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A Time-Expanded Network For The Biomedical Sample Transportation Problem

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Red Tiempo-Expandio para el Problema de Transporte de Muestras Médicas

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Abstract

In this research, a new formulation over a time-expanded network for the biomedical sample transportation problem is proposed. The biomedical sample transportation problem is a vehicle routing problem arising in the context of healthcare logistics. The difficulty of this transportation problem is related to the lifespan of the samples because the duration of a route is limited by the time available to complete the transportation of the sample before it perishes. To test the quality and the behavior of the time-expanded formulation, experiments are conducted over a set of real-life inspired instances from the Quebec's laboratory network under the management of the *Ministère de la Santé et des Services sociaux* (Ministry of Health and Social Services). The results expose the convenience of used a time-expanded network to model the biomedical sample transportation problem.

Keywords: time-expanded network, vehicle routing problem, biomedical samples, healthcare logistic

Resumen

En esta investigación, se propone una nueva formulación sobre una red expandida en el tiempo para el problema de transporte de muestras médicas. El problema de transporte de muestras médicas es un problema de ruteo de vehículos que surge en el contexto de la logística en salud. La dificultad de este problema de transporte está relacionada con la vida útil de la muestra porque la duración de una ruta está limitada por el tiempo disponible para completar el transporte de la muestra antes de que perezca. Para probar la calidad y el comportamiento de la formulación expandida en el tiempo, se realizan experimentos en un conjunto de instancias inspiradas en la operación de la red de laboratorios de Quebec bajo la administración del *Ministère de la Santé et des Services sociaux* (Ministerio de Salud y Servicios Sociales). Los resultados exponen la conveniencia de utilizar una red expandida en el tiempo para modelar el problema de transporte de muestras médicas.

Palabras clave: red tiempo-expandido, problema de ruteo de vehículos, muestras médicas, logística en salud.

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1 Introduction

In the context of healthcare, biomedical tests play an important role to support physicians in the accurate diagnosis of diseases. For this reason, every day hundreds of biomedical samples or specimens are taken from patients and need to be analyzed in a fast and efficient way. Large hospitals can perform and analyze the tests in place. However, establishments like community healthcare centers, clinics, retirement homes, etc., serve patients on a daily basis but are not equipped to analyze the samples collected. This creates a need to plan the transportation of the samples from collection facilities to a central laboratory for analysis. The biomedical samples are perishable items and must be analyzed in a limited time frame. This time limit is usually shorter than the collection period of a center. Therefore, it is necessary to schedule multiples visits during the day at each collection center, in order to bring the samples back to the laboratory before they perish.

This research is inspired by a case study of the laboratory network in the Province of Quebec (Canada). To improve the service of the laboratory network, the *Ministère de la Santé et des Services sociaux (MSSS)* started the reorganization project of medical biology laboratories (*OPTILAB*) in 2011. In the frame of *OPTILAB* the *MSSS* divided the Province of Quebec into administrative regions (or service areas) searching a centralized service. For each administrative region, the *MSSS* selects one regional hospital as the central laboratory (Lab) to analyze the samples collected in the assigned facilities, noted Specimen Collection Centers (SCCs). In a service area, the laboratory is highly equipped, with a high level of automation, optimization workflow and extended working hours. The SCCs are the point of encounter with the community (e.g., small clinics, community healthcare centers), where samples are collected, but the samples are not analyzed.

Before to *OPTILAB* reorganization, each SCC planned the sample transportation to the laboratory independently, selecting a third-party logistics operator, as well as the number of visits to schedule and the specific times the visits are performed. This decentralized service and the lack of a standard structure caused a difficult administration with high logistics costs. The *MSSS* centralization policy was proposed to define a standard in transportation decisions and to reduce logistics and operational costs with the potential of economies of scale. In order to define a standard in the transportation of the samples to be analyzed, it becomes necessary to determine a set of routes to performance all transportation requests at minimum cost on each service area. This Vehicle Routing Problem (VRP) aims at deciding

which vehicle handles which request, in which order, so that all routes meet the operational constraints and the lifetime of the samples, while minimizing transportation cost. Anaya-Arenas et al. (2016) denoted this problem as the Biomedical Sample Transportation Problem (BSTP).

The main challenge in the BSTP is related to the sample lifespan. Samples' lifespan is linked to its biological degradation, its nature and the type of tests to be performed on it. After that time, samples deteriorate and may become unusable, increasing laboratory's costs and affecting the service quality. An unusable sample forces patients to make a second collection, delaying both the analysis and the diagnosis and doubling operations costs of the entire process (collecting, transporting, and analyzing), which in addition could be painful for patients. This time limit is an important factor in the routing plan and the transportation cost. A strict constraint with a short time limit implies more visits to each SCC during the day and probably shorter routes. This interdependency between the service quality of the system and the structure of the routes makes of the BSTP a particular and difficult version of the VRP. Thus, new models and tools to efficiently solve the BSTP are necessary to support the planners of the healthcare system.

1.1 Objectives

Past results on the BSTP (Anaya-Arenas et al., 2016) and the VRP with interdependent time windows (Doerner et al., 2008) proved the complexity of the problem and how commercial solvers were ineffective in finding optimal solutions in reasonable short computational time. An efficient metaheuristic has been proposed (Anaya-Arenas et al., 2019), but there is still a challenge in the quality of the solutions obtained, in relation with the optimal solution. These previous works show the necessity of a new efficient formulation for the BSTP that is a complex problem in the field of healthcare logistics. This is what motivates the research, which is expected to improve efficiently the solution for a BSTP with the formulation over a time-expanded network. The present research has the following objectives:

1.1.1 General objective

To create a time-expanded network formulation to solve efficiently the Biomedical Sample Transportation Problem.

1.1.2 Specific objectives

- To propose a mathematical model on a time-expanded network for the biomedical sample transportation problem.
- To solve the proposed model using a commercial solver.
- To test the quality of the formulation solving real-life instances from Quebec's laboratory network.

The document has the following structure: A review of the pertinent literature is presented in Chapter 2. The literature review focuses on two major topics: the VRP with temporal restrictions and blood collection routing. Chapter 3 presents a detailed description of the problem and its characteristics. The mathematical formulation over a time-expanded network for the BSTP is presented in Chapter 4. Chapter 5 describes the preprocessing used to reduce the size of the optimization program. The results of computational experiments applied to real-life instances are presented in Chapter 6. Finally, the Chapter 7 summarizes the main findings of the research, conclusions and recommendations to future works related to this research.

2 Literature Review

Operation Research and Management Sciences (OR/MS) applied to healthcare had addressed different topics, such as operating room planning, nurse staffing and appointment scheduling. Due to this interdisciplinary nature, the literature existent is wide and correspond to different academic fields (see Brailsford & Vissers (2011) for a review of OR/MS applied to healthcare. Hulshof et al. (2012) presents a taxonomic classification of planning decisions in healthcare. Inside this classification the BSTP is in the operational planning axis. Operational planning involves the short-term decision making related to the execution of the healthcare delivery process. At the specific context of Home Health Care operations, nurses are scheduled and routed to perform various services at clients' homes (see Fikar & Hirsch (2017) for a review in this topic). Within a more general context, problems regarding the distribution of goods or services between a central depot and a set of geographically scattered customers are generally known as VRP.

Owing to VRP usefulness in real-life applications, the VRP has been quite studied for more than 50 years. Therefore the literature about VRP and its variants is extensive (see Braekers et al. (2016), Coelho et al. (2016) and Lahyani et al. (2015) for recent reviews). For this reason, the author concentrates the effort in reviewing contributions related with strong temporal restrictions and blood supply routing.

Temporal restrictions arise from real-life settings, such as time-windows on activities, penalized time constraint transgression, time-dependent activities duration or costs, congestion and so on (Vidal et al., 2014). Usually, time restrictions over the customer's service are modeled with time-windows, which leads to the VRP with Time-Windows (VRPTW). The VRPTW aim is to design least-cost routes from one depot to a set of customers, where the routes must ensure that each customer is visited only once by exactly one vehicle within a given time interval (time windows), all routes start and end at the depot, and the total demands on one particular route must not exceed the capacity of the vehicle (Bräysy & Gendreau, 2005b,a). Some practical issues, impose the definition of multiple time-windows for each customer to perform one or several visits (e.g., Amorim et al. (2014); Favaretto et al. (2007); Tricoire et al. (2010)). Nevertheless, the routes' length is usually independent of the moment that a visit is performed. Dumas et al. (1991) presented the pickup and delivered problem with time-windows as a generalization of the VRPTW, where precedence constraint is included. That constraint implies that a given pickup point must be scheduled before its

associated delivery point. Recently, Naccache et al. (2018) proposed the multi-pickup and delivery problem with time-windows that allows pickup required items from different locations to be delivered at one common location. Notwithstanding, a single visit is performed at each pickup location. At healthcare context, specifically in home healthcare planning, Liu et al. (2013) discussed a PDVRPTW where a single visit to a patient's home can be used to deliver (from a hospital and/or a depot) and pick up goods at patients' homes that have to be taken later to the depot or the laboratory. However, Liu et al. (2013) did not consider any restriction on samples' transportation.

In VRPs time depended restrictions are related to the travel times of the arcs. Thus, the travel times vary over the planning horizon (see Gendreau et al. (2015), for a review). Time-dependent restrictions can be treated in a discrete way using time-expanded networks. Time-space networks are one of the most useful modeling approaches for discrete planning and scheduling problems (Fischer & Helmberg, 2014). However, time-expanded networks quickly grow to a large number of arcs in practical situations. Thus, the problem is computationally harder (Skutella, 2009). Boland et al. (2017) proposed a partially time-expanded network algorithm to keep the size of the time-expanded network manageable. To the best of the author's knowledge, this is the first time that a time-expanded network is used to model time-dependent network design problems.

In the field of blood supply, the work of McDonald (1972) was the first in addressed a blood logistic problem. Recently, Osorio et al. (2015) and Pirabán et al. (2019) present comprehensive reviews on blood supply chain management. The main echelons of the blood supply chain are: collection, production, inventory and distribution. The blood specimens transportation is an essential part of the collection and distribution echelons. Because of the nature of blood specimens, a maximum time constraint is common in blood management problems (Baş et al., 2016). Nevertheless, this is not valid in all the cases, for example, Zahiri et al. (2015) address a blood collection and distribution network design problem to allocate temporary and fixed collection facilities over a multi-period horizon. Zahiri et al. (2015) assumed that the length of each period, in the planning horizon, is smaller than the blood's lifetime, thus, maximal arrival time is not considered. Yi (2003) address the problem of blood transport as a variant of VRPTW, applying it to the case study of the American Red Cross. In this case, only one pickup has to be performed at each center. Şahinyazan et al. (2015) present a tour mobile collection system for the Red Cross in Turkey. The aim is to define tours for mobile collection units (bloodmobiles) and shuttles to collect harvested blood. Yücel et al. (2013) included the laboratory's processing rate in the tour's planning to seek a balance between the number of samples processed in a day and the transportation costs. However, in the problems presented by Şahinyazan et al. (2015) and Yücel et al. (2013) the routes are not restricted by any time constraints. On their side, Ghandforoush & Sen (2010) and Mobasher et al. (2015) considered a maximal arrival time at the laboratory.

Nevertheless, just a single pickup at each location is performed. Multiple pickups in sample transportation problems are considered by Anaya-Arenas et al. (2016) and Naji-Azimi et al. (2016). Anaya-Arenas et al. (2016) presented the BSTP, where each sample collection center required several visits and each one of them must happen inside a given time-windows and the samples' lifespan limit the duration of the routes. Naji-Azimi et al. (2016) present a variant of the BSTP, where they minimized the number of trucks' arrivals to the laboratory in each time slot. However, in both cases, it is assumed that the samples deterioration begins when the sample leaves the SCC. Therefore, the available transportation time is not time-dependent.

To the best of of the author's knowledge, only Doerner et al. (2008) and Anaya-Arenas et al. (2019) consider that the samples' deterioration begins once it is collected from the patients. Therefore the available transportation time depends on the moment in which the pickup is performed. Doerner et al. (2008) defined a blood collection problem as VRP with multiple and interdependent time-windows. Assuming a continue production and fixed deterioration times, the author defined a minimum number of pickups, where the time of a pickup at a given customer determines the maximum time available for the subsequent pickup at the same customer. Anaya-Arenas et al. (2019) in addition to the interdependence between pickups, defines the collection centers' opening hours. Even if their context is similar to this research, in their cases, the minimum number of pickup of each SCC is previously defined. Contrarily, in our case, The author seek to define the optimal number of pickups for each SCCs according to the desired level of service. Therefore, we can conclude that none of the previous works attempt to simultaneously define (1) the optimum number of pickups, (2) the moments at which pickups are performed, and (3) consider the available transportation time as time-dependent.

Chapter 3 presents the problem definition of biomedical sample transportation problem inspired by Quebec's laboratory network. In the problem definition, the characteristics that make of this a complex problem (e.g., multiple-pickups, route length limitation) are discussed.

3 Problem Definition

In this chapter, the BSTP is formalized as a transportation problem, inspired by the laboratory network in the Province of Quebec. As it was mentioned, the decision of centralization of the service and the reorganization of the resources generates the necessity of transportation the samples between the SCCs and the laboratory.

Each SCC has a collection period, during this time the patients are attended and the samples are collected. Although the samples are collected in an almost continuous way during the collection period, a SCC generates the transportation requests discretely. According to a desired level of service, a SCC generates one or more transportation requests during its workday. The level of service of a SCC is related to the number of patients to serve, the type of test to perform, the results response time, etc. i.e., the quality of the desired service. The level of service is defined by the network operators according to guidelines of the *MSSS*. According to the level of service, during the time horizon, several transportation requests are assigned to each SCC. Therefore, each SCC has two important characteristics, the collection period and the number of transportation requests to be served.

A transportation request, made by a SCC, is a group of samples that are ready to be transported to the laboratory. Each transportation request has a moment where it is generated and, linking with the lifespan of the samples, a time limit to arrive at the laboratory. Note that taking the time limit until the laboratory is a simplification where the operational time in the laboratory is ignored. To be transported, the samples are grouped into standard coolers. These coolers are too small as compared to the vehicles capacity, therefore, the capacity constraints are ignored.

In the service area, the laboratory also is the point where all the routes must start and end. Moreover, due to the samples' transportation is provided by private carriers the number of vehicles required to transport the samples is not a constraint on the problem. However, it is expected to minimize the number of vehicles used to reduce transportation costs.

Due to the biological origin of the samples, the samples have a lifespan. This is a period in which the sample is useful and the tests can be performed with it. Usually, the sample lifespan is shorter than the SCC's collection period. Thus, with a single visit or pickup is not possible to fulfill all the SCC's transportation request. Therefore, multiple-pickups are

necessary for each SCC. Moreover, the decision of performing a pickup has an impact on the other pickups to be made in the SCC. For example, if a pickup is realized early in the collection period, the next pickup must be made early too and more pickups are necessary. On the other hand, if the pickup is realized late in the collection period, least pickups will be necessary compared with the first case. However, the time available to completed the route couldn't be enough. Thus, multiple-pickups and interdependent-pickups are characteristic of this transportation problem.

The other important characteristic of the BSTP is the limitation in the routes' duration. In this transportation problem, the duration of a route is limited by the time available to complete the transportation requests in the vehicle. Therefore the route must end before reach any of the samples' time limits. Furthermore, the way that the route's duration is affected, depends on the moment that pickup is made. That is, if the pickup is made close to the moment that the transportation request is generated, the time available to complete the transportation request will be greater than the time available if the pickup is made close to reach the time limit of the transportation request. Moreover, the decision of a new pickup in the route changes the duration of the route, when the new group of samples has less time available to reach the laboratory.

To illustrate the characteristics of the BSTP, let's consider a SCC with four transportation requests, evenly distributed along the collection period, as it is shown in Figure 3.1. Now, if the first pickup is made at t_1 , the time limit to reach the laboratory is $TL_1 - t_1$ and the second pickup must be performed no later than TL_2 . Now, if the first pickup is made at time t_1^* , the time limit to reach the laboratory is $TL_1 - t_1^*$, which is fewer than $TL_1 - t_1$, and the second pickup must be performed before than TL_3 , which is after TL_2 . In the same way, the decision of the second pickup affects the third pickup. Moreover, the third pickup could be necessary or not necessary.

Figure 3.1 shows two different sequences of pickup for a SCC. In the first sequence, three pickups are made, and the vehicle has enough time to visit other SCC and reach the laboratory before the time limit. The second sequence only has two pickups but after each pickup the time available to reach the laboratory if fewer than in the first sequence. Thus, the decision to make a pickup affects, the routes' longitude, the time available for the next pickup, and the number of pickups needed.

The BSTP is a VRP with strong temporal constraints, where the aim is to create a transportation plan to pickup the samples from the SCCs and take them to the laboratory, minimizing transportation costs and warranting that all transportation requests are fully satisfied. Figure 3.2 shows an instances for the BSTP, with three SCCs and seven transportation request.

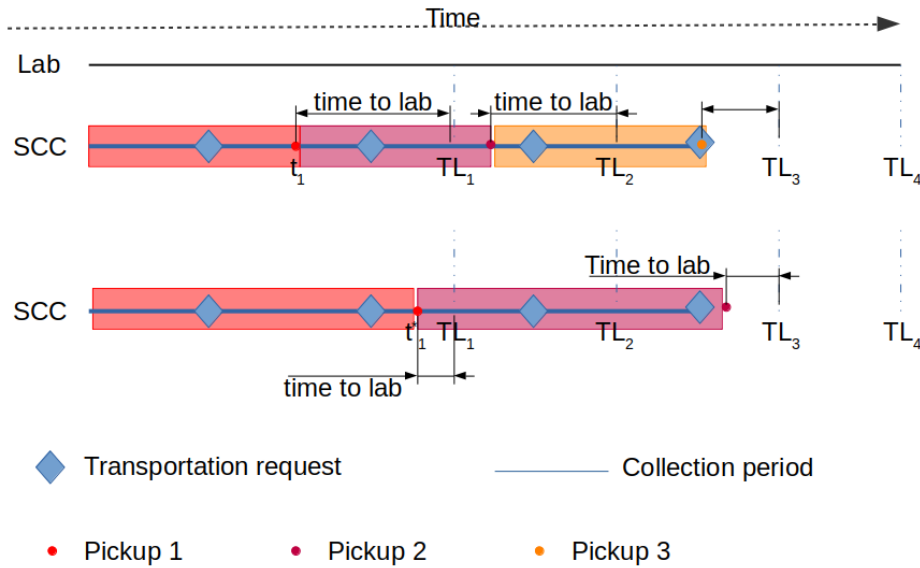


Figure 3.1: Pickup schedule example for a SCC.

Figure 3.2 also illustrates a solution for the instance. In the solution, two routes are used to handle all the transportation requests. The vehicle of the first route leaves the laboratory to the SCC3 and picks the first SCC's transportation request at the same time that it is made. Then, the vehicle goes to the SCC2 and picks up the second group of samples. The next stop of the route is the SCC1, to do the third pickup of the route. In the last pickup of the route, two transportation requests are fulfilled. Finally, the vehicle arrives at the laboratory, to deliver the samples before they reach the time limit. In the second route, the first pickup is in the SCC1, the second pickup is in the SCC3, and the last in the SCC2. Noted that the route length is restricted by the time limit of the second pickup in the route, and the vehicle arrives to the laboratory respected this time limit. In this example is important to highlight that the SCC1 has three transportation requests and only two pickups are necessary to handle these transportation requests.

In Chapter 4 the mathematical formulation used to model the BSTP is presented. This mathematical formalization uses a time-expanded network in which the time horizon is discretized into periods. This discretization allows to simplify the strong temporal restrictions in the problem.

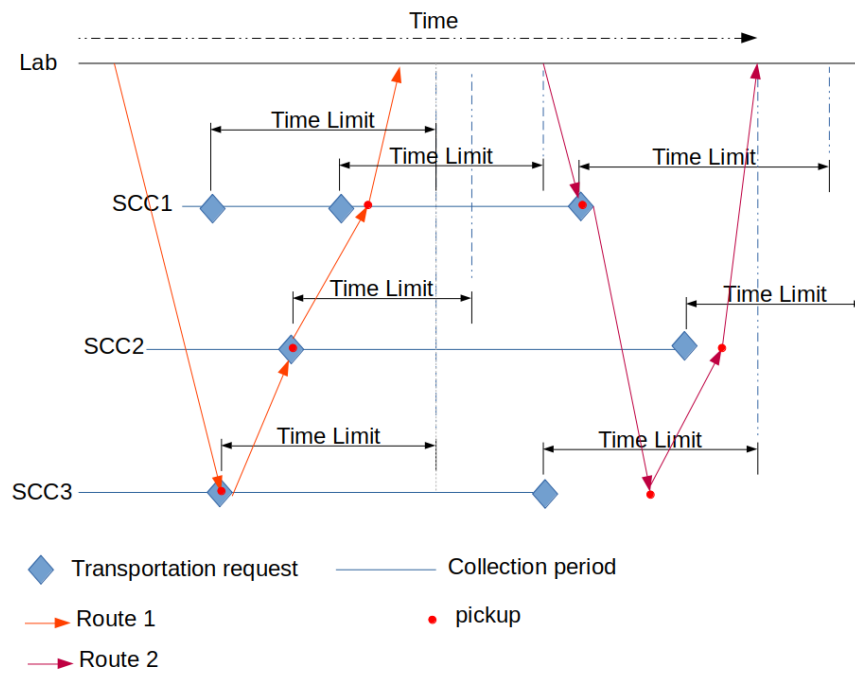


Figure 3.2: Biomedical Sample Transportation Problem example.

4 Mathematical Formulation

This chapter formalizes the problem described in Chapter 3 as a mathematical program over a time-expanded network. Each instance in the BSTP has m vehicles available to transport the samples between SCCs and laboratory. Let be $G = (V, A)$ a network with a node set $V = \{v_0, v_1, \dots, v_n\}$ that represents the geographical location of the laboratory (v_0) and the set of n SCCs, and the arc set $A = (u, v)$ for all $u \in V$ and $v \in V$ where $u \neq v$. The length of each arc $(u, v) \in A$ is the travel time (τ_{uv}) between $u \in V$ and $v \in V$. The travel times include loading and unloading times. The network G is called the “flat” network because it only contains information about the space. The time-expanded network $G^E = (V^E, A^{EH} \cup A^{ED})$ is derived from G and the division of the time horizon (TH) into a set of periods $T = (0, \Delta t, 2\Delta t, \dots, TH)$, where Δt is the length of the period. The set of expanded nodes $V^E = V \times T$, has a time copy of each node $v \in V$ for each time period $t \in T$. For a given node $i = (v, t) \in V^E$, we define $v(i), t(i)$ the physical and temporal components associated with node i . The arc set A^{EH} contains arcs (i, j) for all $i \in V^E, j \in V^E$, where $v(i) = v(j)$ and $t(j) = t(i) + \Delta t$. That is, the horizontal arcs of the time-expanded network. The arc set A^{ED} contains arcs (i, j) for all $i \in V^E, j \in V^E$, where $v(i) \neq v(j)$ and $t(j) - t(i) = \tau_{v(i)v(j)}$. In other words, the arc set A^{ED} contains the diagonal arcs of the expanded network. Note that to expand the network, the travel times have to be rounded up to multiples of Δt , so that any feasible solution on the time-expanded network can be converted to a feasible solution on the flat network. Let be $A^E = A^{EH} \cup A^{ED}$ the set of expanded arcs (i, j) .

Let be K the set of transportation requests, where each $k \in K$ has an origin node $o_k \in V$, a destination node $d_k \in V^E$, and a time limit $T_L(k)$ to be completed. Thus, the transportation request generated by $v(o_k)$ at the instant $t(o_k)$, must arrive to the Lab $v(d_k) = v_0$ no later than $t(d_k) = t(o_k) + T_L(k)$.

The decision variable y_{ij} , represents the number of vehicles that is traveling from $i \in V^E$ to $j \in V^E$. Let be x_{ij}^k represent whether a transportation request $k \in K$ travels from $i \in V^E$ to $j \in V^E$. Let be c_{ij} the cost function define as follows:

$$c_{ij} = \begin{cases} t(j) - t(i) & \text{if } (i, j) \in A^E \setminus \{v(i) = v(j) = v_0\}, \\ 0 & \text{else.} \end{cases} \quad (4.1)$$

The following conventions are used to present the model compactly. Let be $l_0 \in V^E$ the laboratory ($v(l_0) = v_0$) at the time 0 ($t(l_0) = 0$), and $l_{TH} \in V^E$ the laboratory ($v(l_{TH}) = v_0$) at the time $t(l_{TH}) = TH$. Let be $\delta^+(i)$ the sum of all arcs $(j, i) \in A^E$ that arrive to node $i \in V^E$, and $\delta^-(i)$ the sum of all arcs $(i, j) \in A^E$ that leave the node $i \in V^E$. Finally, the BSTP can be modelled over a time-expanded network as follows:

$$\text{MINIMIZE} \quad \sum_{(i,j) \in A^E} c_{ij} \cdot y_{ij} \quad (4.2)$$

Subject to:

$$y(\delta^-(l_0)) \leq m \quad (4.3)$$

$$y(\delta^+(i)) - y(\delta^-(i)) = 0 \quad \forall i \in V^E \setminus \{l_0, l_{TH}\} \quad (4.4)$$

$$x(\delta^-(o_k)) = 1 \quad \forall k \in K \quad (4.5)$$

$$x(\delta^+(d_k)) = 1 \quad \forall k \in K \quad (4.6)$$

$$x^k(\delta^+(i)) - x^k(\delta^-(i)) = 0 \quad \forall k \in K, i \in V^E \setminus \{o_k, d_k\} \quad (4.7)$$

$$x_{ij}^k \leq y_{ij} \quad \forall (i, j) \in A^{ED} \quad (4.8)$$

$$y_{ij} \in \mathbb{N} \quad \forall (i, j) \in A^E \quad (4.9)$$

$$x_{ij}^k = \{0, 1\} \quad \forall (i, j) \forall k \in K, \in A^E \quad (4.10)$$

The objective function (4.2) minimizes the total time traveled by all the vehicles. Constraint (4.3) ensures that no more than m vehicles are used, and leave the node l_0 . Constraints (4.4) ensure that the vehicles' flow conservation is satisfied in all the nodes, so all the vehicles that leave l_0 , arrive to l_{TH} . Constraints (4.5) state that the transportation request k becomes available at node o_k . Constraints (4.6) ensure that the transportation request k arrives at node d_k . Due to $t(d_k) = t(o_k) + T_L(k)$, these constraints ensure that the $T_L(k)$ of k is respected. Constraints (4.7) are the transportation requests' flow conservation constraint, so the transportation request k does not stay in a node different to d_k . These constraints avoid that a transportation request stays in another place different to the laboratory at time d_k . Constraints (4.8) ensure that a transportation request only can leave a physical node $v \in V$ in a vehicle. these are the link between vehicles and transportation requests. Constraints (4.9) and (4.10) are the variables definition.

Due to the time constraints in the problem are simplified to flow conservation constraints, the time-expanded network formulation for the BSTP simplifies the management of the

transportation requests and its time limit. Furthermore, the model optimizes the number of pickups of each SCC according to the level of service desired. Nevertheless, expansion in time increases the size of the network and, consequently, the number of variables. Therefore, the computational efforts are greater as the time granularity gets finer.

A preprocessing is used with the objective of reducing the number of variables of the model. The preprocessing exploits the characteristics of the BSTP and the simplification of the time restrictions over a time-expanded model. Chapter 5 describes the preprocessing proposed to reduce the number of variables in the time-expanded model.

5 Acceleration Techniques

The disadvantage of the use of a time-expanded model is the increase in the number of nodes and arcs of the network, especially for large time horizons with a fine time discretization. To reduce the impact of the time expansion, for the BSTP model a preprocessing approach is implemented. This preprocessing takes advantage of the problem characteristics to reduce the number of expanded nodes, the expanded arcs and the variables associated with them.

For a better explanation of the preprocessing, the following example is used. Figure 5.1 shows the network of 1 central laboratory (v_0) and 4 SCCs (v_1 , v_2 , v_3 and v_4), where the travel times (τ_{uv}) is presented in units of time.

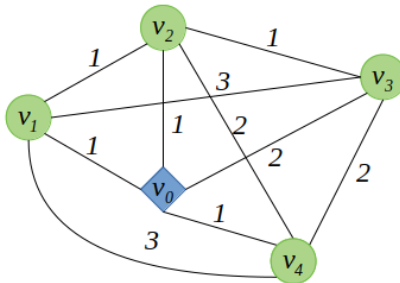


Figure 5.1: “Flat” network example.

When the network presented in Figure 5.1 is expanded into 6 periods (t) of 1 unit of time length, the expanded network presented in Figure 5.2 is obtained. Then, in the time-expanded network the number of the nodes increase from 5 to 30 (i.e., $|V| = 5$ and $|V^E| = 30$) and the number of arcs increase from 10 to 114 (i.e., $|A| = 30$ and $|A^E| = 114$).

The first part of the preprocessing exploits the fact that all the routes must start and end in the laboratory. Therefore in the time-expanded network, the earliest arrival of a vehicle to a node $i \in V^E$ is at time $t = \tau_{v_0v_i}$. Then a node $i \in V^E$ with $t(i) < t = \tau_{v_0v_i}$ will not be part of a feasible solution, due to none vehicle could reach those nodes. On the other hand, if all the routes end in the laboratory, all the vehicles must be in the laboratory at the end of the time horizon, and the latest moment that a vehicle can be in node $j \in V^E$ is

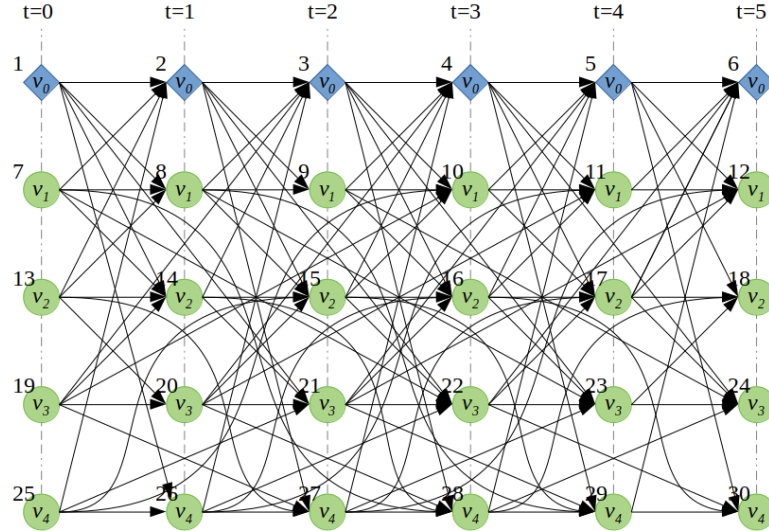


Figure 5.2: Time-expanded network example.

at time $t = TH - \tau_{v_j v_0}$. Thus a node $i \in V^E$ with $t(j) > t = TH - \tau_{v_j v_0}$ will not be part of a feasible solution, because none vehicle could reach the laboratory after TH . Then, the nodes i and j that are not part of a feasible solution, do not have to be created. Hence, in the preprocessing, before creating a time expanded network, the conditions of feasibility are checked, and only the nodes that could be part of a feasible solution are created. Hence, in the preprocessing, before creating a time-expanded node, the conditions of node feasibility are checked, and only the nodes that could be part of a feasible solution are created. In the example network, nodes $i = \{7, 13, 19, 20, 25\}$ are not part of a feasible solution due to $t(i) < \tau_{v_0 v_i}$, and the nodes $j = \{12, 18, 23, 24, 30\}$ are not part of a feasible solution due to $t(j) > t = TH - \tau_{v_j v_0}$. Therefore, those nodes are not created in the preprocessing, and the number of time-expanded nodes is reduced from 30 to 20. Since less time-expanded nodes are created, less time-expanded arcs are created too. With the preprocessing the number of time-expanded arcs is reduced from 114 to 60. The time-expanded network created in the preprocessing is depicted in Figure 5.3.

After reducing the number of expanded nodes, the next step of the preprocessing is related to the number of variables x_{ij}^k of the model. Strictly for each transportation request k exists a variable x_{ij}^k for each arc $(i, j) \in A^E$. Continuing with the example proposed, let be $k = 1$ a transportation request that is available to be transported at the time-expanded node 14 and must arrive at the time-expanded node 5 (i.e., $o_1 = 14 = (v_2, 1)$ and $d_1 = 5 = (v_0, 4)$). Thus, without preprocessing, the number of variables x_{ij}^1 is 114, and after the first prepro-

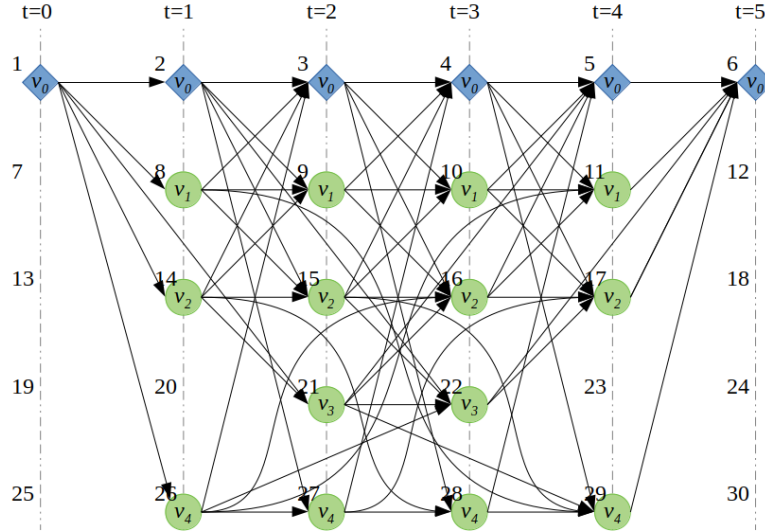


Figure 5.3: After preprocessing time-expanded network example.

cessing step, the number of variables x_{ij}^1 is 60. Due to each transportation request has a specific origin node (o_k) and destination node (d_k) in the time-expanded network, only the arcs $(i, j) \in A^E$ that be part of a path between o_k and d_k are in a feasible solution. In other words, only the paths, in the time-expanded network, that have as origin o_k and destine d_k , can be part of a feasible solution. Thus, for each k it is possible to select a subset of A^E such that (i, j) is in a feasible solution. That subset is denoted as A^{EK} for each k . The subset A^{E1} for the example proposed is showed in Figure 5.4. In the same way, it is possible to define for each k a subset of expanded nodes V^{EK} that are in a feasible solution, if the expanded node is in the path between o_k and d_k . The final part of the preprocessing is to define to define the subset A^{EK} and V^{EK} for each k .

Continuing with the example, after the preprocessing, the number of variables x_{ij}^k is reduced from 112 to 18. Thus, the preprocessing reduces the variables of the model from 224 to 78, that is, a reduction of 65% of the variables.

Furthermore, besides the reduction in the number of variables, the preprocessing has another effect over the time-expanded model for the BSTP. As it is observed in Figure 5.4, after the subset V^{E1} and A^{K1} are created, when the transportation request $k = 1$ leaves the node $o_1 = (v_2, 1)$, and the flow conservation constraint is fulfilled in all the nodes between o_1 and d_1 , the transportation request will be at the node d_1 . Therefore, in the time-expanded model, Equation 4.6 constraints are redundant, because in the subset V^{EK} flow conservation

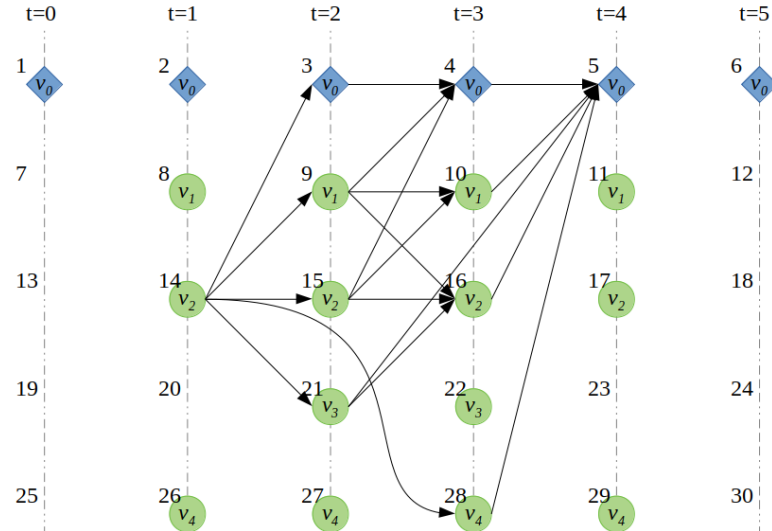


Figure 5.4: Possibles paths between expanded nodes 14 and 5.

constraint ensure that the transportation request k arrives at node d_k .

Numerical experiments were executed to assess the time-expanded formulation presented in Chapter 4. Also, the effectiveness of the preprocessing technique described in this chapter is tested with the numerical experiments. These experiments were conducted over a set of real-life instances. Chapter 6 discusses the results obtained from these experiments.

6 Numerical Experiments

This chapter presents and discusses the results of experiments conducted to test the quality and the behavior of the time-expanded formulation. These experiments are realized over a set of real-life inspired instances from the laboratory network under the management of the *MSSS*. In the first section, the author provides the description of the instances used, in terms of the number of transportation request, its origin time, destination time, geographical and temporal sparsity. Afterward, Section 6.2 presents the results obtained with the commercial solver Gurobi. Sensitivity analysis over a set of parameters relevant to managers is discussed in Section 6.3.

6.1 Instances

To know the transportation needs of each service area, the *MSSS* conducted a detailed survey spanning from June to August 2013 to the SCCs and laboratories in each service area. In the survey, each SCCs and laboratories were required to provide, for each day of the week, information such as opening and closing hours, number and type of samples, and kind of tests needed. Interested readers can consult Anaya-Arenas et al. (2016) for a detailed description of the survey. The differences, between the days of the weeks, lead to a different type of instances. For example, due to fewer SCCs are opened on weekends, the instances related to weekends have fewer SCCs. Furthermore, days with more demand lead to instances with more transportation requests. Another factor that influences in the variety of instances is the different geographical and demographic characteristics between the administrative regions. According to that, two types of instances could be distinguished:

- **Rural instances:** Correspond to service areas that cover a large territory, with a large distance between the laboratory and the SCCs, increasing the travel time. Moreover, few roads connect the SCCs, forcing the vehicles to go through predefined routes.
- **Urban instances:** Correspond to services areas close to main cities of the Quebec province. These services areas, cover less territory, but due to population density, more number of SCCs are necessary. Therefore, the distance between the laboratory and

the SCCs is lower and the density of the roads is greater. Therefore, the travel time in the urban instances is shorter and the paths between SCCs are larger.

Anaya-Arenas et al. (2016), after analyzing the demand and taking into account the services areas characteristics, defined the benchmark instances for the BSTP. However, due to the differences between the time-expanded formulation and previous formulation proposed for the BSTP, these instances were adapted to the thesis' problem. The main difference is related to the fact that a transportation request is not a pickup in the time-expanded formulation. Therefore, for each instance was necessary to define the number of transportation request and the origin time for each transportation request ($t(o_k)$). How these characteristics were defined, is described below.

- **Number of transportation requests:** To solve instances faithful to *MSSS* reality, the number of transportation requests was defined equal to the minimum number of pickups defined by Anaya-Arenas et al. (2019). However, again this does not mean that a transportation request becomes a pickup. This is because a single pickup could satisfy two or more transportation requests of the SCC if $T_L(k)$ is respected for each transportation request.
- **Transportation request origin time:** In each instance, $t(o_k)$ was defined according to the number of transportation requests of the SCC. First, under the assumption that the samples collection's rate is constant, it is assumed that the transportation requests are distributed equitably along the collection period. For example, if a SCC has 3 transportation request, the origin times will be at 1/3, 2/3 and 3/3 of the collection period. To ensure that all the specimens are transported, the last transportation request of each SCC is assigned at the end of the SCC collection period. Figure 6.1 illustrates the transportation request distribution used to defined the transportation requests origin time for a SCC with one, two, three, and up to k transportation requests.

A total of 49 instances were adapted and arbitrary grouped into *Small-Medium* and *Large* instances. The *Small-Medium* group has 37 instances (I01-I37), with up to 18 SCCs and a maximum of 41 transportation requests to complete. The *Large* group includes 10 instances (I38-I47), with up to 49 SCCs and a maximum of 142 transportation requests to complete. The size in terms of the number of SCCs and transportation requests of each instance is illustrated in Figure 6.2. While Figure 6.3 shows the distribution of the travel times for each instance. The travel time distribution is usefully to describe the instances according to the geographical and demographic characteristics.

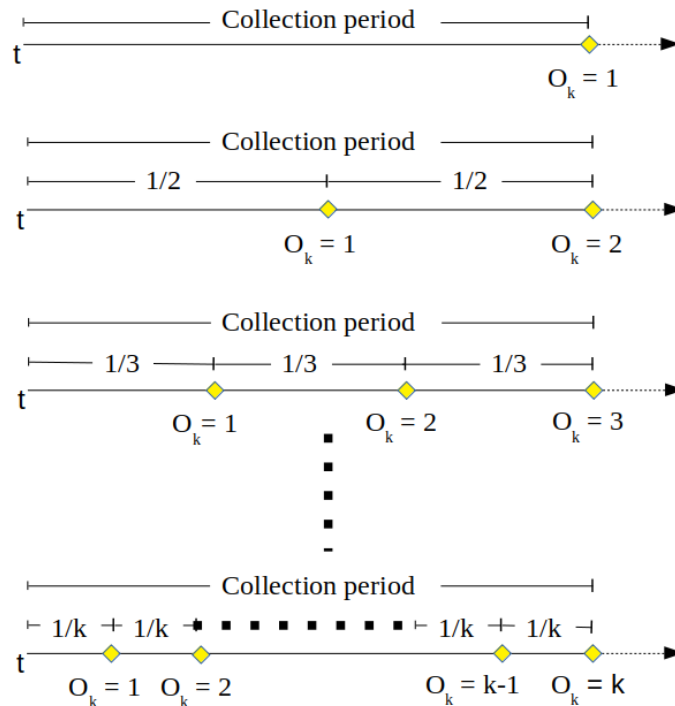


Figure 6.1: Transportation requests distribution.

Figure 6.3 shows: 1) That the instances I21, I22, I29, I30, I31, I33, and I38 are urban instances, where the average travel time is lower than 30 minutes, and the maximum travel time is lower than 60 minutes. 2) That instances I47, I48, and I49 are rural instances of service areas that cover a large territory. Hence, in these instances, the average travel time is close to the 100 minutes, and the maximum travel time is greater than 250 minutes.

6.2 Computational results

This section presents the computational results obtained for the 47 instances. To solve the instances with the new formulation an algorithm was coded on Julia and JuMP (Dunning et al., 2017). The experiments were executed on a PC with an AMD A12 processor and 8 GB of RAM. The experiments were conducted (1) to know the limits of the new formulation in terms of number of SCCs, number of transportation requests and time horizon that can be solved by Gurobi in a reasonable time. (2) To evaluate the strengths of the new formulation over the previous formulation for the BSTP.

As discussed above, the instance size is defined by the number of SCCs and the transporta-

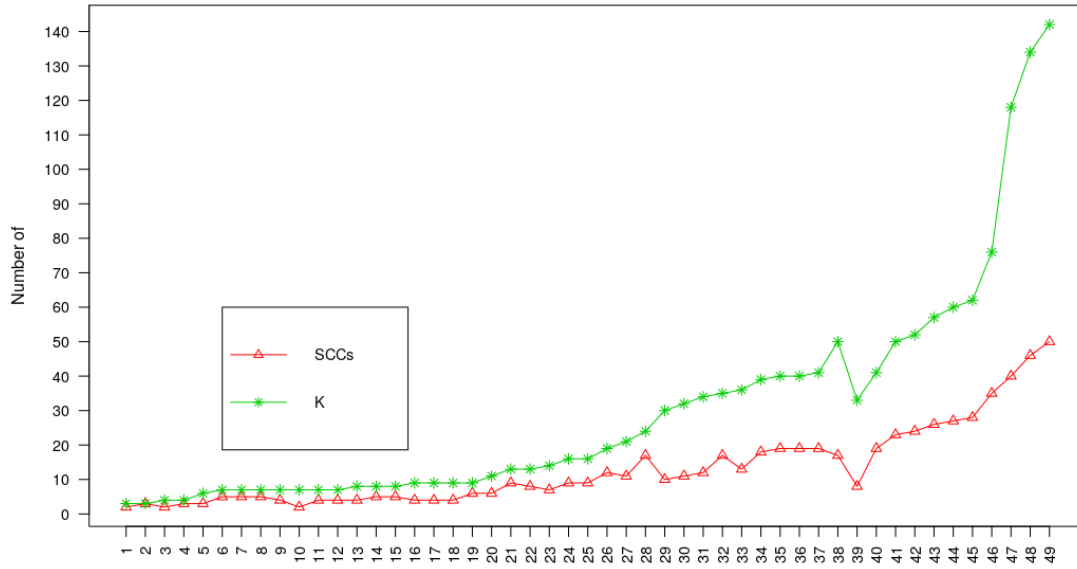


Figure 6.2: SCCs and Transportation requests for the 49 instances.

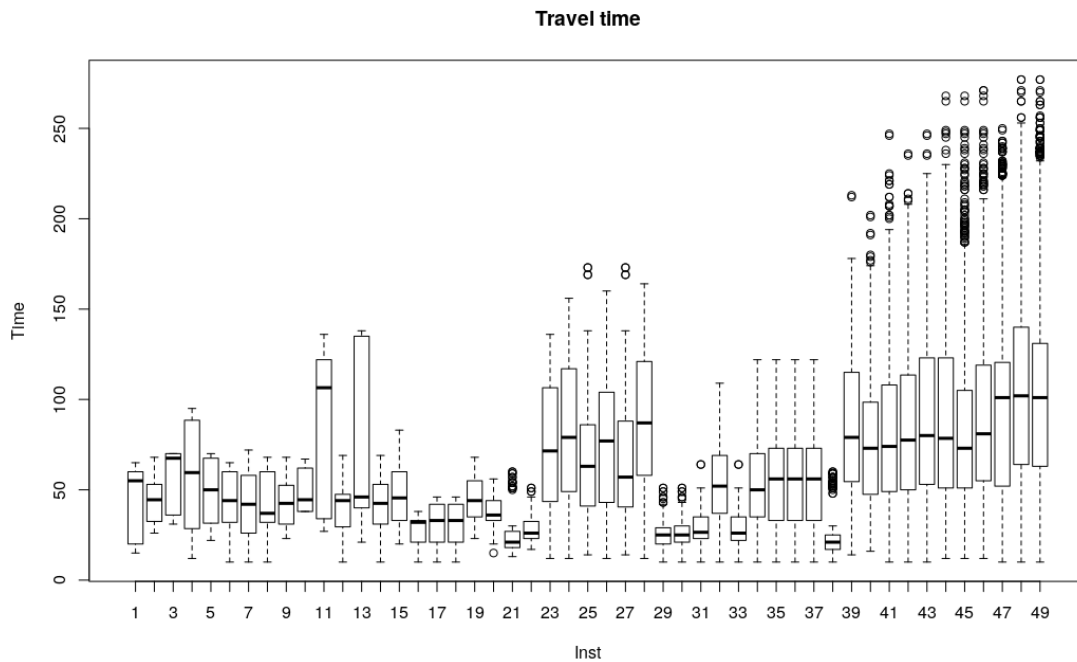


Figure 6.3: Box-plots for travel time between the nodes in the instances

tion request. Nevertheless, in the time expanded formulation the instance size also is defined by Δt . The author evaluates three values for Δt (15, 10, 5 minutes). All the instances were solved using Gurobi with a computational time limit of 3600 seconds. The results for *Small-Medium* and *Large* instances are showed in Table 6.1 and Table 6.2 respectively. In both tables, the first four columns define the instances, the instance type rural (R) or Urban (U), the number of SCCs, the number of transportation requests (K) and the time horizon (TH) in minutes. Columns 5 to 8 show the results when Δt is 15 minutes, columns 9 to 12 show the results for Δt of 10 minutes, and columns 13 to 16 show the results of 5 minutes Δt . In all the cases, the number of periods of the instance is showed (Column T), and the results are presented in terms of the objective function value (column $Obj.$), the computational time (Column $Sec.$) in seconds, the optimality gap (Column $Gap\%$) as reported by Gurobi and the different between the computational with and without preprocessing (Column Δs). For *Small-Medium* the model size (number of variables times number of constraints) reduction after preprocessing is presented in the (Column Δms). This value is calculated as: $\Delta ms = (ms_{before} - ms_{after})/ms_{before} * 100$

For all *Small-Medium* instances, Gurobi found an optimal solution. When $\Delta t = 15$ the average computational time is 9.3 seconds, with a maximum solution time of 89.2 seconds, and minimum solution time less than 0.1 seconds. For $\Delta t = 10$ the average solution time is 27.0 seconds, with a maximum up to 157.8 seconds, and minimum solution time less than 0.1 seconds. With $\Delta t = 5$ the average solution time is 138.8 seconds, with the maximum and minimum solution time equal to 1591.2 and 0.1 seconds respectively. The differences between the solution times, for the different values of Δt , are explained by the fact that the number of variables increases when Δt decreases, and the model is harder to solve by Gurobi.

Besides the effect that the Δt value has over the solution time, Δt also affect the value of the objective function found by Gurobi. Thus, in the *Small-Medium* instances the average objective function values are 857.8, 805.4 and 754.2, for Δt of 15, 10 and 5 minutes respectively. This is explained by the approximation in the travel time when the network is expanded. For example, an arc with a travel time of 22 minutes with 15 minutes Δt is modeled by an arc with a length of 2 periods. Thus, the arc has a cost of 30 minutes instead of 22 minutes. However, if a Δt of 5 minutes is used, the cost of the same arc is 25 minutes.

Results for *Small-Medium* instances show the advantages of using the preprocessing, because without preprocessing, the computational time increases on average by 13.3, 17.3 and 23.9 seconds, for Δt of 15, 10 and 5 minutes, respectively. For each Δt , The maximum computational times reduction are 142.3, 134.9, 284.9 seconds, and the minimum reductions are 0.0, 0.0, -5.6 seconds. Furthermore, with preprocessing is possible to solve instances that without preprocessing are not possible to solve, due to memory limitations. The model size is reduced by 96%, on average, with the preprocessing.

Table 6.1: Computational results for *Small-Medium* instances.

<i>Ins</i>	<i>SCCs</i>	<i>K</i>	<i>TH</i>	$\Delta t = 15$			$\Delta t = 10$			$\Delta t = 5$								
				<i>T</i>	<i>Obj.</i>	ΔS	<i>T</i>	<i>Obj.</i>	ΔS	<i>T</i>	<i>Obj.</i>	ΔS						
01R	1	3	540	36	285	88	0.0	0.0	54	270	87	0.0	-0.1	108	260	86	0.1	0.0
02R	2	3	450	30	210	87	0.0	-0.1	45	200	87	0.1	0.0	90	195	87	0.1	-0.1
03U	1	4	645	43	390	93	0.0	-0.1	65	360	93	0.0	-0.1	129	345	92	0.1	-0.1
04R	2	4	1090	73	285	94	0.1	-0.1	109	270	94	0.1	-0.2	218	265	94	0.2	-0.3
05U	2	6	590	40	435	95	0.0	-0.1	59	400	94	0.1	-0.2	118	380	94	0.2	-0.3
06R	4	7	1050	70	465	97	0.1	-0.4	105	460	97	0.9	-0.5	210	425	97	0.7	-2.0
07R	4	7	1050	70	450	97	0.2	-0.4	105	430	97	0.3	-0.6	210	415	96	0.8	-1.7
08R	4	7	1020	68	450	97	0.2	-0.3	102	430	97	0.3	-0.5	204	415	96	0.7	-1.9
09R	3	7	720	48	390	96	0.3	-0.2	72	370	96	0.4	-0.2	144	345	95	0.8	-0.9
10U	1	7	1014	68	465	97	0.0	-0.2	102	440	97	0.1	-0.2	203	420	96	0.1	-0.4
11R	3	7	1290	86	855	98	0.1	-0.4	129	810	98	0.3	-0.6	258	780	98	0.7	-1.7
12U	3	7	450	30	240	92	0.1	-0.2	45	240	92	0.3	-0.2	90	220	92	0.9	-0.3
13R	3	8	705	47	900	97	0.2	-0.2	71	870	98	0.2	-0.3	141	850	97	0.7	-1.2
14U	4	8	450	30	285	94	0.3	-0.2	45	280	93	0.6	-0.2	90	265	93	3.0	0.9
15R	4	8	900	60	540	97	0.1	-0.4	90	520	97	0.3	-0.6	180	490	97	0.6	-1.9
16U	3	9	630	42	300	94	0.4	0.0	63	240	93	0.3	-0.2	126	215	93	1.3	-0.4
17U	3	9	690	46	270	95	0.4	-0.1	69	240	95	0.4	-0.2	138	220	94	5.8	1.0
18U	3	9	680	46	330	95	0.1	-0.2	68	240	95	1.4	0.4	136	195	94	1.0	-0.4
19R	5	9	930	62	585	97	0.2	-0.6	93	550	97	0.4	-1.2	186	520	97	1.2	-3.7
20R	5	11	670	45	510	97	0.3	-0.7	67	450	96	0.9	-0.7	134	435	96	3.8	-1.8
21U	8	13	720	48	450	96	4.3	0.1	72	390	95	8.9	-0.1	144	375	95	38.9	-9.4
22U	7	13	840	56	495	97	1.9	-0.7	84	440	96	8.7	-2.1	168	415	96	28.3	5.6
23R	6	14	750	50	1170	99	0.2	-1.4	75	1040	99	0.9	-2.0	150	1060	99	1.3	-5.0
24R	8	16	765	51	1290	99	0.4	-2.1	77	1200	99	0.7	-4.0	153	1180	99	1.8	-8.2
25R	8	16	1140	76	1320	99	1.1	-3.7	114	1240	99	2.4	-6.5	228	1235	99	6.3	-23.2

Table 6.1: Continued.

<i>Ins</i>	<i>SCCs</i>	<i>K</i>	<i>TH</i>	<i>T</i>	$\Delta t = 15$					$\Delta t = 10$					$\Delta t = 5$				
					<i>Obj.</i>	Δms	<i>Sec.</i>	ΔS	<i>T</i>	<i>Obj.</i>	Δms	<i>Sec.</i>	ΔS	<i>T</i>	<i>Obj.</i>	Δms	<i>Sec.</i>	ΔS	
26R	11	18	780	52	1455	99	1.3	-5.0	78	1330	99	16.8	-8.3	156	1340	99	23.1	-58.5	
27R	10	21	1140	76	2025	99	1.8	-6.9	114	1940	99	4.9	-11.6	228	1910	99	15.0	-53.1	
28R	16	24	780	52	2490	99	1.8	-19.5	78	2240	99	19.4	-35.5	156	2250	99	51.8	-171.9	
29U	9	30	960	64	855	98	6.6	-9.9	96	720	98	24.5	-12.3	192	715	98	139.5	-13.0	
30U	10	32	960	64	900	98	13.5	-14.8	96	760	98	95.3	0.8	192	745	98	400.6	33.1	
31U	11	34	960	64	975	98	21.9	-18.1	96	820	98	80.8	5.5	192	795	98	415.7	-164.2	
32U	16	35	1095	73	1650	99	33.5	-24.0	110	1520	99	119.0	-105.9	219	1395	99	414.0	-284.9	
33U	12	36	960	64	1005	98	24.9	-27.4	96	850	98	104.8	-36.8	192	825	98	764.2	-18.6	
34U	17	39	1065	71	1650	99	31.5	-66.3	107	1490	99	80.0	-81.5	213	1405	99	409.6	NA ¹	
35U	18	40	1125	75	1755	99	45.7	-142.3	113	1600	99	109.4	-97.9	225	1490	99	491.4	NA	
36U	18	40	1125	75	1755	99	62.8	-50.8	113	1630	99	157.8	-134.9	225	1500	99	321.8	NA	
37U	18	41	1125	75	1860	99	89.2	-92.9	113	1730	99	156.3	-106.6	225	1615	99	1591.2	NA	
<i>Average</i>				857.8	97	9.3	-13.3	784.1	96	27.0	-17.5	754.2	96	138.8	-23.9				

¹ Run out of memory

Table 6.2: Computational results for *Large* instances.

<i>Ins</i>	<i>SCCs</i>	<i>K</i>	<i>TH</i>	$\Delta t = 15$			$\Delta t = 10$			$\Delta t = 5$								
				<i>T</i>	<i>Obj.</i>	<i>Gap%</i>	<i>Sec.</i>	ΔS	<i>T</i>	<i>Obj.</i>	<i>Gap%</i>	<i>Sec.</i>	ΔS	<i>T</i>	<i>Obj.</i>	<i>Gap%</i>	<i>Sec.</i>	ΔS
38U	16	50	710	48	1080	0.0	2939.3	-660.7	71	890	3.5	3600	0	142	NA	NA	3600	NA ²
39R	7	33	810	54	3645	0.0	1.4	-5.3	81	3570	0.0	9.9	-9.6	162	3415	0.0	17.1	NA
40R	18	41	560	38	2535	0.0	12.9	-38.8	56	2360	0.0	260.2	-52.7	112	2085	0.0	915	NA
41R	22	50	660	44	3375	0.0	29.2	-86.1	66	3150	0.0	798.7	111.6	132	2570	0.0	1918.5	NA
42R	23	52	660	44	3330	0.0	20.2	-79.5	66	3270	0.0	747.9	94.6	132	2590	0.0	1593.4	NA
43R	25	57	820	55	4410	0.0	108.4	-514.4	82	4210	0.0	308.9	NA ²	164	3600	4.7	3600	NA
44U	26	60	780	52	4905	0.9	3600	NA ²	78	4310	2.4	3600	NA	156	NA	NA	3600	NA
45U	27	62	780	52	4965	0.0	731.1	NA	78	4210	2.9	3600	NA	156	NA	NA	3600	NA
46U	34	76	780	52	5805	2.65	3600	NA	78	NA	NA	3600	NA	156	NA	NA	3600	NA
47R	39	118	1050	70	NA	NA	3600	NA	105	NA	NA	3600	NA	210	NA	NA	3600	NA
48R	45	134	1050	70	NA	NA	3600	NA	105	NA	NA	3600	NA	210	NA	NA	3600	NA
49R	49	142	1065	71	NA	NA	3600	NA	107	NA	NA	3600	NA	213	NA	NA	NA	NA
<i>Average</i> ¹				3783	0.4	1226.9	-607.7		3246	1.0	1615.7	28.8		2852	0.9	1940.7	NA	NA

¹ Solved instances² Run out of memory

Table 6.2 shows that *Large* instances are hardest to solve, where for some instances Gurobi does not find a solution within the 3600 second time limit, even for $\Delta t = 15$. Notwithstanding, with the time-expanded formulation, big size instances are possible to solve optimally. Figure 6.4 presents the optimality gap in percent, reported by Anaya-Arenas et al. (2019) with the Iterative Local Search (ILS), and the gap obtained with the time-expanded formulation (TE) with Δt of 10 minutes, for the instances 28 to 38.

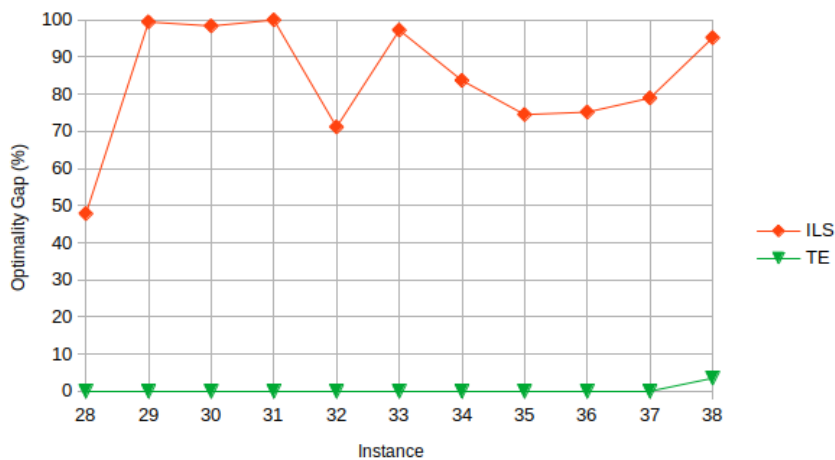


Figure 6.4: Optimality gap for instances 28 to 38

Figure 6.4 shows that with previous formulations were not possible to solve optimally the instances 28 to 38. Yet, with the time-expanded formulation is possible to reach an optimal solution, for similar instances, in the number of SCCs and transportation requests. Moreover, with the optimality gap as reference, the quality of the solutions obtained with the time-expanded formulation is better than the solutions reported with previous formulations.

However, the difficult to solve an instance increases when the number of SCCs and transportation requests increase. Some instances with fewer SCCs and transportation requests are harder to solve, for example, Instance 38. Why does this happen? To answer the question is important to remember that the instances are real instances and these instances are urban or rural. Then, a rural instance has a longer travel time between the nodes than an urban instance. When the travel times in the instance are short, the number of expanded arcs increases and, in the same way, the universe of feasible solutions. Instance 38 is an urban instance in which the average travel time does not exceed 30 minutes. Compared with other large instances, these travel times are the shortest (see Figure 6.3). Therefore, this instance is harder to solve than other instances with more SCCs and transportation requests.

6.3 Managerial insights

In this section, the author discusses a sensitivity analysis over a set of parameters relevant to managers. These parameters are the number of transportation requests of each SCC and the time limit of each transportation request ($T_L(k)$). The number of transportation requests and $T_L(k)$ is determined as a function of the desired service level for each SCC. Thus, a change in service level implies changes in the number of transportation request or $T_L(k)$.

The instances 20 to 37 are solved with changes in $T_L(k)$ for each k in the instance, and for a different number of transportation requests. As discussed previously, the value of Δ used has effects over the Gurobi's solution. To avoid these effects in the sensitivity analysis, for both factors, Δt is fixed on 10 minutes.

- **$T_L(k)$ sensitivity analysis:** The time limit to handled a transportation request is linked with the type of sample to be transported. Also, the time limit is related to the equipment that each SCC has to preserved the sample, after it is taken from the patient. To know the effects of changes the $T_L(k)$, the instances were solved with $T_L(k) = -2, -1, +0, +1, +2$ hours respect to the real T_{Limitk} . Thus, $+0$ instances represent the real instances. However, in any case, the $T_L(k)$ could be less than the travel time between the SCC and the laboratory.

Table 6.3 shows the results for the different values of $T_L(k)$. The first three columns define the instance, the number of SCCs and the number of transportation request. Columns 4 to 7 are the results of $T_L(k) = +0$, where the results presented are objective function value (column *Obj.*), the optimality gap (column *Gap%*) as reported by Gurobi. The number of routes used in the solution (column *Rou.*) and the computational time (column *Sec.*) are in seconds. The results for the other values of $T_L(k)$ are presented in terms of the difference in percentage with the values obtained for $T_L(k) = +0$, with an exception for the optimality gap. In all cases, the percentage difference is calculated as $\Delta_{Value} = (Value - (+0_{value})) / (+0_{value})$.

When the time limit decreases, the number of routes in the solution increases. On average the number of routes used is 152.6% and 49.1% greater, for -2 and -1 hours time limit, respectively. This is because the vehicles have less time to reach the laboratory, and consequently, can visit less SCC in a route. Therefore, a reduction in the capacity of preserved samples implies greater transportation costs. In the same way, the stronger restriction for the length of the routes makes that the universe of solutions been small. Hence, the solution is easy to find by Gurobi and the computational time decreases.

Table 6.3: $T_{Limithk}$ sensitivity analysis results

<i>InstSCCsK</i>	+0			-2			-1			+1			+2											
	<i>Obj.</i>	<i>Gap%</i>	<i>Row. Sec.</i>	ΔO	<i>Gap%</i>	ΔR	ΔS	ΔO	<i>Gap%</i>	ΔR	ΔS	ΔO	<i>Gap%</i>	ΔR	ΔS									
20	5	11	450	0.0	0.0	4	0.9	66.7	0.0	125.0	-91.0	24.4	0.0	25.0	-71.9	-20.0	0.0	-25.0	1054.5	-26.7	0.0	-50.0	405	
21	8	13	390	0.0	0.0	3	8.9	53.8	0.0	166.7	-96.4	25.6	0.0	66.7	-55.1	-10.3	0.0	0.0	339.8	-17.9	0.0	-33.3	631	
22	7	13	440	0.0	0.0	5	8.7	56.8	0.0	80.0	-97.4	31.8	0.0	20.0	-74.5	-18.2	0.0	-20.0	38.3	-25.0	0.0	-40.0	196	
23	6	14	1040	0.0	0.0	5	0.9	39.4	0.0	100.0	-89.5	26.0	0.0	40.0	-72.9	-21.2	0.0	-20.0	281.9	-28.8	0.0	-40.0	1088	
24	8	16	1200	0.0	0.0	5	0.7	55.8	0.0	180.0	-79.4	41.7	0.0	80.0	86.5	-14.2	0.0	-20.0	878.7	-26.7	0.0	-20.0	3617	
25	8	16	1240	0.0	0.0	6	2.4	23.4	0.0	133.3	-84.8	13.7	0.0	33.3	-77.6	-4.8	0.0	-16.7	397.8	-16.1	0.0	0.0	466	
26	11	18	1330	0.0	0.0	6	16.8	51.9	0.0	150.0	-98.3	27.1	0.0	50.0	-96.0	-17.3	0.0	-33.3	153.4	-30.8	0.0	-33.3	484	
27	10	21	1940	0.0	0.0	10	4.9	31.4	0.0	90.0	-81.6	18.6	0.0	0.0	-78.3	-11.9	0.0	-30.0	121.7	-21.1	0.0	-40.0	695	
28	16	24	2240	0.0	0.0	8	19.4	49.6	0.0	125.0	-97.5	42.9	0.0	87.5	-96.3	-27.2	0.0	-37.5	2436.7	-42.0	0.0	-50.0	2729	
29	9	30	720	0.0	0.0	7	24.5	87.5	0.0	128.6	-97.2	54.2	1.8	28.6	14606.2	-12.5	0.0	-14.3	532.7	-23.6	0.0	-28.6	2315	
30	10	32	760	0.0	0.0	7	95.3	85.5	0.0	128.6	-99.3	50.0	0.0	42.9	760.1	-13.2	0.0	-14.3	392.1	-23.7	0.0	-28.6	609	
31	11	34	820	0.0	0.0	7	80.8	96.3	0.0	157.1	-99.0	48.8	0.0	42.9	373.2	-9.8	0.0	0.0	3990.7	-24.4	0.0	-28.6	1057	
32	16	35	1520	0.0	0.0	10	119.0	98.0	0.0	170.0	-98.7	44.1	0.0	70.0	-90.6	-16.4	0.0	-20.0	276.0	-31.6	0.0	-40.0	1699	
33	12	36	850	0.0	0.0	7	104.8	98.8	0.0	171.4	-98.5	48.2	0.0	57.1	1348.5	-10.6	2.7	0.0	3340.4	-24.7	0.0	-28.6	1713	
34	17	39	1490	0.0	0.0	10	80.0	135.6	0.0	230.0	-97.6	57.7	0.0	60.0	-75.8	-12.8	0.0	-20.0	3449.8	-31.5	0.0	-40.0	4118	
35	18	40	1600	0.0	0.0	10	109.4	126.3	0.0	220.0	-97.7	53.1	0.0	70.0	-81.5	-11.9	2.2	-20.0	3197.5	-24.4	8.0	-40.0	3204	
36	18	40	1630	0.0	0.0	11	157.8	122.1	0.0	190.9	-98.4	50.9	0.0	54.5	-88.0	-12.9	1.4	-27.3	2185.8	NA	NA	NA	2182	
37	18	41	1730	0.0	0.0	11	156.3	117.3	0.0	200.0	-98.4	48.6	0.0	54.5	-87.8	-17.9	0.0	-27.3	1643.8	NA	NA	NA	2203	
<i>Average:</i>	1188.3	0.0	7.3	55.1	77.6	0.0	152.6	-94.5	539.3	0.1	49.1	896.0	-14.6	0.4	-19.2	1372.9	-26.2	0.5	-33.8	1633.8				

The results of increasing the time limit show that the number of routes used in the solution is lower than the routes used in the base case (+0). The reduction in the number of routes is 50% for instances 20 and 28 when the time limit increases in 2 hours. The average reduction in the number of routes is 33.8% and 19.2% for $T_L(k) = +2$ and $T_L(k) = +1$ respectively. Thus, when the samples can be preserved more time, fewer pickups are needed for each SCC, the length of the routes are longer, and consequently, the transportation cost is lower. However, the instances are harder to solve, and for the instances, 36 y 37 Gurobi reaches the time limit before finding a solution.

- **Sensitivity analysis of the number of transportation requests:** To know the effects of increasing the number of transportation requests, the transportation requests of the instances 20 to 37 were increased as 1) minimum 2 (m2) transportation requests for SCCs in the instance. 2) minimum 3 (m3) transportation requests for each SCCs. 3) two times (2t) the number of transportation requests in the original instance (o). Table 6.4 shows the number of routes used in the solution of the instances for each case (o,m2,m3,2t). The table first two columns define the instance and the number of SCCs. Columns 3 to 7 are the results of original case (o), where the results presented are number of transportation request (column K), objective function value (column $Obj.$), the optimality gap (column $Gap\%$) as reported by Gurobi, The number of routes used in the solution (column $Rou.$) and the computational time (column $Sec.$) are in seconds. The results for the other values of transportation request are presented in terms of the difference in percentage with the values obtained for **o**, with exception for the number of transportation request and the optimality gap, where the results are presented as the difference with the values obtained for **o**.

In Table 6.4 is observed that increasing the number of transportation requests increases the number of routes used in the solution. In the m2 scenario, the average increase in the number of routes is 16.4%, with a maximum increase of 60.0% and minimum of 0.0%. The m3 scenario has 33.8% of average increase in the number of routes whit a maximum of 100% (i.e., two times the number of routes) and minimum of 0.0%. Results for 2t scenario show that the average number of routes in the solution increase by 43.1%. In this scenario, in one instance the number of routes did not increase, that is, increasing the length of the routes it is possible to satisfy double of demand with the same number of routes.

In scenario 2t, each SCC makes a transportation request every 30 to 60 minutes, which is a situation closer to the reality of samples collected in a continues way (or semi-continues) if the sample preparation time is taken into account. Thus, with the

Table 6.4: sensitivity analysis of the number of transportation request

<i>Inst SCCs</i>	<i>O</i>				<i>m2</i>				<i>m3</i>				<i>2t</i>								
	<i>K</i>	<i>Obj.</i>	<i>Gap%</i>	<i>Row.</i>	<i>Sec.</i>	<i>K</i>	<i>Obj.</i>	<i>Gap%</i>	<i>Row.</i>	<i>Sec.</i>	<i>K</i>	<i>Obj.</i>	<i>Gap%</i>	<i>Row.</i>	<i>Sec.</i>	<i>K</i>	<i>Obj.</i>	<i>Gap%</i>	<i>Row.</i>	<i>Sec.</i>	
20	5	11	450	0.0	4	0.9	2	6.67	0.0	25.0	72.0	7	33.33	0.0	25.0	7.2	11	26.67	0.0	25.0	2.7
21	8	13	390	0.0	3	8.9	6	7.69	0.0	33.3	15.3	14	17.95	0.0	66.7	13.4	13	25.64	0.0	66.7	39.6
22	7	13	440	0.0	5	8.7	3	0.00	0.0	0.0	0.9	11	13.64	0.0	20.0	3.0	13	20.45	0.0	0.0	42.3
23	6	14	1040	0.0	5	0.9	3	23.08	0.0	20.0	-0.4	8	61.54	0.0	40.0	0.6	14	35.58	0.0	40.0	0.8
24	8	16	1200	0.0	5	0.7	5	38.33	0.0	60.0	0.1	12	92.50	0.0	100.0	2.4	16	59.17	0.0	100.0	1.3
25	8	16	1240	0.0	6	2.4	3	18.55	0.0	16.7	2.0	11	75.00	0.0	50.0	8.3	16	51.61	0.0	83.3	-0.4
26	11	18	1330	0.0	6	16.8	9	28.57	0.0	50.0	-6.9	19	63.91	0.0	83.3	-6.7	18	39.85	0.0	50.0	-6.8
27	10	21	1940	0.0	10	4.9	4	3.09	0.0	10.0	6.2	13	9.28	0.0	0.0	0.1	21	54.64	0.0	50.0	-0.2
28	16	24	2240	0.0	8	19.4	13	45.54	0.0	50.0	27.9	27	87.95	0.0	87.5	264.2	24	50.45	0.0	62.5	21.2
29	9	30	720	0.0	7	24.5	1	0.00	0.0	0.0	42.9	5	4.17	0.0	14.3	121.2	30	25.00	0.0	14.3	2084.6
30	10	32	760	0.0	7	95.3	1	-1.32	0.0	0.0	-38.3	6	1.32	0.0	14.3	19.3	32	23.68	0.0	28.6	2218.7
31	11	34	820	0.0	7	80.8	1	-1.22	0.0	0.0	14.1	7	2.44	0.0	14.3	77.4	34	26.83	4.0	28.6	3522.9
32	16	35	1520	0.0	10	119.0	4	11.18	0.0	10.0	-16.0	19	11.84	0.0	20.0	101.6	35	22.37	0.0	40.0	471.2
33	12	36	850	0.0	7	104.8	1	-1.18	0.0	0.0	68.3	8	3.53	0.0	14.3	65.7	36	27.06	3.8	42.9	3499.5
34	17	39	1490	0.0	10	80.0	3	4.70	0.0	10.0	6.4	18	17.45	0.0	20.0	581.2	39	32.89	4.2	40.0	3524.3
35	18	40	1600	0.0	10	109.4	4	3.75	0.0	10.0	16.9	20	14.38	0.0	20.0	1691.3	40	25.00	0.0	40.0	3092.3
36	18	40	1630	0.0	11	157.8	4	3.68	0.0	0.0	-50.0	20	12.88	0.0	9.1	1744.2	40	30.67	0.0	36.4	3288.3
37	18	41	1730	0.0	11	156.3	3	2.31	0.0	0.0	166.9	19	12.14	0.0	9.1	3380.5	41	29.48	1.4	27.3	3448.4
<i>Average.</i>		1188.3			7.3	55.1		10.5		16.4	18.3	29.7			33.8	448.6	33.7			43.1	1402.8

time-expanded formulation was possible to optimally define the number of pickups of each SCC, under a scenario where the samples are collected in a continuous way, warranted that the $T_L(k)$ is respected for all the samples.

7 Conclusions and recommendations

7.1 Conclusions

- The mathematical model over a time-expanded network proposed for the BSTP allows to discretize time horizon into periods. This allows to control the samples' lifespan with expanded nodes if they are into the time limit for the transportation request. Hence, it is possible to make the lifespan constraint with as a flow conservation constraint.
- The mathematical formulation in terms of transportation requests, instead of pickups, make possible to define the optimal number of pickup needed for each SCC. With previous formulations used to solve the BSTP, the number of pickups are an input and the optimal number of pickups is not possible to determine with these formulations.
- The proposed formulation was tested with a commercial solver. With the commercial solver Gurobi it was possible to solve the proposed model for different sizes of instances, and different values of Δt . Moreover, the solver Gurobi finds optimal solutions in a reasonable time in most of the instances.
- With the time expanded formulation it was possible to find an optimal solution for real-instances of the Quebec's laboratory network. Furthermore, the time-expanded model finds optimal solutions for instances that had not previously solved optimally. Moreover, using the optimally gap as a reference, the time-expanded formulation results report better solution, because with the time expanded formulation it was possible to find solutions with a gap less than ten percent in the instances solved.
- The disadvantage of time-expanded formulations is the multiplication of nodes in the network. Due to that, the number of variable and the size of these type of model are big. Nevertheless, a preprocessing that allows reduce the size of the time-expanded model. To do that, some characteristics of the BSTP are exploited. Thanks to this preprocessing, it was possible to solve instances that without the preprocessing had not been possible to solve.

7.2 Recommendations

The numerical experiments conducted in this research expose the convenience of using a time-expanded network to model the BSTP. However, network expansion makes that some instances cannot be solved. With the preprocessing used to reduce the size of the optimization program, the negative impact of the time expansion is reduced, but even so, the reduction is insufficient for the largest instances. It is expected that with better solutions methods joined with network size reduction, largest instances for the BSTP could be solved.

Although the time-expanded formulation reports better results than previous formulations, the time discretization has impact in the quality of the solution. This lose of quality is due to the approximation introduced by the mapping of parameters involving time (Boland et al., 2019). High quality approximation to the continuous-time problem is obtained with short time intervals, but typically it induce a computationally intractable model.

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