middle of the year. Graphics of $k$ vs. $c$ of all sites are in Appendix F.

Fig. 2-6 shows the empirical pdfs of $c$ and $k$ of sites in Germany (DE-Hai), Malaysia (MY-
2.4 Results and Discussion

Figure 2-6: Histograms of $c_{PAR}$ and $k_{PAR}$ of (a-b) DE-Hai, (c-d) MY-PSO and (e-f) SN-Dhr. Blue bars are the rainy days and red bars the dry days.

PSO), and Sudan (SN-Dhr). All pdfs reveal a certain degree of bimodality, or at least, some asymmetry in respect to the mean, as mentioned before by Hollands and Suedecke (2013); Tovar-Pescador (2008); Assunção et al. (2003); Ibáñez et al. (2003); Jurado et al. (1995); Skartveit and Olseth (1992); Olseth and Skartveit (1984) (see Appendix C). This figure also shows that the pdfs of $c$ and $k$ have similar shapes, during both rainy and dry days. I classify the pdfs of $c$ and $k$ in three types: Bimodal, Unimodal I (unimodal with low dispersion), and Unimodal II (unimodal with high dispersion). Sites with bimodal distributions are located in the extratropical northern hemisphere (see Fig. 2-6(a-b)), with the exception of a site in the United States (US-Fep), sites with Unimodal II are located in tropics and subtropics (see Fig. 2-6(c-d)), and Unimodal I in tropics (see Fig. 2-6(e-f)). The same behavior is observed for the two indices ($k$ and $c$). The above let us suspect that in high latitudes the behavior of both indices is bimodal, in low latitudes it is unimodal with low dispersion, and in mid-latitudes it is unimodal with high dispersion. The bimodality in the extratropics may be related to the notable differences in the atmosphere conditions in the various climatic seasons.

Looking for spatial patterns in the pdf shapes of daily $c_{SW}$ and $k_{PAR}$, the sites are classified following the Köppen classification and the Holdridge life zones (Holdridge, 1947, 1967). According to the Köppen classification, bimodality occurs in the oceanic, humid continental and Mediterranean climates, unimodal pdfs with low dispersion in the tropical monsoon, tropical savanna and tropical rain-forest climates, and unimodal pdfs with high dispersion in the semi-arid, temperate, subtropical humid and desert climates. Holdridge life zones have the advantage of allowing to analyze schematically the link between climate and long-term behavior of $c$ and
as shown in Fig. 2-7. Red symbols represent the sites with bimodal pdfs, yellow symbols the unimodal I pdfs, and purple symbols the unimodal II pdfs. Bimodality is concentrated in the cool temperate and boreal regions on subhumid and humid provinces, Unimodal II in humid tropical and warm temperate or subtropical regions, and Unimodal I in tropical and warm temperate or subtropical, in humidity provinces between arid and subhumid. US-Fpe has Unimodal I pdf and is located in the cool temperate region, but it continues to stay on the same range of humidity provinces as the other sites with this type of pdf. Sites as those in Belgium show slightly different behaviors (moist forest and wet forest zones), even if they are located very close. This can be attributed to local conditions. The above allows inferring the high influence of the type of climate in the statistical behavior of $c$ and $k$.

I noted that in the moist forest area with humid provinces between sub-humid and humid, and regions between cool temperate to warm temperate or subtropical, sites with the three defined pdfs types converge. It may occur since each zone comprises a wide range of annual precipitation and potential evapotranspiration, and these or other variables can result in completely different behaviors of the stochastic component of radiation. Besides, this occurs in this life zone and not in the others, since in the moist forest zone, the three regions (red, yellow and purple) that concentrate each type of pdf come together.

Because of bimodality is attributed to clear and overcast skies conditions (Tovar-Pescador, 2008; Olseth and Skartveit, 1984), by separating the values of $c$ and $k$ into rainy and dry days, bimodal pdfs are split into two unimodal ones, with the exception of AT-Neu, DE-Geb, and DE-Hai (see Fig. 2-6(a-b)), where the pdfs of dry days continue to have a bimodal distribution. This can be explained by the presence and absence of clouds on dry days since the relationship between rainfall and clouds is not always direct. Data for rainy and dry days are divided by months to define the $c$ and $k$ seasons. Fig. 2-8 shows the cdf of each month for rainy (blue lines) and dry (red lines) days of DE-Hai, MY-PSO, and SN-Dhr. The seasons of the clearness and the clear-sky indices are defining by using these curves and the comparison matrices that are shown in Appendix D. The comparison matrices test each monthly cdf with the cdfs of the other months by KS and AD Goodness of Fit Tests, and p-values are plotted into their cells. The definition of the seasons of $k$ and $c$ is very clear in extratropical latitudes sites since the cdfs of months with low values of the indices are notoriously thrown to the left side, and the cdfs of other months to the right side, both for rainy and dry days (see Fig. 2-8(a)). The matrices of comparison corroborate it. Nevertheless, in sites in or near the tropics, the monthly cdfs are not markedly grouped (see Fig. 2-8(e)), and matrices of comparison do not indicate clear patterns.

In most sites, it is possible to define two seasons: Season 1 (Season 2) includes the period in which high (low) values of $k$ and $c$ are more likely. In the extratropical northern hemisphere, Season 1 (Season 2) occurs approximately between November (March) and February (September). We note that in many sites there is a transition between seasons, but in this analysis, it is not taking into account. Furthermore, at most sites located in or near the tropics, there is no precise visual differentiation of seasons, so we define only the seasons clearly shown in the
Figure 2-7: Holdridge’s life zones. Red symbols represent the sites with bimodal pdfs, yellow symbols the unimodal I pdfs, and purple symbols the unimodal II pdfs. After FAO (2012).
Figure 2-8.: Monthly cdfs of $c_{PAR}$ and $k_{PAR}$ for rainy (blue curves) and dry (red curves) days of (a-b) DE-Hai, (c-d) MY-PSO and (e-f) SN-Dhr.

comparison matrices. Table 2-2 shows the beginning and the end of $c$ and $k$ seasons for the sites analyzed.

The beginning and the end of seasons do not change notably with $c$ and $k$, but they do with the occurrence or non-occurrence of rainfall. In some sites in Italy (IT-MBo) and the United States (US-Esm and US-NC2), the seasons of $c$ and $k$ are the same, while in the other sites, Season 1 is usually shorter than Season 2 for dry days. When analyzing the differences in the beginning and end of seasons according to the Holdridge life zones, there are no observed clear patterns. This may indicate that these differences are due to local factors.

Fig. 2-9 shows the empirical and the fitted beta cdfs of DE-Hai, MY-PSO, and SN-Dhr. Beta distribution fits better $k$ and $c$ for rainy days (see Appendix E), since dry days tend to have a distribution concentrated in the mode, and this type of pdf does not work well when dispersion is low. Furthermore, the same problem occurs in several sites located near the tropics, where the variability of the indices is low regardless of the seasons of $c$ and $k$ and the rainfall occurrence. Because in these cases the standard deviation is very low, it is reasonable to adopt $c_{PAR}$ and $k_{PAR}$ as constants. $\chi^2$ test parameters, beta parameters ($\beta_1$ and $\beta_2$), and Q-Q plots are shown in Appendix E.
### Table 2-2.: Beginning and end of $k$ and $c$ seasons.

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<th>Dry Season 1</th>
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</table>
2.5. Summary and Conclusions

I have analyzed the stochastic behavior of photosynthetically active radiations through the clearness \((k)\) and clear-sky \((c)\) indices. Both indices remove the astronomical seasonality, but \(c\) also removes the seasonality of the optical mass, describing in a cleaner way the effect of clouds, water vapor, aerosols, pollution, and the other stochastic components of the atmosphere. Due to the multiple refractions of light by snow, \(c\) and \(k\) can have values greater than 1 in sites with seasonal snow, during the periods in which it occurs.

By locating the sites in Table 2-1 into the Holdridge life zones scheme (see Fig. 2-7), I noticed that the statistical behavior of \(c\) and \(k\) is highly influenced by the climate type, showing bimodal distributions in sites located in the extratropical latitudes, unimodal distributions with low dispersion in sites close to the humid tropical regions, and unimodal distributions with high dispersion in sites with arid to subhumid climates. In most of the sites with indices following bimodal pdfs, bimodality vanishes when data is divided by rainy and dry days as a proxy of cloudiness and water vapor, corroborating that each mode is related with clear and overcast skies conditions. On the seasonality of \(k\) and \(c\), we found a clear definition of the seasons of \(c\) and \(k\) in extratropical sites, since the climatic seasons are well defined, unlike tropical sites where the local conditions play a determining role. Variations in the beginning and the end of the seasons of \(c\) and \(k\) do not follow a notorious spatial pattern, and it is possible to see different behaviors in similar geographical locations, which can be also attributed to local conditions.

The shape of the pdfs of \(c_{PAR}\) and \(k_{PAR}\) for all days, and when data are separated by rainy
and dry days, is similar. However, when data are analyzed month by month, and the seasons of $c$ and $k$ are defined, the results vary with the occurrence of rainfall in most sites. The differences between $c$ and $k$ are more accentuated in the extratropical northern hemisphere since the path length that energy must pass through varies considerably during the year. $k$ has higher autocorrelation than $c$, indicating that the air mass of the atmosphere has seasonality, and $k$ does not manage to remove it.

Beta distribution achieves correctly fit the empirical pdfs of $c$ and $k$ in almost all sites, except in some located near the tropics, and several cases of dry days because they have narrow empirical pdf. In these conditions, it is reasonable to assume the indices constant.

This chapter advances in the understanding of the dynamics of radiation on the Earth's surface in the PAR spectral band, showing spatial patterns of the stochastical component of PAR related to the climate types and the Holdridge life zones. Besides, it fits the stochastic component of PAR under seasonally fixed conditions to beta distributions. This type of distribution broadly allows maintaining mathematical tractability. The results obtained in this analysis are useful in study areas related to ecohydrology, meteorology, glaciology, agroclimatology, etc. Furthermore, this methodology can be used in any site and to any radiation wavelengths range between 0.5 and 1991.5 nm, as long as the same processes are maintained.
3. Soil moisture and evapotranspiration dynamics forced by stochastic rainfall and radiation

Abstract: I analyze numerically the effects of radiation stochasticity on soil moisture and evapotranspiration dynamics with the ecohydrological model of Rodríguez-Iturbe et al. (1999) and Laio et al. (2001), and the Priestley-Taylor equation. As the ecohydrological model gives a steady-state probability density function of soil moisture, it is necessary to define periods in which both precipitation and radiation can be considered stationary. Radiation is modeled as proposed in chapter 2. Results of simulations with stochastic and constant radiation show slight differences in soil moisture dynamics and notable variations on evapotranspiration. I propose a semi-analytical solution of evapotranspiration dynamics forced by stochastic rainfall and radiation. This solution is based on the ecohydrological model, the Bayes’ theorem, the Priestley-Taylor equation and, the derived distribution technique. The methodologies are applied in the same sites studied in chapter 2.

3.1. Introduction

Stochastic fluctuations play a crucial role in many dynamical physics, economy and biology systems (Jacobs, 2010; Cai and Lin, 1992; Cook, 1985; Barcons and Garrido, 1983; Benzi et al., 1982; Nicolis, 1982; Hasselmann, 1976). Fluctuations are divided into internal and external. Internal fluctuations are inherent to the system, while the external ones are due to the environmental randomness (Horsthemke and Lefever, 2006; Berger, 1988; Schenzle and Brand, 1979; Robock, 1978). External fluctuations can have very different behaviors according to their temporal regime and intensity distribution, moreover, multiple types of fluctuations may coexist (Daly and Porporato, 2006b; Koutsoyiannis, 2003; Berger, 1988).

The climate-soil-vegetation is an extensively studied system affected by environmental fluctuations. Soil water content is a key player in this system, and its interaction with climate conditions is mainly controlled by rainfall \( P \) and evapotranspiration \( ET \). \( P \) is the input on the soil water balance, while \( ET \) is the most significant soil water loss mechanism (Daly and Porporato, 2006a). Actual evapotranspiration strongly depends on available water and energy, since plants close their stomata in the absence of water and its rate is regulated by the energy (Monteith, 1995). Available energy is directly related to solar radiation at the Earth’s surface,
which has a marked seasonality and a stochastic behavior given by the aerosols (that absorbs and scatter), gases that compose the atmosphere, clouds, etc.

As long as the processes occurring in the climate-soil-vegetation system are those shown in Fig. 3-1, and rainfall and radiation are the fluctuations forcing the system, soil moisture dynamics is expressed by the stochastic differential equation (Rodríguez-Iturbe et al., 1999):

$$nZ_r \frac{ds(t)}{dt} = P(t) - I(t) - Q[s(t),t] - ET[s(t),R(t)] - L[s(t)], \quad (3-1)$$

where $s$ is the soil water content, $n$ is the soil porosity, $Z_r$ is the rooting depth, $s$ is the soil water content, $P(t)$ is the rainfall rate, $I(t)$ is the rainfall rate intercepted by canopy, $Q[s(t),t]$ is the rate of surface runoff generation, $ET[s(t),R(t)]$ is the evapotranspiration rate, $R$ is the available radiation, $L[s(t)]$ is the deep infiltration or leakage rate, and $t$ is the time.

Figure 3-1.: Representation of mechanisms occurring in the climate-soil-vegetation system. After Laio et al. (2001).

Eq. 3-1 forced by daily stochastic rainfall has been analytically studied since Eagleson (1978e) (e.g. Daly and Porporato, 2006a; D’Odorico and Porporato, 2004; Laio et al., 2002, 2001; Milly, 2001; Rodríguez-Iturbe et al., 1999; Milly, 1993), but in all these models, except in the model
of Daly and Porporato (2006a), only rainfall is modeled as a stochastic process (shot noise). Daly and Porporato (2006a) analytically solved a simplification of Eq. 3-1 when both rainfall and ET are stochastic. P is also modeled as a shot noise as Rodríguez-Iturbe et al. (1999) while ET is modeled as the mean value plus a white Gaussian noise. By comparing the pdf of s for deterministic and stochastic ET, they found only small variations in the variance, while shapes and means remained very similar.

The objective of this chapter is to evaluate the effect of the stochastic component of radiation (R) in soil moisture (s) and evapotranspiration (ET) dynamics. For this, I simulate numerically the dynamics of both variables by using the models of infiltration and water losses proposed by Rodríguez-Iturbe et al. (1999) and Laio et al. (2001), without the simplifications made by Daly and Porporato (2006a). Taking into account the results found by Daly and Porporato (2006a), I focus on the behavior of evapotranspiration when it is forced by stochastic radiation and rainfall. The Laio et al. (2001) equation is used to relate ET and soil moisture, and the Prestley-Taylor equation to relate ET and R. Radiation is described as the multiplication of a value representing the mean of seasonal radiation at the top of the atmosphere (TOA), and a stochastic value given by the model presented in chapter 2. Finally, a semi-analytical model of evapotranspiration dynamics forced by stochastic rainfall and radiation is proposed. This model is based on the derived distribution technique and the model of Rodríguez-Iturbe et al. (1999).

### 3.2. Data

I carry out the analysis for the sites in Table 2-1 using daily observation of precipitation, pressure, and temperature data from the FLUXNET dataset (Baldocchi et al., 2001; Olson et al., 2004). The radiation at the top of the atmosphere (PAR0) is that obtained from the SSI (see chapter 2.3), and the pdfs of cPAR are those shown in Annex E. For simulations, I assume grass vegetation cover and loam and loamy sand soils and use the parameter values published by Ridolfi et al. (2003). Soil parameters are in Table 3-1, Zr = 30 cm and ∆ = 1 cm.

### 3.3. Methods

The effects of radiation on soil moisture and evapotranspiration dynamics are numerically model by using the ecohydrological model of Rodríguez-Iturbe et al. (1999) modified by Laio.

---

Table 3-1: Soil parameters.

<table>
<thead>
<tr>
<th>Soil</th>
<th>( \bar{\psi}_s ) [kPa]</th>
<th>b</th>
<th>( k_s ) [cm d(^{-1})]</th>
<th>n</th>
<th>( \beta )</th>
<th>( s_h )</th>
<th>( s_w )</th>
<th>( s^* )</th>
<th>( s_{fc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loamy sand</td>
<td>-0.17</td>
<td>4.48</td>
<td>100</td>
<td>0.42</td>
<td>12.7</td>
<td>0.08</td>
<td>0.10</td>
<td>0.24</td>
<td>0.52</td>
</tr>
<tr>
<td>Loam</td>
<td>-1.43</td>
<td>5.39</td>
<td>20</td>
<td>0.45</td>
<td>14.8</td>
<td>0.19</td>
<td>0.23</td>
<td>0.46</td>
<td>0.65</td>
</tr>
</tbody>
</table>
et al. (2001). The pdfs of $s$ and $ET$ for constant and stochastic radiation are compared and a semi-analytical solution to obtain the pdf that describes the dynamics of $ET$ when $R$ has a stochastic component is proposed. This solution is based on the analytical solution given by the ecohydrological model, the $c_{PAR}$ fittings and the derived distribution technique.

### 3.3.1. Soil water model

Rodríguez-Iturbe et al. (1999) and Laio et al. (2001) proposed a daily stochastic model for soil moisture dynamics at a point under seasonally fixed conditions. Here, their approach is used to model rainfall, interception, infiltration, leakage, and runoff, but evapotranspiration is regarded as a function of soil moisture and available energy, both as stochastic processes. In this section, I briefly describe the stochastic model to analyze the components of Eq. 3-1 and solve it. For a detailed description refer to the book of Rodríguez-Iturbe and Porporato (2004) and the papers by Rodríguez-Iturbe et al. (1999) and Laio et al. (2001).

#### Rainfall and interception

Daily precipitation is modeled through a marked Poisson process with arrival rate $\lambda$ (Eagleson, 1972). The pdf of time intervals ($f_T$) between rainy days $\tau$ is exponential with mean $\lambda^{-1}$:

$$f_T(\tau) = \lambda e^{-\lambda \tau}, \quad \text{for } \tau \geq 0. \quad (3-2)$$

Marks correspond to the rainfall depth of rainy days ($h$) and are modeled as an independent exponentially distributed random variable with mean $\alpha$:

$$f_H(h) = \frac{1}{\alpha} e^{-\frac{1}{\alpha} h}, \quad \text{for } h \geq 0. \quad (3-3)$$

$\alpha$ and $\lambda$ are assumed to be time-invariant quantities during the modeling period (growing season or climate season), i.e., rainfall is considered as a stationary stochastic process. Rainfall rate ($P_n$) is linked to the probability distributions expressed by Eqs. 3-2 and 3-3 as the marked Poisson process (Rodríguez-Iturbe and Porporato, 2004):

$$P_n(t) = \sum_{i=1}^{\infty} h_i \delta(t - t_i), \quad (3-4)$$

where $\delta(\cdot)$ is the Dirac delta function, $h_i$ is the sequence of random rainfall depths distributed as Eq. 3-3 and $[\tau_i = t_i - t_{i-1}, \ i = 1, 2, 3...]$ is the interarrival time sequence of a stationary Poisson process of frequency $\lambda$. The Interception by the canopy is modeled through a threshold
(\Delta) such that only rainfall above \Delta reaches the soil. The censored rainfall is a Poissonian process with rate \lambda':

$$\lambda' = \lambda \int_{\Delta}^{\infty} f_H(h) \, dh = \lambda e^{-\frac{\Delta}{\alpha}}.$$  \hspace{1cm} (3-5)

The depths $h'$ of the censored rainfall process have the same exponential distribution as the original marks $h$ (Rodríguez-Iturbe et al., 1999). Then, the new Poisson process is

$$P_n(t) - I(t) = \sum_{i=1}^{\infty} h'_i \delta(t - t'_i),$$  \hspace{1cm} (3-6)

where $[\tau'_i = t'_i - t'_{i-1}, i = 1, 2, 3,...]$ is the interarrival time sequence of a stationary Poisson process with frequency \lambda', and I is the interception.

**Infiltration and runoff**

Surface runoff is generated via a saturation excess (Dunne mechanism) that occurs when the infiltrated water saturates the soil profile. When rainfall depth is less than or equal to the available soil water storage, all the water from rainfall infiltrates. Infiltration is thus a function of the amount of rainfall and soil moisture, being a stochastic and state-dependent component. Both its magnitude and temporal occurrence are controlled by soil moisture dynamics (Rodríguez-Iturbe and Porporato, 2004).

**Evapotranspiration and leakage**

There is a close link between the atmosphere and vegetation. The sensible and latent heat fluxes from vegetation result in variations in the atmosphere state and, at the same time, plants respond to changes in air temperature and humidity (Monteith and Unsworth, 2013). Plants decrease their rate of transpiration in the absence of light or water so that both radiation and soil moisture are variables directly related to transpiration (Monteith, 1995). Evaporation ($E$) and transpiration ($T$) are governed by different mechanisms, the first one is mainly controlled by abiotic factors, while $T$ also depends on biotic factors such as leaf stomatal control (Larcher, 1995). However, for simplicity, they are typically modeled together and named
evapotranspiration ($ET$). Laio et al. (2001) relate $ET$ and $s$ through the equation:

$$ET (s) = \begin{cases} 
0, & 0 < s \leq s_h \\
E_w \frac{s-s_h}{s_w-s_h}, & s_h < s \leq s_w \\
E_w + (E_{max} - E_w) \frac{s-s_w}{s^*-s_w}, & s_w < s \leq s^* \\
E_{max}, & s^* < s \leq 1.
\end{cases}$$

The term $E_{max}$ represents the maximum evapotranspiration of the vegetation in the presence of unlimited water and can be estimated using physical based expressions such as Penman-Monteith and Priestley-Taylor equations (Rodríguez-Iturbe and Porporato, 2004). $E_{max}$ expresses the evapotranspiration rate for all soil moisture above the point of incipient stomata closure ($s^*$). When $s < s^*$, $ET$ decreases linearly because of limitations of soil moisture, until it reaches the wilting point ($s_w$). Below $s_w$, transpiration ceases and $ET$ decreases at a slower rate ($E_w$) until it reaches the hygroscopic point ($s_h$), where it becomes zero. $E_w$ represents the daily evaporation rate.

As $E_{max}$ can be calculated through equations such as Penman-Monteith, this variable is indirectly related to available energy. There are several models in the literature expressing evapotranspiration in terms of radiation (e.g. Penman, 1947; Priestley and Taylor, 1972; McNaughton and Black, 1973; Shuttleworth and Wallace, 1985). In this analysis, I adopt the Priestley-Taylor (PT) equation because it is the approach widely used in hydrology that requires fewer parameters, and it is not a function of stomatal conductance. Priestley-Taylor equation is expressed by:

$$ET (R) = \frac{1}{\lambda_w \rho_w \Delta_e} \left( \frac{\alpha_{PT} \Delta_e R}{\gamma_p} \right),$$

where $\lambda_w$ is the latent heat of vaporization (I used 2.26 MJ kg$^{-1}$), $\rho_w$ is the water density (I used 998.2 kg m$^{-3}$), $\alpha_{PT}$ is the constant of Priestley-Taylor (1.26), and $\gamma_p$ is the psychometric constant, given by:

$$\gamma_p = \frac{c_p P_a}{\lambda_w e},$$

where $c_p$ is the specific heat at constant pressure (1.013×10$^{-3}$ MJ kg$^{-1}$ °C$^{-1}$), $P_a$ is the atmospheric pressure, and $e$ is the ratio molecular weight of water vapor/air (0.622). I only analyze the effects of stochasticity of $R$ whereby air temperature ($T_a$) and $P_a$ are assumed as constants during the period evaluated. Fig. 3-2 represents $ET (s, R)$ for a particular set of parameter values. According to the PT equation, $ET$ increases linearly with the radiation and the atmospheric saturation deficit. As the slope of saturation of vapor pressure ($\Delta_e$) increases
with air temperature, when the numerator remains constant, $ET$ is negatively related to $T_a$. Unlike Penman-Monteith, PT only considers the diabatic term.

Losses by leakage ($L$) occur when the soil moisture is higher than field capacity ($s_{fc}$). The maximum percolation rate equals the saturated hydraulic conductivity ($k_s$) and decreases rapidly when the soil begins to dry, as expressed by (Laio et al., 2001):

$$L(s) = \frac{k_s}{e^{\beta(1-s_{fc})} - 1} \left[ e^{\beta(s-s_{fc})} - 1 \right], \text{ for } s_{fc} < s \leq 1.$$  \hspace{1cm} (3-10)

where $\beta$ is a fitting coefficient that is a function of the soil type.

**Radiation**

Taking into consideration that vegetation only absorbs energy in the range comprised for the photosynthetically active radiation (PAR) (Dickinson et al., 1986), available radiation for $ET$ is modeled as expressed in:

$$R(c_{PAR}) = \mu_{PAR0} \left[ \lambda' c_{PARw} + (1 - \lambda')c_{PARd} \right],$$  \hspace{1cm} (3-11)

where $\mu_{PAR0}$ is the mean of irradiation at the top of the atmosphere (TOA) in the PAR range, and $c_{PARw}$ and $c_{PARd}$ are random variables representing the clear-sky index for wet and \ldots
dry days, respectively. These variables are modeled as Beta distributions:

\[ f_c(c_{PAR}) = \frac{c_{PAR}^{\beta_1} (1 - c_{PAR})^{\beta_2}}{B(\beta_1 - \beta_2)}, \]

(3-12)

where \( x = w \) or \( d \) in wet and dry days, respectively, and \( B(\beta_1 - \beta_2) = \frac{\Gamma(\beta_1)\Gamma(\beta_2)}{\Gamma(\beta_1 + \beta_2)} \). The fitted parameters \( (\beta_1, \beta_2) \) are those showed in Appendix E. As mentioned in Chapter 2, Beta distribution is selected because of its doubly limited domain, the different shapes that it can adopt, and its roughly mathematical treatability.

It is noticed that Eq. 3-11 assumes the dependence between radiation and precipitation as a weighted average involving the probability of a rainy day \( \lambda \) and the probability of a non-rainy day \( 1-\lambda \). However, this equation results in a different histogram of \( R \) than when radiation is simulated using a random number of the pdf of \( c_{PARw} \) on rainy days, and a random number of the \( c_{PARd} \) pdf on dry days (see Fig.), that is to say:

\[ R(c_{PARx}) = \begin{cases} 
\mu_{PAR0} c_{PARw}, & P > 0 \\
\mu_{PAR0} c_{PARd}, & P = 0 
\end{cases} \]

(3-13)

As seasonality of PAR is not modeled, I simulate periods in which \( PAR_0 \) has slight variations (see section 3.3.2).

**Soil-drying process**

Deterministic decays of soil moisture during inter-storm periods are calculated from the previous wetting and drying historical processes (Laio et al., 2001).
Soil moisture and evapotranspiration dynamics forced by stochastic rainfall and radiation

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Steady-state probability of soil moisture

The pdf of soil moisture \( f_s \) under the steady-state conditions is derived from the Chapman-Kolmogorov forward equation. The general form of the solution is (Rodríguez-Iturbe et al., 1999; Laio et al., 2001; Rodríguez-Iturbe and Porporato, 2004):

\[
 f_s(s) = \frac{C}{\rho(s, R)} e^{-\gamma s + \lambda' \int \frac{du}{\rho(u)}}, \quad \text{for} \quad s \geq s_h, \tag{3-14}
\]

where \( \gamma = \frac{nZ_r}{\alpha} \), \( \rho \) is the sum of the soil moisture losses (ET and L) normalized by \( nZ_r \), and \( C \) is a constant that can be obtained by imposing the normalized condition:

\[
 \int_{s_h}^{1} f_s(s) \, ds = 1. \tag{3-15}
\]

The limits of integration in Eq. 3-14 must be chosen so that there is continuity in \( f_s(s) \) for the different components of the soil moisture losses (Eqs. 3-7 and 3-10). The solution is thus given by:

\[
 f_s(s) = \begin{cases} 
 C & s_h < s \leq s_w \\
 C \eta \left( \frac{s - s_h}{s_w - s_h} \right) (s - s_h) & s_w < s \leq s_{cr} \\
 C \eta \left( 1 + \left( \frac{\eta}{\eta_w} - 1 \right) \frac{s - s_w}{s - s_h} \right) (s - s_w) & s_{cr} < s \leq s_{fc} \\
 C \eta e^{-(\beta + \gamma) s + \beta s_f c} & s_{fc} < s \leq 1,
\end{cases} \tag{3-16}
\]

where \( \eta = \frac{E_{\text{max}}}{nZ_r} \) and \( \eta_w = \frac{E_w}{nZ_r} \), \( T_1 = \lambda' \frac{s_w - s_h}{\eta_w} \), \( T_2 = \lambda' \frac{s - s_w}{\eta - \eta_w} \), \( T_3 = \frac{\lambda'}{\beta (\eta - \eta_w)} \), and \( T_4 = \lambda' \frac{s_f c - s^*}{\eta} \).

### 3.3.2. Definition of stationary periods

In order to avoid modeling rainfall and radiation seasonality, I define periods in which it is realistic to assume the parameters describing those processes as stationary. The criteria to define the periods are different for sites located in the extratropical northern hemisphere and those near or in the tropics. For sites in the extratropical northern hemisphere, the overlap of the seasons of the clear-sky index \( c_{\text{PAR}} \) and the growing season is initially obtained. As in several sites, the photosynthetically active radiation at the top of the atmosphere \( PAR_0 \) varies significantly in this overlap, I define a sub-period in which \( PAR_0 \) has a standard deviation equal or less than 0.5 MJ m\(^{-2}\). The growing season is assumed to occur between April and October, neglecting its variations with latitude, longitude, altitude, and type of crop. As at sites in or near the tropic \( PAR_0 \) has little variations, the dispersion criterion is not necessary.
3.3 Methods

Additionally, the growing season is determined by the rainfall seasons, therefore I define the period to be modeled as the overlap of the seasons of \( c_{PAR} \) and the consecutive months in which the monthly mean amount of rainfall is similar.

### 3.3.3. Numerical simulations

Eq. 3-1 is simulated numerically by using Eqs. 3-2 to 3-11 with constant and stochastic valued of \( R \). \( E_{max} \) is calculated as Eq. 3-8 and \( R \) as Eq. 3-11. When \( R \) is constant, the term \( c_{PAR} (t) \) is replaced by the mean of the pdf describing \( c_{PAR} \), which is given by:

\[
\mu_{c_{PARx}} = \frac{\beta_{1x}}{\beta_{1x} + \beta_{2x}}.
\]  

(3-17)

Simulations have a temporal step of 1/100 days and a length of 500 years.

### 3.3.4. Semi-analytical steady-state probability of evapotranspiration

Using the Bayes’ theorem and the law of total probability, the pdf of \( ET \) can be expressed as:

\[
f_{ET} (ET) = \int_0^{ET} f_{EM} (ET | E_{max}) f_{E_{max}} (E_{max}) dE_{max}.
\]  

(3-18)

The pdf of \( ET \) given \( E_{max} \), i.e., \( f_{EM} (ET | E_{max}) \) is obtaining using the derived distribution technique. This approach corresponds to the change of variables theorem and was initially used in hydrology by Eagleson (Eagleson, 1972, 1978a,b,c,d,e,f,g). The derived distribution technique manages to transform the pdf of one or more independent variables (\( x \)) into the pdf (\( g \)) of a dependent variable (\( y \)) by using the analytical function relating them (\( h(X) \)), as:

\[
G_Y (y) = P_r (h (X) \leq y) = \int_{\{x|h(x)\leq y\}} g_X (x) dx
\]  

(3-19)

where \( G \) is the cdf of the dependent variable and \( P_r \) its cumulative probability.

For this case, the independent variable (\( x \)) is \( s \), the dependent variable (\( y \)) is \( ET \) and the function relating them (\( h(X) \)) is Eq.3-7. The pdf of \( s \) is expressed by Eq. 3-14 gives the pdf of \( s \), and the pdf of \( ET | E_{max} \) by:

\[
f_{EM} (ET | E_{max}) = \begin{cases} 
  C \frac{s_w - s_h}{\eta_w E_w} \left( \frac{ET}{E_{max}} \right)^{T_2 - 1} e^{-\gamma \left( \frac{ET}{E_{max}} \right) + s_h}, & 0 \leq ET < E_w \\
  \frac{C}{\eta_w T_5} \left[ 1 + \left( \frac{n}{n_w} - 1 \right) \left( \frac{ET - E_w}{E_{max} - E_w} \right) \right]^{T_2 - 1} \cdot e^{-\gamma[T_5(ET - E_w) + s_h]}, & E_w \leq ET < E_{max} \\
  \frac{C}{\eta_w} \left( \frac{n}{n_w} \right)^{T_2} \left( e^{-\gamma + \frac{n}{n_w} (1-s^*)} - e^{-\gamma s^*} \right), & ET \geq E_{max}.
\end{cases}
\]
where \( T_5 = \frac{s^*-s_w}{E_{max} - E_w} \).

The pdf of \( E_{max} \) \( (f_{E_{max}}(E_{max})) \) is obtained using the derived distribution technique again. First, I find the pdf of \( R \) (dependent variable) \( (f_r(R)) \) from the pdfs of \( c_{PAR} \) (independent variable) retrieved in chapter 2 and Eq. 3-11 (analytical function relating \( R \) and \( c_{PAR} \)):

\[
f_r(R) = \frac{1}{\text{PAR}_0} \left[ \frac{\lambda'(\frac{R}{\text{PAR}_0})^{\beta_{1w} - 1} (1 - \frac{R}{\text{PAR}_0})^{\beta_{2w}}}{B(\beta_{1w}, \beta_{2w})} + (1 - \lambda') \left( \frac{R}{\text{PAR}_0} \right)^{\beta_{1d} - 1} (1 - \frac{R}{\text{PAR}_0})^{\beta_{2d}} B(\beta_{1d}, \beta_{2d}) \right],
\]

(3-21)

where sub-indices \( w \) and \( d \) indicate the \( c_{PAR} \) distributions during rainy and wet days, respectively. Then, from Eq. 3-21 (dependent variable) and Eq. 3-8 (analytical function relating \( R \) and \( E_{max} \)), the distribution of \( E_{max} \) (dependent variable) is derived as:

\[
f_{E_{max}}(E_{max}) = \frac{c_1}{\text{PAR}_0} \left[ \frac{\lambda'(\frac{c_1E_{max}}{\text{PAR}_0})^{\beta_{1w} - 1} (1 - \frac{c_1E_{max}}{\text{PAR}_0})^{\beta_{2w}}}{B(\beta_{1w}, \beta_{2w})} + (1 - \lambda') \left( \frac{c_1E_{max}}{\text{PAR}_0} \right)^{\beta_{1d} - 1} (1 - \frac{c_1E_{max}}{\text{PAR}_0})^{\beta_{2d}} B(\beta_{1d}, \beta_{2d}) \right],
\]

(3-22)

where

\[
c_1 = \frac{\lambda_v \gamma_w (\Delta_c + \gamma_p)} {\alpha_{PT} \Delta_e}.
\]

### 3.4. Results and Discussion

Fig. 3.3.2 shows the overlap of the seasons of the clear-sky index \( (c_{PAR}) \) defined in chapter 2 and the growing season in a site in Germany (DE-Hai), and the overlap of the seasons of \( c_{PAR} \) and the periods with similar precipitation in sites in Malaysia (MY-PSO) and Sudan (SN-Dhr). In sites located in the extratropical northern hemisphere, the growing season only overlaps the season 2 of \( c_{PAR} \), i.e., the season in which low values of \( c \) are more likely, being possible to define only one period with stationary parameters of radiation \( (R) \) and precipitation \( (P) \). After applying the criterion of dispersion of \( PAR_0 \) in this overlap, the stationary periods are shortened by at least one month (see Fig. 3-4(b)) in most sites analyzed. As the definition of stationary periods in sites in or close to the tropic is conditioned by the precipitation, in sites where it is possible to distinguish two seasons of \( c_{PAR} \), it is viable to define two stationary periods (see Fig. 3.3.2(c-d)). In sites like SN-Dhr (see Fig. 3.3.2(e-f)) where only one season of \( c_{PAR} \) is distinguished, there is just one stationary period. The figures of the stationary periods of all sites analyzed are shown in Annex G. Table 3-2 shows the values used in simulations and modeling of the mean of \( PAR_0 \), temperature, and pressure. I note that in cases where
3.4 Results and Discussion

![Figure 3-4: Overlap of seasons of $c_{PAR}$ (red and purple polygons), growing season (line fill polygons) on monthly annual cycle of (a,c and e) precipitation and (b,d and f) $PAR_0$ of (a-b) DE-Hai, (c-d) MY-PSO, and (e-f) SN-Dhr. Blue lines indicate the selected stationary periods to be modeled.](image)

...there are two stationary periods, Table 3-2 shows the values of the one that has a higher mean rainfall height.

Fig. 3-5 shows the histograms of evapotranspiration (1st and 2nd columns) and soil moisture (3rd and 4th columns) of stochastic and constant radiation and loamy sand and loam soils in sites located in Germany (DE-Hai), Malaysia (MY-PSO), and Sudan (SN-Dhr). The pdfs of $ET$ when $R$ is constant (1st, 3rd, and 5th rows) have a pulse in $E_{max}$ (that is also constant), whereas when $R$ follows a stochastic distribution (2nd, 4th, and 6th rows), the probability of this pulse is redistributed in the other values of $ET$. As expected, the means of the histograms of $ET$ for both cases of $R$ remain the same, since the constant value of $R$ coincides with the mean of the pdf of $R$ (see Eq. 3-17). The histograms of both cases have different shape and dispersion. The above is valid for sites where it makes sense to assume $R$_ stochastic (see Fig. 3-5(a-b, e-g, i-j, and m-n)), while in sites as SN-Dhr where $c$ has low dispersion, the pdfs of $ET$ are the same (see Fig. 3-5(q-e and u-v)). The pdfs of loamy sand and loam soils have very similar behavior, varying only the standard deviation.

The shape, mean and standard deviation of the pdfs of $s$ for constant radiation (1st, 3rd, and 5th rows) and stochastic radiation (2nd, 4th, and 6th rows) for both soil types are very similar in all sites studied. This can be explained by the modulation of fluctuations by the climate-soil-vegetation system. Soil moisture responds to climatic fluctuations with more delay than phenomena such as evapotranspiration and runoff, dampening fluctuations more than $ET$ (see Fig. 2 of Entekhabi et al. (1996)). The mean of the pdf of $s$ of loamy sand soils is lower than the mean of loam soils. A loamy sand soil has higher saturated hydraulic conductivity ($k_s=100$ cm d$^{-1}$), allowing the water to come out faster by means of leakage, besides water limitations of $ET$ begin at a very low value of $s$ ($s^*=0.24$), being able to evapotranspire at...
3 Soil moisture and evapotranspiration dynamics forced by stochastic rainfall and radiation

Table 3-2: Parameters for numerical simulations.

<table>
<thead>
<tr>
<th>Site</th>
<th>$\alpha$ [cm]</th>
<th>$\lambda$ [d$^{-1}$]</th>
<th>$\mu PAR_0$ [MJ m$^{-2}$]</th>
<th>$T_a$ [deg C]</th>
<th>$P_a$ [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.54</td>
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<td>14.43</td>
<td>9.08</td>
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<td>0.57</td>
<td>15.82</td>
<td>18.23</td>
<td>9.97</td>
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<td>15.99</td>
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<tr>
<td>CG-Tch</td>
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<td>15.04</td>
<td>12.72</td>
<td>9.38</td>
</tr>
<tr>
<td>US-NC2</td>
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<td>0.37</td>
<td>15.81</td>
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</tr>
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<td>23.62</td>
<td>9.72</td>
</tr>
</tbody>
</table>
3.5 Summary and Conclusions

$E_{max}$ rate in a greater range of soil moisture values. The results of the simulations of all sites are in Annex H.

Efforts are concentrate on the modeling of evapotranspiration forced by stochastic radiation due to the differences between the pdfs of $s$ for stochastic and constant radiation are non-significant, as noted by Daly et al. (2004b), and the pdfs of $ET$ indicate notable changes in most sites analyzed. Fig. 3-6 shows the comparisons of the numerically simulated cdfs and the semi-analytical solutions given by Eqs. 3-18, 3-20 and 3-22 of DE-Hai, MY-PSO, and SN-Dhr. In most of the sites, the semi-analytical solution agrees to the simulated distribution, with mean squared errors (MSE) around 0.002 (cm $d^{-1})^2$. Sites with rainy days modeled with stochastic radiation and dry days with constant radiation (because the dispersion of $c_{PAR_d}$ is very low) have higher errors (see 3-6(b)). The colors in Fig. 3-6 indicate the two types of soil analyzed (violet loamy sand and green loam), showing similar MSE.

3.5. Summary and Conclusions

This chapter presented a simple way to evaluate the effects of stochastical radiation on soil moisture and evapotranspiration dynamics using the ecohydrological model of Rodríguez-Iturbe et al. (1999) and Laio et al. (2001). As the solution of the pdf of $s$ is given for seasonally fixed conditions, both the parameters describing rainfall and radiation must be stationary. To guarantee this, the stationary periods were determined based on the seasons of $c_{PAR}$, growing and the rainfall. In sites in the extratropic northern hemisphere, the stationary period coincides with the overlap of season 2 of $c_{PAR}$ and the growing season, shortening them in the cases where PAR at the top of the atmosphere has high variability. In sites near or in the tropics, where daily $PAR_0$ has low variability, one or two periods were defined as the overlapping of $c_{PAR}$ seasons and the consecutive months with similar total rainfall. Rainfall parameters, and the means of PAR at TOA, temperature, and pressure were established for the defined stationary periods, and evapotranspiration and soil moisture were numerically simulated at the sites in Table 2-1.

According to the results obtained and under the assumption of the ecohydrological model, the stochastic component of radiation does not play a fundamental role in the soil moisture dynamics, but in evapotranspiration. This is probably due to the modulation of climate fluctuations by the climate-soil-vegetation system, having greater lags and damping in soil moisture than in evapotranspiration. The pdfs of $ET$ with $R$ constant shows an atom of probability on $E_{max}$ value, whereas when $R$ is stochastic, this probability is redistributed in the other values of $ET$. Finally, a semi-analytical solution to analyze the dynamics of evapotranspiration when both precipitation and radiation are stochastic is proposed. This solution is based on Bayer’s theorem, the solution of the soil water balance equation given by Rodríguez-Iturbe et al. (1999), the Priestley-Taylor equation, and the derived distribution technique. The semi-analytical solution is in good agreement with the numerical simulations in most cases evaluated.

This chapter confirms the result obtained by Daly et al. (2004b) which indicates that the
Figure 3-5: Histograms of evapotranspiration (1st and 2nd columns) and soil moisture (3rd and 4th columns) for constant and stochastic radiation in (a-d, e-h) DE-Hai, (i-l, m-p) MY-PSO and (q-t, u-x) SN-Dhr.
stochasticity of $E_{max}$ (directly related to radiation) has a slight effect on the long-term dynamics of soil moisture under the assumptions of the Rodríguez-Iturbe et al. (1991a) model. On the other hand, when analyzing the dynamics of evapotranspiration, the stochasticity of radiation must be taken into account.

Advances in the understanding of evapotranspiration dynamics are important because $ET$, in addition to control the plant conditions, it impacts on the main physical processes related to water and energy, regulating salinity, gas emission, and nutrient cycles in the soil (Daly and Porporato, 2005; Porporato et al., 2003a; Shani and Dudley, 2001).
4. Coupled dynamics of photosynthesis, transpiration and soil moisture

Abstract: Transpiration and photosynthesis are phenomena occurring simultaneously, since plants open their stomata to capture CO₂, losing water by transpiration. In this chapter, transpiration and available energy are related through the stomatal conductance, using the Penman-Monteith transpiration equation, the Leuning’s stomatal conductance approach, the C₃ photosynthesis model by Farquhar et al. and information from the FLUXNET database.

Transpiration is calculated in the 28 sites studied in the previous chapters, and the results are integrated at a daily scale, finding an expression of transpiration in terms of the available radiation. Analyzing the behavior of transpiration when it is limited by the radiation, it is concluded that the model of Rodríguez-Iturbe et al. and Laio et al. is capable of representing the soil water dynamics in energy-limited ecosystems as long as the $E_{\text{max}}$ term is calculated as a function of the available energy, and parameters of both rainfall and radiation are stationary.

4.1. Introduction

Evapotranspiration ($ET$) is the variable coupling the balances of water and energy in the climate-soil-vegetation system (Eagleson, 1978e; Medlyn et al., 2017; Fisher et al., 2017; Shan et al., 2019). $ET$ can be divided into two regimes according to its relationship with the soil moisture ($s$): an energy-limited regime when $ET$ is independent of $s$, and a water-limited regime when $ET$ is very sensitive to $s$. The first regime has been associated with humid regions with large and recurrent precipitations that result in high values of soil moisture. Values of $s$ usually exceed the threshold value in which water stress begins ($s^*$ in the Rodríguez-Iturbe et al. (1999) model), being the solar radiation ($R$) the variable limiting $ET$. The second regime is associated with arid and semi-arid regions where water is scarce (Eagleson, 1982; Seneviratne et al., 2010), being vegetation in constant water stress. There are also seasonal environments in which the availability of water fluctuates sharply, leading to $ET$ being limited by water or by energy.

Transpiration ($T$) and photosynthesis are processes taking place simultaneously since when plants take up carbon dioxide (CO₂) to photosynthesis, they lose water through transpiration (Daly et al., 2004a; Yu et al., 2004). Both water and CO₂ movements through the plant are governed by rules analogous to those of electricity flow (Larcher, 1995), therefore they can be represented by circuit diagrams (e.g. Daly et al., 2004a; Daly and Porporato, 2005; Man-
4.1 Introduction

Figure 4-1.: Analogous circuit diagrams of a) transpiration and b) carbon assimilation.

zoni et al., 2011; Bartlett et al., 2014; Shan et al., 2019) as those in Fig. 4.1. Water flow follows a path of decreasing water potential from the soil ($\psi_s$) to the atmosphere ($\psi_a$) (see Fig. 4.1(a)), and CO$_2$ flow a path of decreasing concentration (see Fig. 4.1(b)). Both movements are restricted by the resistances (reciprocal of conductance) interposed by each part of the soil-climate-vegetation system, but it is the stomatal resistance that couples transpiration and photosynthesis. Stomatal aperture responds to a wide variety of internal and external conditions such as the quantity of available Photosynthetically Active Radiation (PAR), leaf and air temperature, air vapor pressure deficit, soil water content and atmospheric CO$_2$ (Bonan, 2016). Although the description of the stomatal conductance ($g_s$) is the most critical factor in modeling transpiration (Daly et al., 2004a; Daly and Porporato, 2005), following the hypothesis indicating that plant maximize carbon gain for a given rate of water loss (Farquhar, 1973; Cowan and Farquhar, 1977), it is possible to predict the stomatal response to environment variables (Buckley et al., 2017; Kurc and Small, 2007; Leuning, 1995). However, as the analytical solution for optimal $g_s$ implies potentially unrealistic assumptions (Buckley et al., 2017), it is frequent to adopt simplified and semi-analytical equations of $g_s$ (e.g. Jarvis, 1976; Farquhar, 1989; Leuning, 1990; Collatz et al., 1991; Leuning, 1995; Gao et al., 2002; Dewar, 2002; Tuzet et al., 2003; Yu et al., 2004).

The link between $g_s$ (or $T$), and soil moisture has been widely studied in water-limited ecosystems (e.g. Laio et al., 2001; Porporato et al., 2001; Rodríguez-Iturbe et al., 2001; Rodríguez-Iturbe and Porporato, 2004; Daly et al., 2004a,b; Manzoni et al., 2013) but no much in energy-limited ecosystems. Consequently, the aim of this chapter is to analyze the relations of ET and available radiation, and ET and $s$ when radiation is the limiting variable. $R$ directly drives the fundamental plant physiological processes involving in transpiration, i.e., photosynthesis, stomatal conductance, and leaf temperature. Besides, it indirectly influences secondary processes such as plant growth, seedling generation, structure and emission of gases (Monteith,
Required energy affects photosynthesis, transpiration, and stomatal opening through light receptors driving CO₂ fixation and lower intercellular CO₂ concentration (Yu et al., 2004), and determining the diabatic component of transpiration (Monteith and Unsworth, 2013). Hence, to properly study the effects of radiation on transpiration (T), the relations among carbon assimilation (Aᵣ), stomatal conductance (gₛ) and transpiration (T) must be taken into account. Leuning’s equation (Leuning, 1990, 1995) is used to model the stomatal conductance since it is simple, agrees with measurements of gₛ under different conditions (Leuning, 1995; Buckley et al., 2003), and relates directly the stomata behavior to Aᵣ. Penman-Monteith equation is adopted to model transpiration, noting that in this case, unlike the previous chapter, its use is appropriate because the behavior of gₛ is contemplated. Leuning’s equation gives gₛ at the leaf level, so the leaf index area (LAI) is used to scaling it to the canopy level. This approach neglects the plant physiology responses to the gradients between the vegetation and the bulk environment above the canopy (Ehleringer et al., 1993).

In the sites where there are measurements of CO₂ assimilation, they are modeled by using the Farquhar model. Finally, to extend the model of Rodríguez-Iturbe et al. (1999) and Laio et al. (2001) (see section 3.3.1) to energy-limited ecosystems, the dependence of gₛ and T on radiation at the daily level is integrated, relating T and PAR through a simple expression. The effect of available energy in soil moisture dynamics is analyzed with dimensionless numbers.

### 4.2. Data

Half hourly resolution data of air temperature (Tₐ), atmospheric pressure (Pₐ), vapour pressure deficit (Δₑ), photosynthetic photon flux density (PPFD), net ecosystem CO₂ exchange (NEE), CO₂ air concentration, and soil moisture (s) in the sites in Table 2-1 are those published in the FLUXNET dataset (Baldocchi et al., 2001; Olson et al., 2004). NEE data contain positive values during the day (assimilation), and negative values during the night (respiratory loss) (Drake and Read, 1981), therefore the positive values of these series are used as Aᵣ. The parameters to apply Penman-Monteith and Leuning equation are in Table 4-1, and the parameters to apply the Farquhar model in Table 4-2. These values are those published by Daly et al. (2004a).

### 4.3. Methods

The Penman-Monteith equation, Leuning’s stomatal conductance model, and a simplified energy balance model are numerically and simultaneously solved to evaluate transpiration and stomatal conductance at an hourly scale. Then, these results are integrated at a daily scale to obtain the daily relation of T and the available energy. This relation is used to extend the Rodríguez-Iturbe et al. (1999) model to energy-limited ecosystems and to analyze the effect of radiation on soil moisture dynamics.
4.3 Methods

Table 4-1.: Parameters for the stomatal and transpiration models.

<table>
<thead>
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<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<td>Eq. 4-2</td>
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<tr>
<td>$c_a$ [$\mu$mol mol$^{-1}$]</td>
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<td>Atmospheric CO$_2$ concentration</td>
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<tr>
<td>$c_p$ [J kg$^{-1}$ K$^{-1}$]</td>
<td>1013</td>
<td>Specific heat of air</td>
</tr>
<tr>
<td>$D_x$ [Pa]</td>
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<td>Eq. 4-2</td>
</tr>
<tr>
<td>$e$</td>
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<td>Ratio molecular weight of water vapour/dry air</td>
</tr>
<tr>
<td>$g_a$ [mm s$^{-1}$]</td>
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<td>Atmospheric conductance</td>
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<td>$g_b$ [mm s$^{-1}$]</td>
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<td>$\rho_a$ [kg m$^{-3}$]</td>
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Table 4-2.: Parameters for the C$_3$ photosynthesis model.

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<td>$H_{Ko}$ [J mol$^{-1}$]</td>
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4.3.1. Transpiration

Penman-Monteith (PM) equation (Monteith, 1965; Monteith and Unsworth, 2013) is widely used in hydrology, and relates transpiration ($T$) and stomatal conductance. PM is expressed as:

$$T = \frac{(\rho_a c_p D g_{ba} + \Delta_e R) g_s LAI}{\rho_w \lambda_w \left[ \Delta_e g_s LAI + \gamma_p (g_{ba} + g_s LAI) \right]};$$  \hspace{1cm} (4-1)

where $\lambda_w$ is the latent heat of vaporization (2.26 MJ kg$^{-1}$), $\rho_w$ and $\rho_a$ are the water (I used 998.2 kg m$^{-3}$) and air (I used 1.2 kg m$^{-3}$) density, respectively, $c_p$ is the specific heat of air (1.013·10$^{-3}$ MJ kg$^{-1}$ K$^{-1}$), $\Delta_e$ is the slope of saturation of vapor pressure, $\gamma_p$ is the psychometric constant, $D$ is the saturation vapor pressure deficit, LAI is the leaf area index, and $g_{ba}$ is the series of leaf boundary conductance ($g_b$) and atmospheric boundary layer conductance ($g_a$). Both $g_a$ and $g_b$ are assumed to be constant. The first term in Eq. 4-1 is the adiabatic component which accounts for the atmospheric saturation deficit, and the second term is the diabatic component of latent heat loss, related to radiation supply. According to the PM equation, $T$ increases linearly with available energy ($R$) and with the atmospheric saturation deficit. As $g_{ba}$ is strongly related to wind speed, when it increases, $T$ also increases, and when the variables in the numerator remain constant, $\Delta_e$ increases with temperature.

4.3.2. Stomatal conductance

Net assimilation and transpiration are processes coupled by the stomatal aperture. Therefore, to analyze the dynamics of transpiration it is necessary a stomatal conductance ($g_s$) model that relates it to net assimilation. For this purpose, the semi-empirical formulation given by Ball et al. (1987) and improved by Leuning (1990, 1995) is used. Leuning’s equation gives $g_s$ as:

$$g_s = 1.6a_1 \frac{A_n}{(c_s - \Gamma^*) \left(1 + \frac{D}{D_x}\right)};$$  \hspace{1cm} (4-2)

where $A_n$ is the carbon assimilation, $D$ is the water vapor saturation deficit, $\Gamma^*$ is the CO$_2$ compensation point, $c_s$ is the carbon concentration at the leaf surface, $D_x$ is a fitted parameter representing the sensitivity of stomata to changes in $D$, and $a_1$ is an empirical constant with typical value around 15. The CO$_2$ compensation point is the CO$_2$ concentration at which the CO$_2$ uptake rate in the photosynthesis equals the CO$_2$ loss rate of respiration (Birmingham and Colman, 1979). $\Gamma^*$ is significantly affected by leaf temperature, and according to Brooks
and Farquhar (1985), they can be related as:

\[
\Gamma^* = \gamma_0 + \left[1 + \gamma_0 (T_l - T_0) + \gamma_2 (T_l - T_0)^2\right].
\]

(4-3)

where \(\gamma_0, \gamma_1\) and \(\gamma_2\) are empirical constants, and \(T_0\) is the reference temperature.

### 4.3.3. Energy balance

As solving Eqs. 4-1 and 4-2, there are three unknowns \((T, g_s, \text{and } T_l)\), it is mandatory to couple another equation that allows solving the system. This purpose is met by the energy equation expressed by:

\[
T_l = T_a + \frac{R - \rho_w \lambda_w T}{c_p \rho_a g_a}.
\]

(4-4)

where \(T_l\) and \(T_a\) are the leaf and air temperatures, respectively.

### 4.3.4. Net carbon assimilation

In sites where there are no measurements of \(A_n\), net assimilation is calculated through the Farquhar’s model (Farquhar, 1973; Cowan and Farquhar, 1977; Farquhar et al., 1980). This is the most frequently used model to quantify the responses of \(C_3\) plants to external perturbations under well-watered conditions. Farquhar’s model is a function of the photosynthetic photon flux density \((Q)\), \(\text{CO}_2\) concentration in the mesophyll cytosol \((c_i)\) and leaf temperature \((T_l)\), and expresses the biochemical demand for \(\text{CO}_2\) as:

\[
A_n = f (Q, c_i, T_l) = \min [A_c, A_q],
\]

(4-5)

where \(A_c\) and \(A_q\) are the photosynthesis rates limited by the Ribulose bisphosphate carboxylase-oxygenase (Rubisco) activity, and by the Ribulose bisphosphate (RuP\(_2\)) regeneration through electron transport, respectively. \(A_c\) and \(A_q\) are given by:

\[
A_c = V_{c,max} (T_l) \frac{c_i - \Gamma^*}{c_i + K_c (1 + \alpha_i/K_o)}.
\]

(4-6)

\[
A_q = J \frac{c_i - \Gamma^*}{4 c_i - 2\Gamma^*},
\]

(4-7)
where $\Gamma^*$ is the CO$_2$ compensation point (see Eq. 4-3), $\alpha_i$ is the intercellular oxygen concentration, $V_{c,max}$ is the maximum catalytic activity of Rubisco in the presence of saturating levels of RuP$_2$ and CO$_2$, and $K_c$ and $K_o$ are Michaelis coefficients for CO$_2$ and O$_2$, respectively. $V_{c,max}$ and $K_x$ are given by:

$$V_{c,max} (T_l) = V_{c,max0} \frac{\exp \left[ \frac{H_vV}{R_gT_0} \left( 1 - \frac{T_0}{T_l} \right) \right]}{1 + \exp \left[ \frac{S_vT_l - H_dV}{R_gT_l} \right]},$$

(4-8)

$$K_x (T_l) = K_{x0} \exp \left[ \frac{H_{Kx}}{R_gT_0} \left( 1 - \frac{T_0}{T_l} \right) \right].$$

(4-9)

where $J$ is the electron transport for a given absorbed photon irradiance, and is equal to \( \min [J_{max} (T_l), Q] \), being $J_{max}$ equal to:

$$J_{max} (T_l) = J_{max0} \frac{\exp \left[ \frac{H_{vol}}{R_gT_0} \left( 1 - \frac{T_0}{T_l} \right) \right]}{1 + \exp \left[ \frac{S_vT_l - H_dJ}{R_gT_l} \right]},$$

(4-10)

The parameters not mentioned are described in Table 4-2.

### 4.3.5. Upscaling from half-hourly to daily time scale

The results obtained with the models of transpiration, stomatal conductance, and net assimilation (when it is necessary) have the same temporal resolution of FLUXNET data, i.e, half-hour. The results and the information from the FLUXNET database are integrated at a daily scale to evaluate the daily dynamics of transpiration (and soil moisture). Daily values of soil moisture, transpiration, and stomatal conductance correspond to the average during the day, while radiation (PAR) and assimilation are sub-daily values accumulated throughout the day.

### 4.3.6. Dimensionless groups

Four dimensionless groups are used to analyze the sensibility of soil moisture dynamics to the available energy, and how this last is related to other parameters of the Rodríguez-Iturbe et al. (1999) and Laio et al. (2001) model. The dimensionless groups are used because they simplify the interpretation and visualization of the results (Bridgman, 1922; Barenblatt, 1996; Gorokhovski and Hosseinipour, 1997; Butterfield, 1999; Barenblatt and Isaakovich, 2003). The sensitivity of the model output to each parameter is evaluated by moving the input parameter within an appropriate range and keeping the other parameters fixed. The four dimensionless
4.4 Results and Discussion

groups are:

\[ \pi_1 = \frac{E_{max}}{\alpha \lambda} \]  
(4-11)

\[ \pi_2 = \frac{nZ_r}{\alpha} \]  
(4-12)

\[ \pi_3 = \frac{k_s}{\alpha \lambda} \]  
(4-13)

\[ \pi_4 = \frac{k_s}{E_{max}} \]  
(4-14)

\[ \pi_1 \] and \[ \pi_2 \] groups have been adopted in previous works to analyze the soil moisture response to rainfall forcing, soil and vegetation changes (e.g. Li, 2014; Feng et al., 2012; Daly and Porporato, 2006a; Porpotato et al., 2004; Rodríguez-Iturbe and Porporato, 2004; Guswa et al., 2002; Milly, 2001; Rodríguez-Iturbe et al., 1999; Milly, 1993). \[ \pi_1 \] is the dryness index of Budyko (1974) and represents the ratio between the maximum evapotranspiration rate and the long-term mean rainfall rate. Values of \[ \pi_1 \] greater than 1.0 indicate that evapotranspiration is limited by the amount of water in the soil, not by atmospheric demand (Guswa et al., 2002). For values of \[ \pi_1 \] near 1.0 there is a balance between water supply and evaporative demand (Milly, 2001). Values of \[ \pi_1 \] close to zero are related to low or no vegetation areas while very high \[ \pi_1 \] values are typical of arid regions. \[ \pi_2 \] is called the storage index and is the ratio between the amount of water that can be stored in the soil (until the rooting dept) and the long-term mean rainfall depth. High values of \[ \pi_2 \] indicate that soil can retain a large portion of rainfall, resulting in moderate changes in the soil moisture over time (Feng et al., 2012). \[ \pi_3 \] and \[ \pi_4 \] are proposed by Guswa et al. (2002). \[ \pi_3 \] is the runoff index and is the ratio of the saturated hydraulic conductivity coefficient and the long-term mean rainfall rate. These authors used this group to identify if runoff generation is due to the Hortonian mechanism. High values of \[ \pi_3 \] evidence that soil infiltration capacity is not likely to be exceeded. Finally, \[ \pi_4 \] is named the infiltration index and is the ratio of the saturated hydraulic conductivity to the maximum evapotranspiration rate. If \[ \pi_4 \] is greater than 1.0, water can move into the soil with a greater rate than the maximum evapotranspiration rate.
4.4. Results and Discussion

4.4.1. Daily dynamics

Fig. 4-2 shows the relationship between available energy and CO₂ assimilation, and available energy and the stomatal conductance in a site located in the extratropics (Germany, DE-Obe) and one in the tropics (French Guyana, GF-Guy). It is noted that this figure does not show the same sites used as examples in previous chapters. This is because the sites shown in this chapter have more sub-daily information, allowing to clearly observe the behavior of the modeled variables. In the graphs of Germany (Fig. 4-2(a,b)), the relationships of photosynthetically active radiation (PAR) and CO₂ assimilation ($A_n$), and PAR and stomatal conductance ($g_s$) are positive for low values of PAR ($\approx 4$ MJ m$^{-2}$) and negative for high values. This can be explained by the phenomenon of photo-inhibition. This phenomenon is the superfluous radiation energy is diverted directly from the photosystems via fluorescence, and above as heat (Larcher, 1995). The above occurs in situations of strong light since this can destroy the plant tissues. Otherwise, at sites in tropics (see Fig. 4-2(c-d)) the relationships of PAR and $g_s$, and PAR and $A_n$ are less clear. This can be explained by the adaptation and the strategies developed by the plants at sites where they usually receive high radiation. It is recalled that the values of PAR analyzed correspond to the PAR reaching the ground surface, and not the PAR absorbed by the plant. The results of all sites are shown in the Annex J.
Figure 4-3.: Relation of daily $PAR$ and $T$ in (a) an extratropical site (DE-Obe) and (b) a tropical site (GF-Guy). The black line represents the proposed equation to relate both variables.

Fig. 4-3 shows the relationship between PAR and transpiration in the same sites in Fig. 4-2, i.e., DE-Obe and GF-Guy. In both types of ecosystems the relationship is direct, since when PAR increases both adiabatic and diabatic terms of Penman-Monteith increase. Radiation affects temperature, and this, in turn, modifies the vapor saturation deficit. Furthermore, if there is enough available energy, the stomata open up as they can fix more CO$_2$, leading to the plant loses water. However, as shown in Fig. 4-2, the relation of PAR and $g_s$ is not always direct, but $g_s$ stabilizes (light-saturated plateau) at a point (Lambers et al., 2008), and may even decrease. The phenomenon of light-saturation is also observed in transpiration ($T$), but not that of the photo-inhibition, or at least for the measured values of PAR in the studied sites.

Since transpiration is modeled using measured data, many factors may be limiting the CO$_2$ assimilation, and consequently the stomatal conductance and transpiration. Therefore, a link between PAR and $T$ must involve the envelope of simulated points relating these variables (see Fig. 4-3). For most sites, this relationship can be expressed by:

$$T(PAR) = T_{max}(1 - e^{-a PAR}).$$

(4-15)

This is a function of the maximum possible value of transpiration (light-saturation) ($T_{max}$) and a fitting parameter that determines the shape of the curve ($a$). This relationship avoids considering the indirect effects of radiation in transpiration ($g_s, T_a, D$, etc.). Fig. 4-3 shows the points of measured PAR and modeled $T$ (orange points), the fitting curves of PAR-$T$ (black lines) and their expressions in two sites in Germany and French Guiana.

### 4.4.2. Extension of the Rodríguez-Iturbe et al. model

The evapotranspiration expression of Laio et al. (2001) model (Eq. 3-7) indicates that transpiration reduces its rate when $s$ is below a given threshold ($s^*$) since plants close their stomata to avoid dehydration. Although the model does not specify the relation of available energy and
transpiration because it is developed for water-limited ecosystems, it is indicated that $E_{\text{max}}$ can be calculated from equations such as Penman-Monteith (Rodríguez-Iturbe et al., 1999; Laio et al., 2001; Rodríguez-Iturbe and Porporato, 2004), which are functions of radiation.

Transpiration can be limited by three factors: soil water, energy and plant capacity (physiology). The maximum rate at which a plant can transpire when it has no external limitations depends on the maximum stomatal conductance, which is directly proportional to pore width (Larcher, 1995). In Fig. 4-4 this rate is represented by the red line and is named $E_{\text{maxmax}}$. Soil moisture only limits transpiration for values below $s^*$ (see left graph in Fig. 4-4), while energy limits for values above $s^*$. Energy also influences the reaction of the plant to water stress. The rate of water loss is proportional to the gradient in water vapor concentration within the plant and the bulk atmosphere (Pallardy, 2008), and a high radiation results in a high vapor-pressure deficit in the air.

When the energy in the atmosphere is high ($R_1$), the plant must react in a more drastic way to the water stress, because it can lose water at a high rate. In this case, plants begin to rapidly close their stomata (from $E_{\text{max1}}$ to $E_w$), as shown in the right dotted line in Fig. 4-4 (from $R_1$), while when energy demand is low ($R_2$), plants can also suffer water stress, but their reaction should be slower (from $E_{\text{max2}}$ to $E_w$) as shown in the left dotted line in Fig. 4-4 (from $R_2$). The above suggests that the evapotranspiration expression of the Laio et al. (2001) model manages to describe the daily $ET$ dynamics in energy-limited ecosystems, and the stationary solution of soil moisture dynamics (see 3-14) is also valid in this kind of environments. This is proper as long as $E_{\text{max}}$ is defined taking into account the available energy, and stationarity is maintained not only in the parameters describing the rainfall, but also those describing the radiation. It is noted that the above is valid under the simplifications of the model of Rodríguez-Iturbe et al. (1991a), e.g., deep water table, soil homogeneity, distribution of infiltration volume into the rooting depth, etc. From Eq. 4-15 and taking into account the maximum evapotranspiration given by the plant physiology, $E_{\text{max}}$ can be defined as:

$$E_{\text{max}}(R) = \begin{cases} T_{\text{max}} \left(1 - e^{-aR}\right), & E_{\text{max}}(R) < E_{\text{maxmax}} \\ \frac{E_{\text{maxmax}}}{E_{\text{maxmax}}}, & E_{\text{max}}(R) \geq E_{\text{maxmax}}. \end{cases} \quad (4-16)$$

where $E_{\text{maxmax}}$ is the maximum transpiration rate of the plant without external limitations, $T_{\text{max}}$ is the maximum transpiration rate limited by radiation ($R$), and $a$ is a fitted parameter describing the shape of the curve relating available energy and transpiration.

Fig. 4-5 shows the response of soil water dynamics to different parameters, and the response of $s$ to PAR when other parameters vary following the dimensionless groups of Eqs. 4-11 to 4-14. For these analysis I consider a loamy sand soil and a grass cover with the parameters in Table. 3-1 and in the caption of Fig. 4-5.

Fig. 4-5(a) shows the pdf of $s$ ($f(s)$) for $\pi_1$ values between 0.1 and 1.4. As the value of $\pi_1$ increases, $f(s)$ moves to the left. Higher $\pi_1$ results in lower soil moisture values in the
long-term, since the losses due to evapotranspiration are greater than soil water gains due to rainfall. These results are consistent to those found by Milly (2001). Milly indicated that the larger the value of the dryness index, the lower the mean of soil water. Fig. 4-5(b) shows \( f(s) \) for \( \pi_2 \) varying between 4 and 20, since natural ecosystems tend to have root zones deep enough to result in values of \( \pi_2 \) larger than 1.0 (Milly, 2001). The higher the \( \pi_2 \) value, the lower the soil moisture. For large values of \( nZ_r \), characteristic of plants with deeper roots such as trees, when \( \alpha \) is fixed and the maximum available soil water storage is varied, the amount of rainfall reaching the soil is distributed into a larger volume (according to the model) resulting in smaller increases in \( s \). For lower values of \( nZ_r \), rainfall is uniformly distributed in a smaller volume, increasing soil moisture rapidly. Very high and very low \( \pi_2 \) values occur when soil storage capacity is much larger or smaller than the rainfall amount, respectively. \( \pi_2 \) values approaching zero are related to highly compacted soil with a low void volume, preventing the entry of significant amounts of water, and \( \pi_2 \) values approaching infinity occur when the soil water storage is large or when rainfall depth is very small.

Fig. 4-5(c) shows the results obtained for \( \pi_3 \) values varying between 50 and 400. As the runoff index increases the water moves rapidly out of the soil, decreasing \( s \). \( \pi_3 \) values approaching zero and infinity occur when the amount of water flowing out the soil is much lower or much greater than the rainfall rate. Fig. 4-5(d) shows \( f(s) \) for \( \pi_4 \) values between 100 and 1000. For low values of \( \pi_4 \), \( s \) remains at high values since the water losses are minor. For high values of \( \pi_4 \) (greater than 550) the mode of the pdfs stabilizes near the field capacity point, changing only its frequency, and consequently the dispersion. When \( k_s \) is much larger than \( E_{max} \), soil loses water by leakage at a very high rate, being the evapotranspiration and its variability less relevant. These results are in agreement with those obtained by Muñoz et al. (2018).

If available energy is high (dotted lines in 4-5) the curves of all dimensionless groups move more rapidly to the left than when PAR is low (solid lines), as the plant transpires at higher rates, maintaining soil moisture lower. The sensitivity of \( s \) is more noticeable for \( \pi \) values...
Figure 4-5: Dimensionless sensitivity analysis of soil water dynamics conditioned by available energy. Parameters are those in Table. 3-1 and $\alpha=2$ cm, $\lambda=0.5$ d$^{-1}$, $\Delta=0$ cm, $Z_r=30$ cm, $T_{max}=0.47$ cm d$^{-1}$, $a=0.384$ m$^2$ MJ$^{-1}$. 
related to low soil moisture since the demand of energy in the atmosphere varies the rate at which the plant decreases its transpiration when it is under water stress. The dimensionless groups including $E_{max}$ as a parameter ($\pi_1$ and $\pi_4$) show less sensitivity to PAR, and the modes of the pdfs of $s$ have always a minor frequency for high available energy. The other dimensionless groups ($\pi_2$ and $\pi_3$) show a more noticeable variation with PAR, completely altering the dynamics of $s$ for some $\pi$ values (e.g., $\pi_2=16$ and $\pi_3=225$). Furthermore, the mode has a high (low) frequency for low values of PAR when it is greater (lower) than $s^*$, decreasing (increasing) the dispersion.

4.4.3. Soil water balance

Fig. 4-6 shows the behavior of the components of the water balance normalized by the average rainfall rate for a loamy sand soil. These components are: canopy interception ($I$), runoff ($Q$), leakage ($L$), evapotranspiration under stressed conditions ($E_s$), and evapotranspiration under non-stressed conditions ($E_{ns}$). Figs. 4-6 (a,b) show the influence of rainfall events frequency ($\lambda$) in each component of the water balance for PAR=3 and 15 MJ m$^{-2}$, respectively. In both cases, the fraction of intercepted water is constant and equal, since it changes in proportion to the rainfall rate. The percentage of runoff increases with $\lambda$ in a similar proportion for both cases. The fraction of water transpired under stressed conditions decreases rapidly until $\lambda \approx 0.3$ d$^{-1}$ for PAR=3 MJ m$^{-2}$ and until $\lambda \approx 0.5$ d$^{-1}$ for PAR=15 MJ m$^{-2}$, being in the first case much lower. The same behavior is observed in the fraction of water transpired under non-stressed conditions ($E_{ns}$). When PAR is low, the percentage of leakage is higher and the percentage of evapotranspired water is significantly lower. This indicates that in water-limited regions more water reaching the soil is lost by evapotranspiration than in energy-limited regions (for these parameter values), becoming more important $Q$ and $L$ than in water-limited ecosystems. The results of these examples are in agreement with field observation and results found in previous studies (e.g. Sala et al., 1992; Entekhabi and Rodríguez-Iturbe, 1994; Golubev et al., 2001; Rodríguez-Iturbe and Porporato, 2004; Robock and Li, 2006; Roderick et al., 2009).

Figs. 4-6(c,d) show the behavior of the water balance when $\lambda$ and $\alpha$ are varied while maintaining constant the total amount of precipitation during a season $\Theta$ ($\Theta = \alpha \cdot \lambda \cdot nd$, being $nd$ the number of days of the growing season). For this figure $\Theta = 60$ cm and $nd = 200$ d. Figs. 4-6(c,d) indicates that the interception increases almost linearly with $\lambda$ while runoff decreases rapidly. According to Laio et al. (2001) this decreasing depends strongly on the ratio between soil depth and mean depth of rainfall events. The opposite behavior of interception and runoff determines a maximum of evapotranspiration at certain values of $\lambda$. As when only $\lambda$ is varied, the main difference in the behavior of the water balance components for high and low PAR is observed in the percentage of evapotranspiration, being remarkably lower for low available energy.
Figure 4-6: Examples of the behavior of the components of the water balance normalized by the total rainfall $\langle P \rangle$ for different values of available energy, loamy sand soil and grass vegetation. The parameters are shown in Table 3-1.
4.5. Summary and Conclusions

In this chapter the dynamics of transpiration as a function of available energy were analyzed. This analysis took into account the coupling of water and CO$_2$ fluxes through the leaf. This was initially done at a half-hourly temporal scale since it is the most detailed resolution found in the FLUXNET database. The results were integrated over the daily scale, obtaining that in the extratropical sites both the net assimilation and the stomatal conductance have a positive relationship with the available energy. This happens until the called light-saturation point, when CO$_2$ assimilation and stomatal conductance stop increasing, even reaching PAR values in which they decrease due to the photo-inhibition phenomenon.

Transpiration is directly related to the stomatal conductance, being the relation between PAR and $T$ positive until a certain point in which transpiration ceases to increase. In the sites analyzed, the decreasing of the stomatal conductance for high values of available energy was not observed in transpiration. In order to parameterize the relationship between the available energy and transpiration, an equation was proposed. This equation describes the envelope of the points representing the transpiration calculated with its respective PAR value. With this parameterization and evaluating the evapotranspiration description of the Rodríguez-Iturbe et al. (1999) and Laio et al. (2001) model, it was found that this model manages to describe the daily evapotranspiration and soil moisture dynamics in energy-limited ecosystems. The above is valid as long as the $E_{\text{max}}$ parameter is calculated taking into account the available energy, the parameters of both rainfall and radiation are stationarity, and the assumptions of the model of Rodríguez-Iturbe et al. (1999) are considered.

Some examples exhibiting the influence of radiation on the soil moisture dynamics were presented. This examples indicate that $s$ can notoriously vary for different values of available energy. Finally, the components of the water balance were studied, founding that evapotranspiration plays a more important role in water-limited ecosystems, and leakage and runoff are more noticeable in energy-limited ecosystems.

It is highlighted that the results obtained in this chapter are only valid on a daily scale and under the assumptions contemplated in the Rodríguez-Iturbe et al. (1999) model, such as a deep water table, stationarity, homogeneous soil, and homogeneous vegetation, etc.
5. Application: frequency of landslides triggered by rainfall

Abstract: Rainfall is the most common triggering of shallow landslides (soil slips). The rainfall thresholds approach is among the widely used methods to determine the landslide frequency. In this chapter, analytical and simulated probabilities of the cumulative rainfall, soil moisture, and factor of safety are presented. This analysis is based on the ecohydrological model of Rodríguez-Iturbe et al. and Laio et al., and the infinite slope model for unsaturated soils. The effects of two types of soil, two types of vegetation and the mean rainfall on the landslide occurrence are evaluated. Furthermore, an analysis of the rainfall characteristics from 0 to 40 days before each landslide is carried out, concluding that a slope with given properties of soil, vegetation and climate can reach a threshold factor of safety by rainfall with different features. This highlight possible failures in the definition of rainfall thresholds to determine the landslide frequency.

5.1. Introduction

Although many analyses have been developed to understand and predict landslides, they continue to be a challenge due to the complex soil processes and environmental conditions (von Ruette et al., 2014; Leonarduzzi et al., 2017). The occurrence of landslides is the result of the reduction of shear strength and the increase of driving forces acting on the sliding mass. This may occur as a consequence of quasi-static factors, or dynamic triggering factors such as rainfall, snowmelt, earthquakes, and human activities (Rosso et al., 2006; Crosta and Frattini, 2008; Simoni, 2009; Gariano and Guzzetti, 2016; Zhao et al., 2019). Rainfall is the most common triggering of shallow landslides (Terlien, 1998; Alvioli et al., 2014; Waswa and Lorentz, 2014; Leonarduzzi et al., 2017; Zhao et al., 2019) and can affect by two mechanisms: i) the precipitated water entering the soil recharges the water table or builds-up a perched water table (Terlien, 1998; Simoni, 2009; von Ruette et al., 2014; Waswa and Lorentz, 2014) increasing the pore pressure, and ii) the wetting front product of the infiltration in the superficial layers of the soil reduces the suction (negative pore pressure) (Lumb, 1975; Fredlund et al., 1978; Alonso et al., 1990; Gallipoli et al., 2003; Kim et al., 2004).

Rainfall thresholds are among the most popular approaches to relate rainfall and landslides frequency (Lu and Godt, 2013; Leonarduzzi et al., 2017; Postance et al., 2018) and are widely used in early warning systems around the world. This method was initialized by Caine (1980)
and has been applied in many studies around the world (see reviews in Guzzetti et al., 2007, 2008; Pardeshi et al., 2013; Gariano and Guzzetti, 2016). Among the precipitation characteristic normally used are the total rainfall amount, peak intensity, event duration, cumulative rainfall, and antecedent rainfall (Pardeshi et al., 2013). Rainfall thresholds for landslides are determined through empirically- (e.g. Caine, 1980; Larsen and Simon, 1993; Glade et al., 2000; Aleotti, 2004; Guzzetti et al., 2007, 2008; Cepeda et al., 2010; Staley et al., 2013; Garcia-Urquia, 2016; Palenzuela et al., 2016; Robbins, 2016; Corsini and Mulas, 2017; Leonardsuzzi et al., 2017) or physically-based (e.g. Frattini et al., 2009; Segoni et al., 2009; Salciarini et al., 2012; von Ruette et al., 2014; Peres and Cancelliere, 2016) models. The most common rainfall thresholds are defined as the minimum intensity and/or duration of rainfall required to trigger a landslide (Aleotti, 2004; Jaiswal and van Westen, 2009; Gianvecchini et al., 2012; Pardeshi et al., 2013; Robbins, 2016; Zhang et al., 2016a), however, as mentioned before, the definition can take into account other variables related to the soil strength. Empirical models use statistical approaches based on historical landslides catalogs and rainfall datasets, and physical-based models employ mechanical models describing the phenomena occurring into the soil mass (Guzzetti et al., 2008; Zhuang et al., 2016).

The rainfall thresholds method undergoes several weaknesses, among which: i) is highly empirical, ii) implies that rainfall and the factor of safety follow a monotonic relation (Crosta, 1998; Reichenbach et al., 1998; D’Odorico et al., 2005; Rahardjo et al., 2007; Aristizábal, 2013), iii) ignores the complex dynamics of the physical processes going between rainfall and slope failure (Terlien, 1998; Casadei et al., 2003; Guzzetti et al., 2007; Rahardjo et al., 2007; Frattini et al., 2009; Simoni, 2009; Berti et al., 2012; Aristizábal, 2013; Waswa and Lorentz, 2014; Chen et al., 2016), iv) it is not possible to generalize the results over broad regions (Terlien, 1998; Casadei et al., 2003; Guzzetti et al., 2007; Lu and Godt, 2013; Chen et al., 2016; Gariano and Guzzetti, 2016; Ho and Lee, 2017), v) depends on data available, vi) initial conditions are not properly taken into account (Frattini et al., 2009; Peres and Cancelliere, 2016). It is noted that bullet points i and ii are weaknesses specifically from empirical-based methods. The above leads to a high false alarm ratio (Nikolopoulos et al., 2014; Bezak et al., 2015; Zhang et al., 2016a), and early warning systems that must be constantly calibrated (Terlien, 1998). Some methodologies based on statistical methods, such as Bayesian, logistic regression, and neuronal network models (e.g. Guzzetti et al., 2007; Segoni et al., 2009; Berti et al., 2012; Staley et al., 2013; Robbins, 2016; Corsini and Mulas, 2017; Leonardsuzzi et al., 2017) have been developed trying to minimize these false alarms.

In this chapter, I focus on the first three weaknesses, since they are largely related to the dynamics of soil moisture. Landslides are not directly caused by rainfall, but by the increase of pore pressure in the soil (Ray and Jacobs, 2007; Aristizábal, 2013). This increase depends on the soil moisture, playing s a critical role in triggering slope failure (Montgomery and Dietrich, 1994; Pelletier et al., 1997; Iversion, 2000; Ray and Jacobs, 2007; Ponziani et al., 2012; Peres and Cancelliere, 2016; Segoni et al., 2018; Zhao et al., 2019). Most of the methodologies applied to obtain the rainfall thresholds include the antecedent rainfall as a proxy of soil moisture.
(Segoni et al., 2009, 2018; Valenzuela et al., 2018) because it is considered the most relevant factor controlling the soil degree of saturation (Campbell, 1975; Crosta, 1998; Godt et al., 2006; Frattini et al., 2009). However, some authors suggest explicit include the antecedent soil moisture because $s$ and antecedent rainfall are poorly correlated (Pelletier et al., 1997; Godt et al., 2006; Segoni et al., 2009). This is due to soil moisture dynamics depend not only of rainfall but other phenomena as evaporation, transpiration, winds, leakage, etc. By explicitly considering $s$ in the definition of thresholds, the predictive capacity is improved (Segoni et al., 2018; Zhao et al., 2019). Soil moisture has been explored in the empirical definition of thresholds by using direct measures, satellite observations, and soil water balances (e.g. Crozier, 1999; Glade et al., 2000; Godt et al., 2006; Ray and Jacobs, 2007; Ponziani et al., 2012; Posner and Georgakakos, 2015; Mirus et al., 2018; Segoni et al., 2018; Valenzuela et al., 2018; Zhao et al., 2019).

In this study, the model of Rodríguez-Iturbe et al. (1999) and Laio et al. (2001) is used to calculate the probability distribution of soil moisture, and the derived distribution technique (Eagleson, 1978a) to calculate the probability distribution of the Factor of Safety (FS) for soil slips. These models are also used to obtain the histograms of FS and $s$ by Monte Carlo simulations. This approach has the advantage of having a clear physical meaning of the processes relating rainfall, soil moisture, soil shear strength and factor of safety, directly account the highly variable soil moisture prior to and during the rainfall events. Furthermore, it gives both the simulated and the analytical pdf of FS given some properties of the slope, its soil, vegetation and rainfall. It is noticed that positive pressures are not considered due to the restrictions of the ecohydrological model and because the objective is to analyze the effect of suction changes on slope stability. Furthermore, as the soil is homogeneous, perched water tables are not developed. The proposed model here does not pretend to predict the occurrence of landslides but to help to understand the relationship between rainfall and FS and soil moisture and FS.

## 5.2. Methods

An approach combining a stochastic rainfall model with a physically-based hillslope model is used. They are presented both the simulated and the analytical pdf of FS given a stochastic rainfall for the five scenarios comprising the climate, soil, and slope parameters shown in Table 5-1. Analytical pdfs are obtained by using the Erlang distribution, the ecohydrological model of Rodríguez-Iturbe et al. (1999), and the infinite slope model. Simulated pdfs are calculated with the model of Rodríguez-Iturbe et al. (1999) discretized, the infinite slope model and the Monte Carlo approach.

The five scenarios evaluate the behavior of FS for two types of soil (loamy sand and loam), two types of vegetation (grass and trees) and two values of long-term rainfall intensities (0.43 and 1.40 cm d$^{-1}$). Scenarios A, B and C comprise loamy sand soil, and C and D, loam soil. Scenarios A, C and D comprise grass cover, and B and D, trees cover. Finally, scenarios from A to D have a long-term rainfall intensity of 0.432 cm d$^{-1}$, and scenario E, of 1.4 cm d$^{-1}$. The
parameters are based on Table 3-1, Table 2 of Ridolfi et al. (2003), Table 2 of Ren et al. (2013) and Table 2 of Clapp and Hornberger (1978). All simulations are performed for 8000 days long period.

5.2.1. Rainfall model

As in the Rodríguez-Iturbe et al. (1999) model, the 1-day rainfall model is a Poisson process with an arrival rate $\lambda$. $\lambda$ can be interpreted as the probability of occurrence of a rainy day, $P(h_1 > 0)$ (Eagleson, 1978a), hence, the probability of a dry day is $P(h_1 = 0) = 1 - \lambda$. The rainfall depth of the rainy days follows an exponential distribution, with pdf and cdf given by:

$$f_{H_n}(h_n) = \frac{1}{\alpha} e^{-\frac{h_n}{\alpha}} \quad \text{for} \quad h_n \geq 0. \quad (5-1)$$

$$F_1(h_n) = 1 - e^{-\frac{h_n}{\alpha}}. \quad (5-2)$$
Denoting $\alpha$ the mean rainfall depth of rainy days, and $h_n$ the daily $n$-day cumulative rainfall depth. The probability of $h_n = 0$ is given by:

$$
\begin{align*}
\Pr(h_n = 0) &= \Pr(h_1 = 0 \land h_2 = 0 \land \cdots \land \Pr h_n = 0), \\
\Pr(h_n = 0) &= (1 - \lambda)^n.
\end{align*}
$$

The arrival rate $\lambda_n$ is the probability of $h_n > 0$, as given by:

$$
\begin{align*}
\Pr(h_n > 0) &= 1 - \Pr(h_n = 0), \\
\Pr(h_n > 0) &= 1 - (1 - \lambda)^n.
\end{align*}
$$

The sum of $n$ independent exponentially distributed random variables with a common mean $\alpha$ follows the Erlang distribution, whose pdf and cdf are (see Grinstead and Snell, 2006; Krishnamoorthy, 2006):

$$
\begin{align*}
\frac{\alpha}{(n-1)!} e^{-\frac{x}{\alpha}} x^{n-1}, \\
\frac{\alpha}{(n-1)!} e^{-\frac{x}{\alpha}}, \\
1 - \sum_{j=0}^{n-1} \frac{1}{j!} e^{-\frac{x}{\alpha}} \left( \frac{x}{\alpha} \right)^j.
\end{align*}
$$

The probability of $h_n$ is given by:

$$
F_{h_n}(h_n) = \Pr(h_n = 0) + \sum_{k=1}^{n} \binom{n}{k} \lambda^k (1 - \lambda)^{n-k} F_E(h_n | \alpha, k),
$$

where the first term corresponds to the probability of having $h_n$ equals to zero (see Eq. 5-3), and the second term to the probability of having $k$ no-null values and $n-k$ null values. The Poisson distribution can be viewed as arising from the binomial distribution (Krishnamoorthy, 2006). Then, replacing Eq. 5-6 in Eq. 5-7, and $\binom{n}{k}$ by $\frac{n!}{k!(n-k)!}$, $F_{h_n}$ is expressed as:

$$
F_{h_n}(h_n) = \Pr(h_n = 0) + \sum_{k=1}^{n} \frac{n!}{k!(n-k)!} \lambda^k (1 - \lambda)^{n-k} \left[ 1 - \sum_{j=0}^{k-1} \frac{1}{j!} e^{-\frac{h_n}{\alpha}} \left( \frac{h_n}{\alpha} \right)^j \right].
$$
5.2 Methods

5.2.2. Water balance and soil moisture

The stochastic rainfall model described in the previous section is used as an input of the ecohydrological model of Rodríguez-Iturbe et al. (1999) (see section 3.3.1). When using this model, it is taking into account the temporal dynamics of soil moisture, by means of mechanistic models relating its response to rainfall and evapotranspiration. ET cannot be ignored in landslides triggered by long duration rainfalls (Van Asch et al., 1999; Guzzetti et al., 2007; Van Asch et al., 2009). This model requires some simplifications in obtaining the factor of safety not related to the stability slope model, such as the failure surface must have a depth equal to or greater than the rooting depth (to the mass balance be valid), soil and soil moisture are homogeneous throughout the slope, and changes in soil suction due to rainfall variability have daily resolution.

5.2.3. Slope stability and non-exceedance of FS

The infinite slope is a widely used approach to evaluate soil slips (e.g. D’Odorico and Fagherazzi, 2003; D’Odorico et al., 2005; Rosso et al., 2006; Tsai and Yang, 2006; Arnone et al., 2011; Lepore et al., 2013; Alvioli et al., 2014; Valentino et al., 2014; Tang et al., 2015; Ho and Lee, 2017) since it allows relating the factor of safety and soil moisture in 1-D. Among its simplifications are a small thickness of the soil relative to the slope length, slope angle constant along the slope height, failure surface parallel to the slope surface, deep water table, rainfall uniform in space, and the resulting lateral forces are neglected. The effect of the slope-parallel unsaturated flows on water pressure variability is negligible at the storm timescale with respect to infiltration (D’Odorico et al., 2005). It is assumed than landslides triggered by rainfall occur due to the water entering the soil as a result of infiltration that reduces the suction. This approach differs from those assuming that the wetting front advance vertically saturating the soil, since water entering the soil is distributed homogeneously until the fixed failure surface. Moreover, the developed methodology neglects the building-up of a perched water table, that can be determinative in some cases, since the soil mass is homogeneous.

The infinite slope approach is applied together with the extended shear strength criterion for unsaturated soils by Fredlund et al. (1978). This criterion relates shear strength with soil moisture through the suction, being s the dynamic element in the FS estimation (e.g. Montgomery and Dietrich, 1994; Iverson, 2000; Godt et al., 2009). Soil moisture can modify soil suction (ψ) and weight (W). When s affects only ψ, the factor of safety is given by:

\[ FS = FS_c + FS_\psi s^{-b} + FS_\phi, \]  

where

\[ FS_c = \frac{d}{\gamma_t Z \sin \theta \cos \theta}, \quad FS_\psi = \frac{\bar{\psi}\gamma_w \tan \phi^b}{\gamma_t Z \sin \theta \cos \theta}, \quad FS_\phi = \frac{\tan \phi'}{\tan \theta}, \]  

(5-9)
where $s$ is the soil water content, $c'$ is the effective soil cohesion, $\phi'$ is the effective friction angle, $\phi^b$ is the rate representing the shear strength changes due to matric suction changes, $Z$ is the depth of the soil, $\gamma_t$ is the soil unit weight, $\gamma_w$ is the water unit weight, $\theta$ is the slope angle, and $\overline{\psi_s}$ and $b$ are experimentally determined parameters to define the soil-water retention curve of Clapp and Hornberger (1978). As mentioned before, the ecohydrological model limits the depth of the failure surface to values equal or greater than the rooting depth $Z_r$. For these analyses, $Z$ is set equal to $Z_r$. When $s$ affects only soil weight, FS is given by

$$FS = \frac{FS_c^s}{1 + n \frac{\gamma_w}{\gamma_d} s} + FS_\phi,$$  

(5-11)

where $\gamma_d$ is the dry unit weight of soil, $n$ is porosity, and

$$FS_c^s = \frac{c'}{\gamma_d Z \sin \theta \cos \theta},$$  

(5-12)

and when $s$ affects both $\psi$ and $W$ by:

$$FS = \frac{FS_c^s - FS_\psi^s s^{-b}}{1 + n \frac{\gamma_w}{\gamma_d} s} + FS_\phi,$$  

(5-13)

where

$$FS_\psi^s = \frac{\overline{\psi_s} \gamma_w \tan \phi^b}{\gamma_d Z \sin \theta \cos \theta}.$$  

(5-14)

The analyses are made for the conditions of $s$ yield a factor of safety less than four. It is generally assumed that a landslide takes place when FS is less than one, however, since for these scenarios the infinite slope model with realistic soil parameters hardly gives such low values, I decide to define as an FS threshold ($FS_f$), a higher value to perform the theoretical analysis. The above can be supported by the heterogeneity of the soil and the variables that are not taken into account in the modeling. Furthermore, the objective of this test is to analyze the influence of rainfall on landslides, and not to predict the occurrence of them.

5.2.4. Probability distribution of factor of safety

The pdf of FS as a function of $s$ is obtained by adopting the derived distribution technique (see section 3.3.4). In this case, the independent variable is $s$ (Eq. 3-14), the dependent variable is FS and the functions relating $s$ and FS are Eqs. 5-9, 5-11 and 5-13. The pdf of FS when $s$ only affects suction is:
where \( f s_1 = FS - FS_c - FS_\psi \), \( T_1 = \lambda' \omega_{s-w} - \omega_{s-h} \), \( T_2 = \lambda' \omega_{s-w} - \omega_{s-w} \), \( T_3 = \frac{\lambda'}{\beta \eta (\eta-m)} \), and \( T_4 = \frac{\lambda' \omega_{s-fc}}{\eta} \). When \( s \) affects only soil weight, \( FS \) is:

\[
 f (FS) = \begin{cases} 
 \frac{C w (FS_c^2 - FS_\psi)}{\omega_{s-h}} \left[ \frac{(w-s_h)fs_2-wFS_\psi}{s-w} \right] T_1 \cdot e^{-\gamma \frac{fs_2-FFS_\psi}{fs_2}} & s_h < \frac{fs_1}{FS_\psi} \leq s_w \\
 \frac{C w (FS_c^2 - FS_\psi)}{\omega_{s-h}} \left[ \frac{(w-s_h)fs_2-wFS_\psi}{s-w} \right] T_2 \cdot e^{-\gamma \frac{fs_2-FFS_\psi}{fs_2}} & s_w < \frac{fs_1}{FS_\psi} \leq s^* \\
 \frac{C w (FS_c^2 - FS_\psi)}{\omega_{s-h}} \left[ \frac{(w-s_h)fs_2-wFS_\psi}{s-w} \right] T_3 \cdot e^{-\gamma \frac{fs_2-FFS_\psi}{fs_2}} + \beta s_{fc} & s^* < \frac{fs_1}{FS_\psi} \leq s_{fc} \\
 \frac{C w (FS_c^2 - FS_\psi)}{\omega_{s-h}} \left[ \frac{(w-s_h)fs_2-wFS_\psi}{s-w} \right] T_4 \cdot e^{-\gamma \frac{fs_2-FFS_\psi}{fs_2}} + \beta s_{fc} & s_{fc} < \frac{fs_1}{FS_\psi} \leq 1, 
\end{cases}
\]

where \( w = \frac{\tau a}{\eta \gamma w} \) and \( fs_2 = FS - FS_\psi \). Finally, when \( s \) affects both \( \psi \) and \( W \), it is not possible to analytically express the pdf of \( FS \) since Eq. 5-13 is not invertible for \( s \), being necessary to use a numerical method. Eqs. 5-15 and 5-16 allow finding the probability density of \( FS \) from the mechanical parameters of the soil, the slope angle, the maximum evapotranspiration rate \( (E_{max}) \), the evaporation rate \( (E_w) \), and the soil moisture thresholds defined in the model of Rodríguez-Iturbe et al. (1999) and Laio et al. (2001).
5.2.5. Numerical simulations

Precipitation is simulated numerically precipitation following the exponential distribution of $h_n$ and $\lambda'$, Eq. 3-1 (using Eqs. 3-7 to 3-10), and Eq. 5-9 by the Monte Carlo technique. Simulations have a temporal step of 1 day and a length of 8000 days. The precipitation results are compared with those given by Eq. 5-8, soil moisture results with Eq. 3-16, and FS results with Eq. 5-15.

5.3. Results and Discussion

Fig. 5-1a shows the pdf of $s$, and Fig. 5-1b, c and d the pdf of FS when $s$ affects the soil suction, when affects the soil weight, and when it affects both $\psi$ and $W$, respectively. It is noted that by using the described models to calculate $s$ and FS, the stochasticity of $s$ only plays a remarkable role in the stochasticity of FS when it affects the soil suction. This test is done for different values of soil parameters, obtaining similar results. Given the above, for the analyzes detailed below, only the case in which $s$ modifies the soil suction is taking into consideration, neglecting the soil moisture effect on soil weight.

Table 5-2 shows the number of days in which FS is less than the defined threshold of FS ($FS_f$) of each scenario and the soil moisture value that results in FS equal to $FS_f$. According to the models used, low values of FS occur more often in slopes covered by trees (scenarios A and
Table 5-2: Results of scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>n. events</th>
<th>$s_{fail}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
<td>0.92</td>
</tr>
<tr>
<td>B</td>
<td>39</td>
<td>0.58</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>D</td>
<td>23</td>
<td>0.75</td>
</tr>
<tr>
<td>E</td>
<td>126</td>
<td>0.95</td>
</tr>
</tbody>
</table>

C) than by grass (scenarios B and D), which can be explained by the greater evapotranspiration in the long-term. High ET values result in low soil moisture values, and consequently in higher FS. This can be contradictory, since the roots of the trees are larger and stronger than those of grasses, increasing the cohesion of the soil (and the shear strength). However, this modeling does not take into consideration this property. In slopes composed of loam sandy soil (scenarios A and B), low values of FS are more frequent than in loam soil (scenarios C and D). Loam sandy has both cohesion and suction lower than loam, with more water being necessary to carry FS to values below $FS_f$. The last scenario (E) corresponds to the same parameters as scenario A but with more long-term precipitation (higher $\alpha$ and $\lambda$ values). The mean rainfall intensities are 0.432 and 1.4 cm d$^{-1}$ in scenarios A and E, respectively. As intensity in E is higher, soil moisture is more likely to be above the value resulting in FS less than $FS_f$ ($s_{fail}$), being days with FS lower than $FS_f$ considerably higher. These results are in agreement with those obtained by Muñoz et al. (2016, 2018).

Fig. 5-2 shows the time series (left panels) and exceedance probability (right panels) of the factor of safety (top), soil moisture (center) and 1-day rainfall (bottom) for Scenario A. Red lines indicate the probability of FS taking a value less than $FS_f$. The establishment of a $FS_f$ and the following analysis is accomplished looking for patterns in the rainfall behavior that result in FS below $FS_f$, in order to physically define the rainfall thresholds resulting in FS lower than $FS_f$. However, although six events occur during the 8000 days simulation of scenario A, not all of them are related to similar precipitation features. Figure 5-3 shows these events, being n-day the number of days in which rainfall is accumulated (from 1 to 40 days). Gray dots represent the n-days cumulative rainfall during the simulation, and the red numbers the n-day cumulative rainfall corresponding to the event. The features of each event when $FS$ reaches a value lower than $FS_S$ are:

- Event # 1 and 3 are triggered by a short period of heavy rainfall. There is no or almost no rainfall in the previous 20 days.
- Event # 2 occurs after 15 rainfall days. There is a heavy rainfall 9 days before and light rainfall events 5 and 6 days before.
- Event # 4 occurs after 7 days of no rain. It could have been triggered by a one-day rain
pulse or by exceeding the 20-day threshold. The rainfall amount the day is relatively large (according to the corresponding probability distribution).

- Event # 5 occurs after a long period of almost constant rainfall.

- Event # 6 occurs after 9 days of rainfall. From 10 to 37 days before there is almost no rain.

According to the above, values of FS lower than $FS_f$ can occur due to rainfall with different features, regardless of whether the parameters of soil, vegetation, geometry, and climate (long-term rainfall) remain fixed. This indicates that when defining fixed values of precipitation quantity (or intensity) or accumulated precipitation during a certain number of days, false positive and false negative are frequent. False positives are understood as the cases in which a fixed precipitation threshold (or some of its characteristics) indicating the occurrence of landslides, may not trigger one of it. For example, a large value of $P_1$ occurs around day 5500 (indeed larger than $P_1$ related to event # 1) and no event occurs that day. On the other hand, false negatives occur when rainfall events that in general do not trigger landslides, do it, as the event # 3 that is synchronized with a $P_1$ value similar to those around 230, 250, 325 and 700 days, where landslides do not occur. Analogous results are obtained to analyze different n-days cumulative rainfall (see K).
5.4 Summary and Conclusions

The driving effect of rainfall on soil strength and possible slope failure follows a complex dynamics that make landslide frequency very difficult to estimate from rainfall data. It is well known that it is soil moisture, and no rainfall, the variable that really triggers the failure process. Yet, the correlation between some rainfall exceedances and some slope failures has built up the idea of a straightforward method to predict landslides. A similar path followed the unit hydrograph theory for streamflows estimation (Rigon et al., 2016). The physical mechanisms of slope failure are masked by the rainfall threshold method when only statistical tools are used. As a consequence, it is common to find both false positives and false negatives when applying this method, especially when it is used with a few previously established (or ‘standard’) time windows. Therefore, it is very important to strengthen the analyses with physically-based models (Casadei et al., 2003; Aleotti, 2004; Aristizábal et al., 2011).

In this chapter, a combination of an ecohydrological model to relate the rainfall to soil moisture, and the infinite slope approach for unsaturated soils to relate soil moisture to the factor of safety was used. Monte Carlo simulations were performed and analytical expressions of the pdf of cumulative rainfall, soil moisture, and factor of safety were proposed. Through five scenarios, the effects of two types of soil, two types of vegetation cover, and two values of mean rainfall were analyzed. The results indicated that it is more probable the occurrence of landslides in slopes constituted by loam sandy than by loam soils since the shear strength due

Figure 5-3.: Landslide events from a run for Scenario A.
to its cohesion and suction is lower. The slopes covered by trees are more prone to landslides
that those covered by grass, because they have a lower evapotranspiration rate, maintaining
the soil moisture higher than grasses. In this analysis, it was taken into account neither the
effect of the contributed strength by the roots nor the weight that vegetation exerts on the
soil. By increasing the mean of rainfall in the long-term, the number of days in which the FS
is lower than the defined threshold of FS, increases remarkably.

By means of the simulations, the characteristics of rainfall from 0 to 40 days before the
occurrence of each event where FS has a value lower than the threshold is analyzed. It was found
that a slope can reach a valued of FS due to multiple rainfall features, regardless of whether
the properties of the soil, vegetation, and climate remain constant. The results indicate that
the widely used rainfall threshold approach can result in both false positive and false negative
alerts, since it is assumed a linear relationship between rainfall and landslides, leading to
serious consequences in its use in early warning systems. Hydrological processes (and not only
the rainfall) play a fundamental role both in the initial conditions and the trigger of landslides.
The need to understand these processes becomes evident to forecast their occurrence in a
reliable way under the actual conditions and induced by climate and land use change (Van
6. Conclusions and future work

In this thesis, I have analyzed the dynamics of soil moisture and evapotranspiration in water- and energy-limited ecosystems. I started modeling and studying the available radiation for plants, analyzing the effects of stochastic and deterministic available energy in the soil-climate-vegetation system, and parameterizing the link between transpiration and photosynthetically active radiation. As an application, I studied landslides frequency when they are related to soil moisture. All analyses, except the application, were performed in 28 sites located around the world. Chapter 2 investigates the stochasticity of radiation in the PAR spectral band by using the clearness and the clear-sky indices, finding that the clear-sky index describes in a better way the effects of the stochastic atmospheric components, such as water vapor, aerosols, pollution, etc. Unlike the clearness index, the sky-index removes besides the astronomical seasonality, the optical mass seasonality. Theoretically, both indices should be in the range [0,1], however, it was found that values greater than 1 are more common in sites with seasonal snow that maximizes the refraction of the light. The empirical pdfs of $c$ and $k$ showed three shapes: bimodal, unimodal with high dispersion, and unimodal with low dispersion. According to the Holdridge life zones and the Köppen classification, bimodal pdfs are associated with oceanic, humid continental, and Mediterranean climates, high dispersion unimodal pdfs with the monsoon, tropical savanna, and tropical rain-forest climates, and low dispersion unimodal pdfs with semi-arid, temperate, subtropical humid and desert climates. The above indicates clear spatial patterns of stochastic radiation associated with the macroclimate. Seasons in which the indices have a similar behavior conditioned by the rainfall occurrence were defined, and the values of $c$ and $k$ in these seasons were fitted to Beta pdfs. Results are in good agreement in almost all sites, excepting some cases in the tropics in which dry days have narrow pdfs. However, since the variability of the indices is so low in these cases, those were assumed as constants.

Chapter 3 defines periods in which it is realistic to assume both rainfall and radiation parameters as stationary (the stationary periods), by using the $k$ seasons defined in chapter 2, the growing season in the extratropics, and the rainfall seasons in the tropics. During the stationary periods, soil moisture and evapotranspiration dynamics were evaluated by numerical simulations, and for constant and stochastic radiation. The results indicate that the stochasticity of radiation does not play a significant role in soil moisture, but in evapotranspiration, which can be explained by the greater lag of soil moisture to modulate climate fluctuations. As evapotranspiration showed to be influenced by stochastic radiation, a semi-analytical solution of evapotranspiration dynamics forced by stochastic rainfall and stochastic radiation was
Chapter 4 studies sub-daily transpiration by coupling models of photosynthesis, stomatal conductance, and transpiration. The results were integrated at a daily level and an expression to relate transpiration and available energy was proposed. After analyzing the factors limiting evapotranspiration and the evapotranspiration description of the ecohydrological model of Rodríguez-Iturbe et al. (1999) and Laio et al. (2001), it was found that this model (under its assumptions) manages to describe the daily soil moisture dynamics in energy-limited ecosystems. The above, as long as the $E_{\text{max}}$ parameter is calculated taking into account the available energy, and the stationary of both rainfall and radiation is maintained.

Finally, chapter 5 studies the frequency of landslides triggered by rainfall using the combination of the ecohydrological model and the infinite slope approach for unsaturated soils. It was analyzed the days in which the factor of safety is lower than a defined threshold of FS in 5 scenarios that vary the type of soil, the type of vegetation cover, and the mean rainfall. Slopes constituted by loam sandy (loam) soil and covered by trees (grass) are more (less) unstable, indicating the important role of maximum evapotranspiration and soil cohesion in slope stability. High rainfall in the long-term results in a notable rise in the number of days with low FS, since it derives in high values of $s$. When analyzing in detail the precipitation before the occurrence of each low value of FS, it was realized that a slope with determined soil, vegetation cover and climate features can failure by rainfall events with very different features. These results suggest that the rainfall threshold approach may result in frequent false positive and false negative alarms because shallow landslides occur due to the decrease of shear strength of soil, which is directly related to soil moisture, and not to the rain.

This thesis is about moving forward in the understanding of radiation, soil moisture, transpiration, and landslides dynamics in both energy- and water-limited ecosystems. However, because of the highly non-linear phenomena occurring in the soil-climate-vegetation, with feedbacks, and high spatiotemporal variability, many questions still remain to be solved. In chapter 2, I only studied the PAR spectral band since the objective was to model the evapotranspiration. However, it would be interesting to study radiation in other spectral bands for other applications such as the availability of energy for human activities.

In the analyses carried out in chapters 3, 4 and 5, I consider stationarity in the parameters describing rainfall and radiation, restricting the solutions to periods in which this assumption is valid. Laio et al. (2002) analyzed the seasonality of the precipitation using an approach based on the Rodríguez-Iturbe et al. (1999) model, but the seasonality of radiation in this model has not been studied, being this a potential future work. The methodologies used in these chapters can be easily modified to analyze the responses of the phenomena occurring in the soil-climate-vegetation system in the face of climate change and climate variability.

As the ecohydrological model is on a daily scale, many phenomena that may be relevant in energy-limited ecosystems, such as the sub-daily variability of radiation and the response of vegetation to it, were neglected. On the other hand, Rodríguez-Iturbe et al. (1999) model assumes a depth water table, ignoring its effects on the soil moisture dynamics, and these
effects can be significant in humid sites, and specifically in landslides application. In places where there are very intense rainfall events, it is less likely infiltration to depend on the soil void ratio, that is, runoff occurring only when soil is saturated (Dunne mechanism), but the water entering the soil is a function of the soil infiltration capacity (Hortonian mechanism).

The study of these problems can be fundamental for the comprehension and generalization of soil moisture dynamics, however, it is possible that many of these problems have no analytical solution. In chapters 2 and 4, I proposed expressions involving the obtain of fitting parameters (to model \( k \) and \( c \), and to relate \( T \) and PAR), but no attempts were made at a physical explanation of them.

The finite slope method was used to calculate the factor of safety, being aware that it has simplifications that in some situations are unrealistic, such as the spatiotemporal soil homogeneity, the predefined failure surface and its parallelism with the slope surface. To obtain more reliable results, improvements must be made in the physical model reflecting the conditions of the problem analyzed. The simplifications taking into consideration in this thesis were carried out trying to maintain analytical solutions. Finally, we notice that the approaches used in this thesis do not intend to predict the actual behavior of any of the studied phenomena, but they seek to help to understand the dynamics and relationships between climate variables and generalize the results.
Nomenclature

\( \alpha \) Mean rainfall depth of rainy days

\( \alpha_{PT} \) Constant of Priestley-Taylor

\( \beta \) Soil fitting coefficient

\( \beta_i \) Fitting parameter of Beta pdf

\( \Delta \) Interception by canopy threshold

\( \gamma \) Solar altitude

\( \Gamma^* \) CO\textsubscript{2} compensation point

\( \gamma_0 \) CO\textsubscript{2} compensation point at \( T_0 \)

\( \gamma_d \) Dry unit weight of soil

\( \gamma_p \) Psychometric constant

\( \gamma_t \) Unit weight of soil

\( \gamma_w \) Unit weight of water

\( \lambda \) Mean time between rainy days

\( \lambda_v \) Latent heat of vaporization

\( \mu_{PAR_0} \) Mean of irradiation at TOA in the PAR range

\( \overline{\psi} \) Suction for saturation conditions

\( \phi' \) Effective friction angle

\( \phi^b \) Rate representing the shear strength changes due to matric suction changes

\( \pi_1 \) Dryness index

\( \pi_2 \) Storage index

\( \pi_3 \) Runoff index
\( \pi_4 \) Infiltration index

\( \psi_a \) Atmospheric water potential

\( \psi_s \) Soil water potential

\( \rho_a \) Air density

\( \rho_w \) Water density

\( \tau \) Transmittance due to absorption of the atmosphere

\( \tau_g \) Mixed gases transmittance

\( \tau_o \) Ozone transmittance

\( \theta \) Slope angle

\( a \) Fitting parameter to relate PAR and \( T \)

\( a_1 \) Empirical constant of Leuning’s model

\( A_c \) Photosynthesis rate limited by Ribulose bisphosphate carboxylase-oxygenase (Rubisco) activity

\( A_n \) Net carbon assimilation

\( A_q \) Photosynthesis rate limited by Ribulose bisphosphate (RuP\(_2\)) regeneration through electron transport

\( ACF \) Autocorrelation Function

\( c \) Clear-sky index

\( c' \) Effective soil cohesion

\( c_a \) Atmospheric CO\(_2\) concentration

\( c_i \) CO\(_2\) concentration in the mesophyll cytosol

\( c_p \) Specific heat of air at constant pressure

\( c_s \) CO\(_2\) concentration at the leaf surface

\( cdf \) Cumulative distribution function

\( D \) CO\(_2\) Saturation vapor pressure deficit

\( D_x \) Empirical constant of Leuning’s model
Conclusions and future work

$e$ Ratio molecular weight of water vapor/air

$E_{\text{max}max}$ Maximum evapotranspiration for unlimited water and energy

$E_{\text{max}}$ Maximum evapotranspiration rate for unlimited water

$ET$ Evapotranspiration rate

$FS$ Factor of safety

$FS_f$ FS threshold

$g$ Mixed gases

$g_a$ Atmospheric conductance

$g_b$ Leaf boundary layer conductance

$g_s$ Stomatal conductance

$g_{ba}$ Series of leaf boundary conductance and atmospheric conductance

$H$ Surface daily global radiation on a horizontal surface

$H_0$ Extraterrestrial daily global radiation on a horizontal surface

$H_b$ Direct global irradiance

$H_d$ Diffuse global irradiance

$H_{cda}$ Cloudless-sky daily global radiation on a horizontal surface

$H_{dJ}$ Deactivation energy for $J_{\text{max}}$

$H_{dV}$ Activation energy for $V_{c,max}$

$H_{Kc}$ Activation energy for $K_c$

$H_{Ko}$ Activation energy for $K_o$

$H_{vJ}$ Activation energy for $J_{\text{max}}$

$H_{vV}$ Activation energy for $V_{c,max}$

$I$ Rainfall rate intercepted by canopy

$I_0$ Extraterrestrial spectral irradiance normal to the rays from the sun

$J$ Electron transport for a given absorbed photon irradiance
$k$  Clearness index

$k_s$  Saturated hydraulic conductivity

$K_{c0}$  Michaelis constant for CO$_2$ at $T_0$

$k_g$  Absorption attenuation coefficient for mixed gases

$k_o$  Absorption attenuation coefficient for oxygen

$L$  Leakage rate

$l_o$  Amount of ozone in cm

$LAI$  Leaf Index Area

$m_r$  Relative air mass

$n$  Porosity

$NEE$  Net ecosystem CO$_2$ exchange

$o$  Ozone

$o_i$  Oxygen concentration

$P$  Rainfall rate

$P_a$  Atmospheric pressure

$PAR$  Phosynthetically Active Radiation

$PAR_0$  Extraterrestrial daily PAR on a horizontal surface

$PAR_{cda}$  Cloudless-sky daily PAR on a horizontal surface

$PAR_{obs}$  Surface daily PAR on a horizontal surface

$pdf$  Probability density function

$PPFD$  Photosynthetic Photon Flux Density

$Q$  Photosynthetic photon flux density

$Q$  Surface runoff rate

$R$  Available radiation

$R_g$  Universal gas constant
Soil moisture

Incipient stomata closure

Field capacity

Hygroscopic point

Wilting point

Sunset

Solar Spectral Irradiance

Transpiration rate

Time

Reference temperature

Air temperature

Leaf temperature

Maximum daily transpiration

Top of the atmosphere

Universal gas constant

Maximum catalytic activity of Rubisco in the presence of saturating levels of RuP$_2$ and CO$_2$

Soil depth

Rooting depth

Michaelis constant for O$_2$ at $T_0$

Entropy term
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