Investment portfolio optimization with predictive control

Optimización de portafolios de inversión aplicando control predictivo

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No price is too high to pay for the privilege of owning yourself.

Friedrich Nietzsche

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Y todas las personas que de alguna u otra forma me ayudaron en este tiempo

Abstract

This work presents the application of a portfolio management strategy based on model predictive control. A brief comparison between single-period and multi-period management strategies is presented.Optimal control theory is used as a management strategy on a multi-period asset price model. A basic explanation of the control theory and its application to the management problem is described. The management strategy of this work considers: constrains for the portfolio assets, the transactions cost and the transfers between a credit account and a free risk asset. The required price estimation is based on a moving horizon estimator used to the estimate the trends of the prices. A practical application of the solution is implemented on 3 stocks of the Colombian market.

Keywords: Portfolio management, model predictive control, Kalman filter, moving horizon estimator, multiperiod models.

Resumen

Este trabajo presenta la aplicación de una estrategia de gestión portafolios basada en control predictivo, una breve comparación entre los de modelos de un periodo y multiperiodo es presentada y utilizando un modelo de gestión multiperiodo, se implementa la estrategia de gestión, esta estrategia se aborda desde la teoría del control óptimo con un explicación básica de los conceptos y su aplicación al problema de administración. La estrategia de gestión del portafolio considera: restricciones para los activos de la cartera, el costo de las transacciones y las transferencias entre una cuenta de crédito y un activo libre de riesgo. La estimación del precio requerido se basa en un estimador de horizonte deslizante utilizando el cálculo de las tendencias de los precios. Una aplicación práctica de la solución se implementa en tres acciones del mercado colombiano.

Palabras clave: Administración de portafolios, control predictivo, filtro de Kalman, estimadores de horizonte deslizante , modelos multiperiodo

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1. Introduction

The market

The term market refers to a group of consumers or organizations interested on a product. This group has the resources to purchase the product, and is permitted by law and other regulations to buy. When a person is interested on a specific product and decides to buy it,this operation is called an investment. In this way an investment can be defined as the current commitment of money or other resources in the expectation of reaping future benefits or returns. These investment decisions are made in an environment where higher returns usually can be obtained only at the price of greater risk. The portfolio of an investor is just the collection of investment assets. Once the portfolio is established, it is updated or "rebalanced" by selling existing securities (assets) and using the proceeds to buy new securities, by investing additional funds to increase the overall size of the portfolio, or by selling securities to decrease the size of the portfolio [4].

Portfolio basics by Markowitz

In 1950 Harry Markowitz [24] defined the process of selecting a portfolio as a two stages process. The first stage is based in the observation and experience and ends with beliefs about the future performances of the securities. The second stage uses the relevant beliefs about the future performance and ends with the choice of the portfolio. Anyway, the final goal is to maximize the returns of the portfolio and minimize the possible losses (risk). Each stock of the portfolio is weighted so the expected value of a weighted sum is the weighted sum of the expected values, each expected value of every stock has a variance associated and it is possible to find a function to describe the variance of the overall portfolio; with this contribution Markowitz described how to combine assets into efficiently diversified portfolios.According to Markowitz, the risk of the portfolio could be reduced and the expected rate of return could be improved if investments having dissimilar price movements were combined. In other words, Markowitz explained how to best assemble a diversified portfolio and proved that such a portfolio would likely do well. This solution does not consider the difficult question of how investors do (or should) form their probability beliefs, and the fact that in general, a market is assumed a stochastic problem, a wide number of implications arise from this assumption: the uncertainty of the future price of the stocks, the modeling of external variables that affects the market, correlations between stocks are among others elements that makes the solution of the problem a difficult task.

Portfolio management strategies

In order to solve the mentioned implications, two big groups for portfolio strategies emerge: the first a passive Portfolio Strategy or also called single-period model, a strategy that involves minimal expectation input, and instead relies on diversification to match the performance of some market index. The second is an active Portfolio Strategy or multi-period model, that uses available information and forecasting techniques to seek a better performance than a portfolio that is simply diversified broadly.

Comparison between models: many single-period portfolio solutions are presented in the literature, the most famous and most widely used is the mean-variance framework suggested by Markowitz [24] already discussed before. In general all the single period models are limited to long-term applications where investors are able to re-balance their portfolio frequently, they are also used in situations where investors face liabilities or goals at specific future dates, and the investment decisions must be taken with regards to the dynamics and the time structure involved.

The multi-period approach may also provide superior performance over the single-period approach, Dantzig and Infanger [8] shows that multi-period portfolio optimization problems could be efficiently solved as multistage stochastic linear problem. This solution assumed the returns of the stocks in the future periods to be independent stochastic parameters which is not always true. The theory in the models include several assumptions such as no transaction costs, identically and independently distributed asset returns, and neither liabilities nor inflows or outflows to be time-dependent. When these assumptions are not satisfied, a multi-period setting is the appropriate framework to handle such a problem. This solution basically divides the single period solution in a sum of minor parts. Multi-period models and the corresponding optimization methods (control methods) were employed in order to address the points raised above [20].

Control theory and portfolio management

As a basic definition, control engineering is the science that causes dynamic systems to behave in a desired manner. So if multi-period models are seen as a dynamic system, it is possible to try to manipulate their behavior in a desired way. Optimization and automatic control solutions have played a vital role in the advancement of engineering and science, and they have an important participation in areas such as: space systems, missile guidance, robotics, chemical process, and automation. *"The automatic control has become an important part and integral part of modern industrial processes and manufacturing"* [29]. Within the field of control theory there are several tools, basically divided into linear and nonlinear control. Among the multiple applications of optimization and control, the control theory has been recently applied to financial issues recently, [7]. If the portfolio management application is formulated as an optimization problem, for a example, a feasible solution to the multi and single-period problems, the control theory and optimization can be applied (see [7, 15, 21, 38, 39, 30, 40, 34]).

Among the tools of control theory and optimization, one suitable method that can handle the multi-period model problem with his implications is the model predictive control (MPC). MPC is an optimal control technique whose objective is to regulate the states and or the outputs of the system towards desired values, by minimizing a cost function inside a feasible region. The entire system is modeled and all control inputs are computed as a solution of the single optimization problem. Such a solution is a sequence of control actions. Given this sequence, its first element is applied to the system and the remaining elements are used as initial condition for further time steps.

"Due to its ability to handle complex systems with input and state constraints, MPC has become one of the most successful advanced control techniques implemented in industry" (see [6, 10, 14, 27, 28]). Moreover, MPC has a good performance even when it is used with rather intuitive design principles and simple models [10]. However, MPC relies on the model accuracy and often also on the availability of sufficiently fast computational resources [37]. In the investment area, in ([12, 11]), Dombrovskiy et all proposed solutions for a portfolio management based on MPC, and the solutions included constraints about the transaction cost and the volume of the assets.

This work is focused on the MPC solution applied to portfolio management with transactions constraints, in the solution named, the feedback in the MPC its a mean value of the trend of the stocks. The feedback function acts like a sensor in the system, that will feedback to the controller the trend of the prices to get the best solution available according the past data. The natural behavior of the prices of the stocks is stochastic, so our objective is to estimate the feedback of the MPC using an estimator like the Kalman filter or a MHE (moving horizon estimator). Both are estimators that works with stochastic systems, and provide the optimal estimation of the feedback variable. These tools are designed to guarantee optimal solutions under certain stochastic characteristics that can naturally be assumed to be present in stock prices.
The Kalman Filter has been used in some financial applications (see [3] and [19]) where both papers employ the Kalman filter as an estimator of variables in economic problems, the first reference shows the modeling of stock returns to economic factors based in neural

GARCH models and Kalman filter against the Risk metrics method. The MHE estimator does not have any reported application to financial problems, so this work will include an initial approach to the implementation of the MHE tools to these problems.

Outline

The Outline of this work begins with an introductory chapter to the portfolio management. In this chapter, a brief review of the basic concepts of the market definition is presented with a description of the financial instruments that a market has and some considerations required to the development of this thesis. Second chapter explains the use of control theory in finance applications, with a small review about control theory, and an explanation about how to extrapolate the concepts of control theory to the investment area. In the third chapter, there is a discussion about of the Kalman filter and MHE that will be used to estimate the trends of the assets in the MPC as an optimal way to predict the trend of the prices. In the four chapter an application case from the Colombian market is presented and finally the conclusion and final remarks of the work are presented.

Motivation

The stochastic behavior of the stock markets extense the portfolio management problem a wide case of study, and many theoretical tools can be applied to solve the problem. Since the Markovian theory of diversification, the question: “How to manage a portfolio” has been explored as was shown in the introduction section. In this work the control theory will be used, as a mathematical tool that had been successful in many applications which have a stochastic behavior, with high levels of uncertainty and other characteristics of the stock markets.

In this way this work wants to propose a new solution to the portfolio management problem, based on control theory with: optimal estimators to make a forecast of the stocks trends and model predictive control tools to make an optimal management of the portfolio.

2. Portfolio management and asset price models

This chapter begins with a brief description about asset classes, then describes a basic approximation of the portfolio management from the mathematical view in a single period model and a general description of the multi-period models, finishing with a brief description of capital asset price models.

2.1. Asset classes

A brief description from asset classes extracted from is: A financial investment, in contrasts to a real investment which involves tangible assets (e.g. land, factories), is an allocation of money to financial contracts in order to make its value increase over time. According to the statements above, an investment can be seen as a sacrifice of current cash for future cash. The two main attributes that distinguish securities are time and risk, where risk also includes inflation. The interest rate or return is defined as the gain or loss of the investment divided by the initial value of the investment. Any investment always contains some sort of risk. Therefore, the higher an investor considers the risk of a security, the higher the rate of return he demands, sometimes referred to as risk premium. The first type of securities considered are treasury bills. They involve loaning on a short-term basis to the U.S. Treasury or any other national treasury. Treasury bills contain almost no risk. Other types of short-term interest rate products are money-market accounts.The second type of securities are bonds. There are two major categories of bonds: government bonds and corporate bonds. Bonds, as treasury bills, involve lending money but on a fairly long-term basis. Bonds result in a cash payments on a regular basis, the coupon of a bond, up to its expiry, i.e., the maturity date, when the final cash payment is made. There is a market for bonds where they can be bought and sold. The price of a bond vary over its lifetime depending on the changes of interest rates. The bond rating system is a measure of a companys credit risk.

Other type of securities are stocks. Stocks, also called shares, represent ownership in a part of a corporation. Stocks pay out part of their profits as dividends. The main reason or investing in stocks is the expected increase in the stock price. [20]

2.1.1. Financial securities

"An investor can choose to purchase directly any of a number of different securities, many of them represent a participation title on a private or government entity. Alternatively, an investor can invest in an intermediary (mutual fund), which bundles together a set of direct investments and then sells shares in the portfolio of financial instruments it holds" [16]. In the the set of possible options to make an investment is useful to see the following classification of the financial securities is presented in the figure 2.1.1 taken from [16].

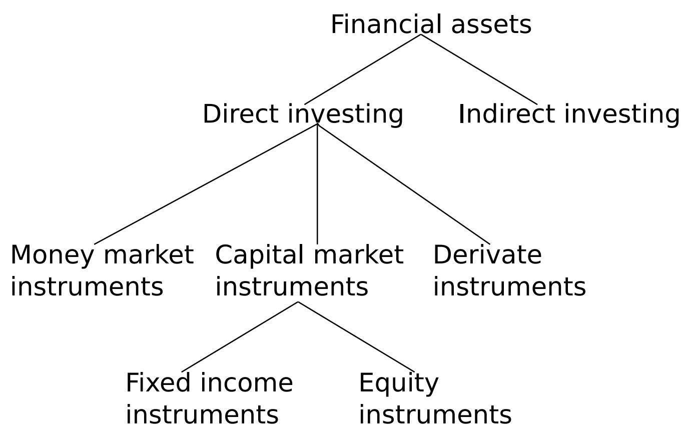

Figure **2-1**.: Classification of financial securities.

The financial securities have many kinds, each financial asset have special features that give a particular behavior in a financial market, this features like: fixed or variable yield, high or low volatility and many others, must be considered at the time of building an investment portfolio. The right selection of the kind of financial security according the owner preferences or the market behavior can be the difference between the failure or success of the portfolio. As example, the money market is a short time asset, that can change the value each minute, have a very high volatility and requires very fast actions about buy and sell but the incomes are effective also in a short time and the possibility of money lose is very high. On the other hand, the government bond instruments have very low volatility but the yield is very small. The possibility of loosing money is practically zero, but requires a big amount of time to give any return to the owner.

Colombian financial securities

The composition of the Colombian financial market is divided in two main groups: intermediate and non intermediate, and they are regulated by the national government, the citizen who wish to invest, normally have access to the stock market through a stock brokerage, which are entities supervised by the Superintendency of Securities and it is located in the non-intermediated market.

The Colombian stock market can be divided in six operations:

- Kind of market
- Kind of operation
- Sector
- Kind of yield
- Kind of operations
- Activity

In order to choose the right operations for the management of the portfolio, some operations will be explained, in order to guide the selection of the securities that will be used in this thesis.

Kind of market The securities works on two levels: primary market and secondary market; the primary market are the securities that that will be negotiated for the first time on the market, on this market the stock brokerage companies buy a big amount of securities, and then, these securities are placed in the secondary market where can be acquired by natural persons. The financial operations made through a stock brokerage had an associated cost of transaction, and the investor must try to minimize this purchase cost by reducing the amount of transaction for each security. The cost of transaction will be considerate in the management problem.

Kind of operation The purchase of securities can be on cash or credit, this work will considerate the buy made to the stock brokerage on cash, and will be possible to make a leverage with a credit account that will have an interest rate associated.

Kind of yield The yield associated to the stocks of the portfolio can be fixed or variable. Fixed income guarantees a yield negotiated at the time of purchase, normally have a low yield but they are a safe investment with low volatility. This fact makes a safer choice to make a secure investment with a very small chance of a money loss in the short time. On the other side, a variable yield stock, normally have a big amount of volatility, but the market rules in general gives in reward to the risk aversion a bigger yield. This kind of instrument normally changes the price several times in a short window of time. Money markets are a good example of this kind of stocks.

This work uses 3 stocks to conform an investment portfolio. The idea is to give 3 options from low variance and low yield to a bigger yield and variance stock. In this way, the portfolio participation can be placed in a secure way of investment or a high risk aversion.

Once that the portfolio stocks are selected, it is necessary to define how to manage the portfolio. Related to this question, next we will present some models used to the portfolio management.

2.2. Management models

When the portfolio assets are selected it is necessary to have a mechanism or rules to do the portfolio management. Now, two management models classifications are presented:

2.2.1. Single-period model

The static models or single period models, are the most used in diversification of an investment portfolio. It provides a single solution for the problem, so , in 1850 Harry Markowitz, proposed in [24] the following relationship:

$$R = X_1R_1 + X_2R_2 + X_3R_3 + ... + X_nR_n \tag{2-1}$$

$$R = \sum_{i=1}^{n} X_iR_i \tag{2-2}$$

where: R is the expected return of the portfolio and it as a combination of different financial instruments R_i weighted each one by X_i. Now the problem is to choose the right value for X_i to get a desired value or objective. This example will minimize the variance (VaR)[1] of the portfolio.
A classic problem in the portfolio management is proposed. Given a portfolio composed by 3 assets called a, b, c $R = X_aR_a + X_bR_b + X_cR_c$, the objective is to minimize the variance (VaR) of the portfolio.

Each asset had a a variance $\sigma(r_i)$ associated, and can be defined the variance of the overall portfolio as:

$$\begin{aligned} \sigma^2(p) &= X_a^2\sigma(r_a) + X_b^2\sigma(r_b) + X_c^2\sigma(r_c) + \\ &\quad 2X_aX_bcov(r_a, r_b) + 2X_aX_ccov(r_a, r_c) + 2X_bX_ccov(r_b, r_c) \end{aligned} \tag{2-3}$$

The solution of this problem will give the expected value $E(R)$, the combination of X_i that minimize the VaR of the portfolio. As follows:

$$\min_{X_i} \sigma^2(p) \tag{2-4}$$

[1]Value at risk

Subject to:

$$E(R) = \sum_{i=1}^{3} X_i E(r_i) \tag{2-5}$$

$$X_1 + X_2 + X_3 = 1 \tag{2-6}$$

$$0 \leq X_i \leq 1 \tag{2-7}$$

The constraints in this problem defines: (2-5) the expected return of the portfolio, as the sum of the weighted assets on 2-6 the sum of the overall portfolio weights must be one and (2-7) each weight of the assets must be greater that zero and less than one. This solution implies some assumptions that are not feasible according the nature of the problem: first assumes constant variances and co-variances of the assets for the model optimization, also it is assumed that all information is contained in the time series (this fact depends on the amount of data available) It also assumes that prices are normal distributed, and it is hardly true, as can be seen since the solution of the problem decides the values of the X_i once, and in a long term investment, the solution cannot be valid along the time for the same problem.

2.2.2. Multi-period models

Multi-period asset allocation problem

At the initial time period an investor have a certain amount of wealth and it is available to make a decision to distribute the amount in i assets $i = 1, ..., n$ the wealth in general is money and can be in a risk free state, normally a bank account that generate a yield. This relationship can be described as an asset $n + 1$, an asset $n + 2$ is used to present the evolution of the borrowed capital, this asset is considered if the investor decides to get a loan for investment. The decision maker has to decide each period how to rearrange his portfolio in order to achieve the best return on his initial investment over time. The problem is presented in discrete time and define time steps $t = 1, ..., T$, e.g. by months, with T being the end of the planning horizon.

This strategy of management, as can be seen in [8], shows that multi-period portfolio management can be efficiently solved as a multistaged stochastic linear programs. In this way, partitioning the problem into a sum of small pieces, solves the main problem of single period and allows to re-balance the portfolio according to the changes of the system over the time. In fact, subject to the duration or dynamics of the asset the length of the time window can be selected to give the most suitable time that guarantees an efficient action over the portfolio participation. The study of consumption-investment problems via stochastic processes in continuous time was initiated by Merton [25]. He considered a model in which the prices of the risky securities were generated by a Brownian motion, and assumed that there were no transaction costs. The optimal strategy in his model consists of making an infinite number of transactions in order to keep the proportions invested in the risky securities equal to a constant vector. This work will not demonstrate the strategies of the authors.

The main reason of the time variability of the portfolio model is the change of the assets prices. This variation is hard to predict, and have a strong relationship with unpredictable variables, i.e: social factors such as national security issues, or the volatility of dollar and euro that can change the value of the assets, In this order, the assets prices can be modeled as a stochastic system, and based on the publications of [8], [25] and the survey of investment problems described in [5] we have the following model:

$$S_i(k+1) = S_i(k)\left[1 + \mu_i(k) + \sum_{j=1}^{n} \sigma(k) w_j(k)\right] \tag{2-8}$$

where S_i is the price of the i-asset, $\mu_i(k)$ is the mean value of the tendency of the i-asset, also know as growth coefficient, w_j is a normal white noise $w_j \sim (0, \sigma)$ and $\sigma(k)$ is a matrix of volatility. This model estimate the next asset value $S_i(k+1)$ as the sum of past value of the asset $S_i(k)$, the product $S_i(k)\mu_i(k)$, this term increases the value of the asset based in the tendency of growth on the price, and finally the sum represents an stochastic term that measure the volatility of the assets.

In the Kalman filter chapter, see cap 4, a implementation of this tool will be used to estimate in a optimal way the value of $\mu_i(k)$.

In [5] a survey of problems and methods contained in various works on consumption-investment problems are presented. This work takes a model (used in another works as: [11, 13] that consider the transaction cost defined by the following equations:

The management portfolio model for the n assets is given by :

$$x_i(k+1) = [1 + \eta_i(k)]\,[x_i(k) + p_i(k) - q_i(k)] \tag{2-9}$$

where $\eta_i(k)$ is the expected return of the assets:

$$\eta_i(k) = \mu_i(k) + \sum_{j=1}^{n} \sigma_{ij}(k) w_j(k) \tag{2-10}$$

and

- $p_i(k)$: Is the sum of transfers from the risk free asset to i-th risky assets.
- $q_i(k)$: Is the sum transfers of the i-th risky assets into risk free asset.

- $p_i(k) > 0$ and $q_i(k) > 0$ and if $x_i < 0$ means a short sale. [2].

The transaction costs paid are defined as: α and β corresponding to buy and sell respectively. Then another equation is defined as evolution of the risk free asset.

$$x_{n+1}(k+1) = \\ [1 + r_1(k)] \left[x_{n+1}(k) - v(k) - (1+\alpha)\sum_{i=1}^{n} p_i(k) + (1-\beta)\sum_{i=1}^{n} q_i(k) \right] \tag{2-11}$$

where

- $r_1(k)$ is a variable given by the banks, and represents the yield of the free risk asset.
- $v(k)$ is the transfer between the bank account and the credit account, if $v(k) > 0$ is a sum of borrowing capital, and if $v(k) < 0$ represents a credit repayment.

The evolution of the borrowed capital is given by:

$$x_{n+2}(k+1) = [1 + r_2(k)] \left[x_{n+2}(k) + v(k) \right] \tag{2-12}$$

- $r_2(k)$ is the interest rate of the loan.

Finally
The portfolio capital can be expressed as:

$$V(k) = \sum_{i=1}^{n+1} x_i(k) - x_{n+2}(k) \tag{2-13}$$

As can be seen, the final capital is the sum of the yield of the stocks minus the borrowed capital, this model also include the α and β values that penalizes the transactions cost in terms of the yield of the stocks. These terms are very useful while working with low yield stocks, since it avoids that the yield given by one asset will be consumed by the transactions cost paid to the financial or trade system.

In [18], a review of capital pricing shows, the properties of single and multi-period models are showed. Then, a brief description of the a new solution called *The single index model* (see [35]) is presented. This solution is computationally more efficient than the Markowitz theory, in 1965 and 1966 Linter [23] and Mossin [26] proposed similar solutions with different analysis. A theory commonly know as the Sharpe-Lintner or Sharpe-Lintner-Mossin "Capital Asset Pricing Model" (CAPM). Lots of research has been done to extend it, but the basic

[2]short selling (also known as shorting or going short) is the practice of selling assets, usually securities, that have been borrowed from a third party (usually a broker) with the intention of buying identical assets back at a later date to return to that third party

ideas constitutes a very important model used on modern finance theory and practice.Even though, the Capital asset pricing model (CAPM) is a very relevant model in the asset prices this model is a static single period model, but given the features of the solution proposed in this work, it was not considered.

3. The control theory role in finance applications

"During the past half-century, many optimization problems have arisen in fields such as finance management, engineering, computer science, production, industry, and economics. Often one needs to optimize (minimize or maximize) certain objectives subject to some constraints. For example, a public utility company must decide what proportion of its earnings to retain to the advantage of its future earnings at the expense of gaining present dividends, and also decide what new stock issues should be made. The objective of the utility is to maximize the present value of share ownership, however, the retention of retained earnings reduces current dividends and new stock issues can dilute owners equity" [7].

The use of financial models with control solutions has grown, and in order to get better and more accurate solutions. The solutions have become more complex making the formulation of analytic solutions a very difficult task, in this way, the computational solutions have become the main tool to the current researched questions. Optimal control theory is able to handle deterministic and stochastic models, even, hybrid models with the two characteristics. The finance problems are a particular case of the mix of the two worlds. his chapter focus its contents on a brief relationship between the two areas control and finance.

3.1. Control in finance

A generic control system representation is shown in the figure 3.1. The system is divided in two blocks, where P is the process and C the controller. The objective is:given a reference signal r which represent the desired value for the process variable called x, the feedback is represented with a -1 value, and the signal e named error is the difference between the desired value and the feedback of the process. Once the controller take actions based on the e signal the u signal denominated manipulated variable, manipulates the input values that can change the value of x on the P process. In this case d and n values are called load disturbance and measurement disturbance respectively. Basically the disturbances are measured or un measured, known or unknown signals that affect the controlled variable. Otherwise known as non-manipulated input variable.

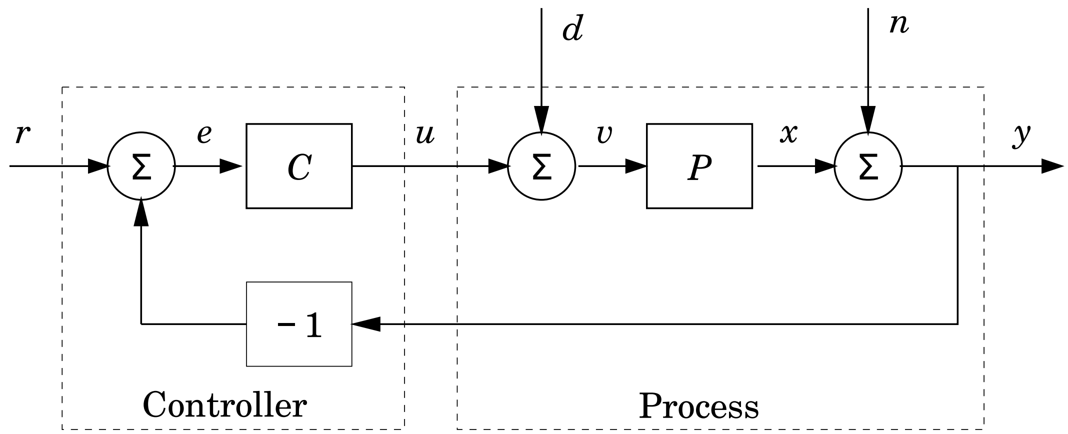

Figure **3-1**.: Generic control with error feedback

Some authors already made an analogy between the model of the figure 3.1 and the portfolio management problem. In Venkat et al [38] reviewed a control theoretical model for dynamic portfolio management see figure **3-2**, this model proposes 3 stages: *Monitor* that estimates timely the return and or the risk of current portfolio at the end of a plan horizon, The *decider* determines the timing of re-balancing according to some criteria and the *regulator* that changes the current portfolio to a more desirable one by solving certain portfolio model.

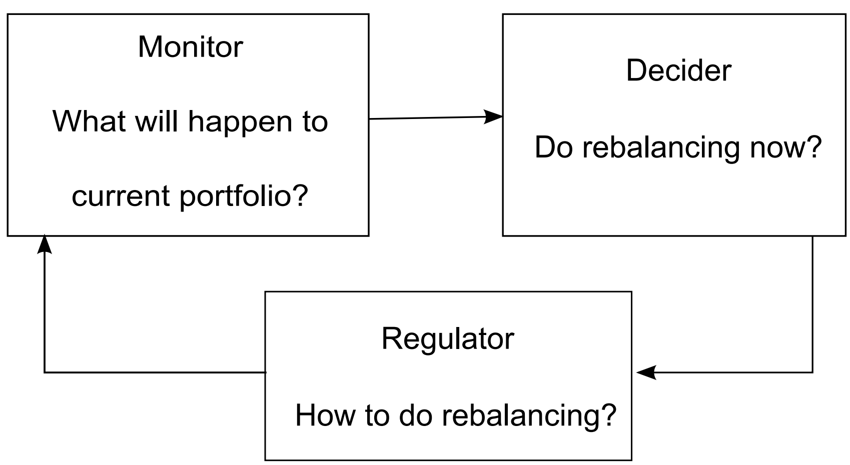

Figure **3-2**.: Control theoretical model for dynamic portfolio management. (Source [38])

As seen, comparing the figures 3.1 and **3-2**we observe a similar structure given by the feedback. Therefore, establishing a start point to explain how the portfolio management can be handled as a control problem; According the model proposed in chapter 2, the variables of the management model are related in the following table:

Control variable	Portfolio variable
r	Expected return of the portfolio
e	Difference between the expected return and the real return of the portfolio
u	The transfers between risk free asset and i-th risky assets in both directions
P	The portfolio is described by the equation 2-9 and equations 2-11, 2-12
C	The controller or regulator
-1	Controller feedback

Table **3-1**.: Relationship between control system variables and portfolio variables

Since the portfolio management model of chapter 2 can be handled as a dynamic system used in the P variable as referenced in table 3.1, the control strategies to solve the problem are be placed in the C variable of the table,a big amount of control methods can be used to solve the problem.

3.2. Control techniques in finance application

Among the many available control techniques, some have been recently used to solve the problem of portfolio management.
Computational intelligence solutions has been tested, but their general implications as the high dependence on the quality of the training data, the lack of guarantees to ensure convergence and optimality, made them undesirable as a tool for general applications of portfolio management. In [17] the problems mentioned before are explained and conclude that the results of the neural network are sub optimal compared with a least square solution.

In [13] the author made an optimal strategy of portfolio management based on stochastic models and made the reference of the control a benchmark portfolio. In this strategy the objective of active portfolio management is to make investment portfolio (IP) more profitable than the given benchmark portfolio and the objective is to minimize a risk function. This solution do not include constraints over the transaction costs and assumes a fixed value in the returns of the assets. The authors at least, leave an open case of study to propose solutions with other control tools. The paper cited in [39] proposes a minimax strategy, this solution is based in the Markowitz basic principle given: *a desired expected return value is possible to minimize the variance, and to a desired variance level is possible to maximize the expected return*. The paper proposes a minimax function that maximizes the expected return and minimizes the variance (that is also called risk aversion). The function have a free parameter that the user can choose, and there is no way to determinate the optimal value. This parameter weights the two terms of the objective function, giving to the investor the ability to choose the degree of aggressiveness of the management strategy. This strategy has the following assumptions in the model: only consider n risky assets (there is not a free risk asset), do not assume transactions cost, there are not restrictions on short-sales. Another control based solution is developed in [38] where the authors defines zones with return of the portfolio and the variance, when the portfolio do not meet the requirements a new optimization based on single period model is made, maximizing the return subject to the desired risk restriction.

As can be seen, the trend in solutions on portfolio management is to guarantee optimal solutions in multiperiod models, and the most of the assumptions in the solutions proposed are related to the problem constraints. With increased computing capacity, optimal control techniques have emerged in many fields, because their general solutions involve high computational costs that now can be provided, but these higher costs are rewarded with robust solutions that can integrate a large number of constraints and stochastic models.

3.2.1. Optimal control model in finance

Optimal control modelling, both deterministic and stochastic, is probably one of the most crucial areas in finance given the time series characteristics of financial systems behaviour. It is also a fast growing area of sophisticated academic interest as well as practice using analytical as well as computational techniques [7].
Consider a financial optimal control model:

$$\min_{x,u} J(u) = \int_0^T f(x(t), u(t), t)dt \tag{3-1}$$

Subject to:

$$x(0) = x_0, \dot{x}(t) = m(x(t), u(t), t) \tag{3-2}$$

$$a \leq u(t) \leq b \tag{3-3}$$

$$0 \leq t \leq T, T > 0 \tag{3-4}$$

where, $x(t)$ are the states, $u(t)$ are the control inputs, $\dot{x}(t)$ describes the dynamics of the financial system and T is the planning horizon since $J(u)$ is a cost function, the objective is to find $(\tilde{x}, \tilde{u})$ that minimizes $J(u)$.

Now based on the equations of section 2.2.2, and the definitions of this chapter , it is possible to propose an optimal solution to the portfolio management problem with optimal control techniques. Given the characteristics of the optimal control proposed on equations 3-1, 3-2 and 3-3, this work proposes a management function that optimizes the portfolio return, with transactions constraints between one risk-free asset and three risky assets. The feedback of the controller will be implemented with an optimal estimator that will made the estimation of the trend of the prices of the assets.

4. Estimation in finance applications

In practice, the finance applications are highly defined by empirical expertise also called "expert knowledge". This knowledge allows the stocks traders to infer based on the data history the future behaviour of the market. Based this expected values, to take the most appropriate decisions according to some criteria. In order to solve the management portfolio problem, the MPC based on the equations of chapter 2 needs to estimate so called "expected return of the assets" $\eta_i(k)$. This estimate can drastically affect the controller's performance,this is , as well, as the stock trader, the MPC makes its decisions inferences based on what will happen and this "future" and it is related to the $\eta_i(k)$ value.

A well know estimation algorithm used in control applications, suitable for systems where the signals have a stochastic or a random component, just like the stock prices, is the Kalman filter. A general description of the Kalman Filter can be seen in appendix A. On the other hand the Moving Horizon Estimation (MHE), is an estimator cable of handling constraints and a time window. So far only kalman filters has been applied in finance. This thesis is the first known case where MHE is applied to finance. The characteristics of MHE motivates its use do not have any reported application on finance and give some additional characteristics, explained forward, that motivate the use of this method on this work as an alternative solution to the problem formulation proposed in [11]. A full detailed literature review of estimators including a full description of the MHE can be seen in [31].

4.1. MHE-based Estimators

Moving Horizon Estimation (MHE) strategies was born as a dual problem of the Model Predictive Control (MPC). Despite of its similarities, MPC technology was developed first in petroleum industry due to the dynamic complexity of the processes and the need of improved control strategies, whereas MHE theory was developed first in academia [2].

The basic strategy of MHE reformulates the estimation problem as a quadratic problem using a moving, fixed-size estimation window. The fixed-size window is needed to bind the computational effort to solve the problem. This is the main difference of MHE with the batch estimation problem (or full information estimator) [1, 2, 31]. Once a new measurement is available, the oldest one is discarded, using the concept of window shifting. Moreover, the main advantage of MHE in comparison with other estimation schemes (like the Kalman Filter) is the straightforward constraint addressing inside the optimization problem, and the possibility to propose the cost function.

4.2. MHE applied to portfolio management

There is not a reported application of the MHE procedure applied to the portfolio management problem. This work implements the MHE as an estimator of the prices, the fixed moving estimation window technique fits perfectly in the stock price estimation if the historical data prices are considered as oursource of information. In this order, the adjust of the $\eta_i(k)$ value in a fixed time window that have past information based on the MHE algorithm, fits with the trade strategy used in the financial markets.

In [36] a full description of the MHE is made. One of the examples present some simple examples to highlight the benefits of the moving horizon estimation approach. And considers a non isothermal gas-phase reactor in which the reversible reaction $2A \rightleftharpoons B$ is taking place. They represents the system with a non linear function and compare the estimation results with an EKF estimation. The paper shows a better performance estimation with the MHE, and this fact motivate the implementation of this procedure to validate the performance in the portfolio prices estimation.

4.2.1. MHE Prediction model

For the development of the MHE estimator in this work, a new price evolution equation is proposed. The stock prices evolution can be written as:

$$S_i(k+1) = S_i(k) + S_i(k)\eta_i(k) \tag{4-1}$$

with $i = 1..n$
$S_i(k)$ is the i-th asset price and $\eta_i(k)$ is the price asset trend. And:

$$\eta_i(k+1) = \eta_i(k) \tag{4-2}$$

In order to use a MHE approach it is necessary to obtain a linear model of 4-1 and 4-2

$$x(k+1) = \begin{bmatrix} S_i(k+1) \\ \eta_i(k+1) \end{bmatrix} = \begin{bmatrix} S_i(k) + S_i(k)\eta_i(k) \\ \eta_i(k) \end{bmatrix} + w(k) \tag{4-3}$$

And the model have the state space form:

$$x(k+1) = f(x) + w(k) \tag{4-4}$$

$$y(k) = Cx(k);\ C = [1\ \ 0] \tag{4-5}$$

In order to use a MHE approach is necessary to linearize the equation 4-3 around the $x^*(k)$ point:

$$\Delta x(k+1) = f(x^*(k)) + A(x^*(k))\Delta x(k) \tag{4-6}$$
$$\Delta y(k) = Cx^*(k) + C\Delta x(k) \tag{4-7}$$

4.2.2. MHE approximation

The MHE model can be written:

$$\hat{x}(k+1) = A(k)\hat{x}(k) + G(k)w(k) \tag{4-8}$$
$$\hat{y}(k) = C(k)\hat{x}(k) + v(k) \tag{4-9}$$

where $\hat{x}(k) \in \mathbb{R}^n$ and $w(k) \in \mathbb{R}^w$ are the linearized state and uncertainty respectively, $v(k) \in \mathbb{R}^p$ is the linearized measurement noise. The estimation of the whole state in equations 4.2.2 can be formulated as:

$$\phi_T^* = \min_{x_0, \{w_k{}_{k=0}^{T-1}\}} \phi_T\left(x_0, w_k{}_{k=0}^{T-1}\right) \tag{4-10}$$

with:

$$\phi_T\left(x_0, \{w_k{}_{k=0}^{T-1}\}\right) = \sum_{k=0}^{T-1} \parallel y(k) - \hat{y}(k) \parallel_Q^2 + \parallel w(k) \parallel_R^2 \tag{4-11}$$

This cost function can be rewritten in a alternative way as:

$$\phi_T\left(x_{T-N}, \{w_k{}_{k=T-N}^{T-1}\}\right) = \sum_{k=T-N}^{T-1} \parallel y(k) - \hat{y}(k) \parallel_Q^2 + \parallel w(k) \parallel_R^2 \tag{4-12}$$

when N is the length of the fixed time window of the MHE.

4.2.3. Results

In order to prove the MHE estimation, three stocks prices used in the MPC application case where estimated. As can be seen in figures **4-1**, **4-2** and **4-3** the price estimation of the MHE fits to the real price of each stock, due that the MHE estimate the actual estate based on the past data, the first value of each estimation is the initial value X_{T-N} given to the estimator system.

The values used are $R = 0$ and the Q and initial values for each stock are presneted table 4.2.3

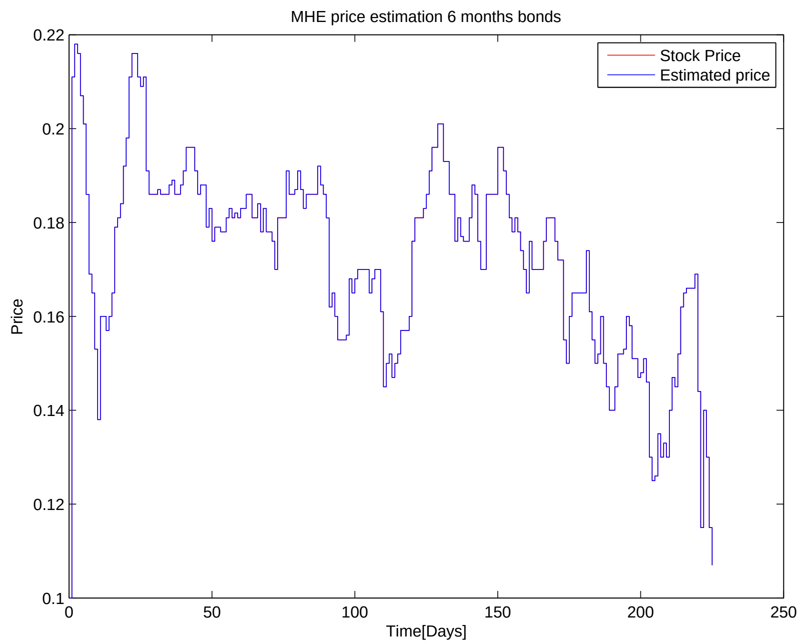

Figure **4-1**.: MHE price estimation for the 6 months bonds asset

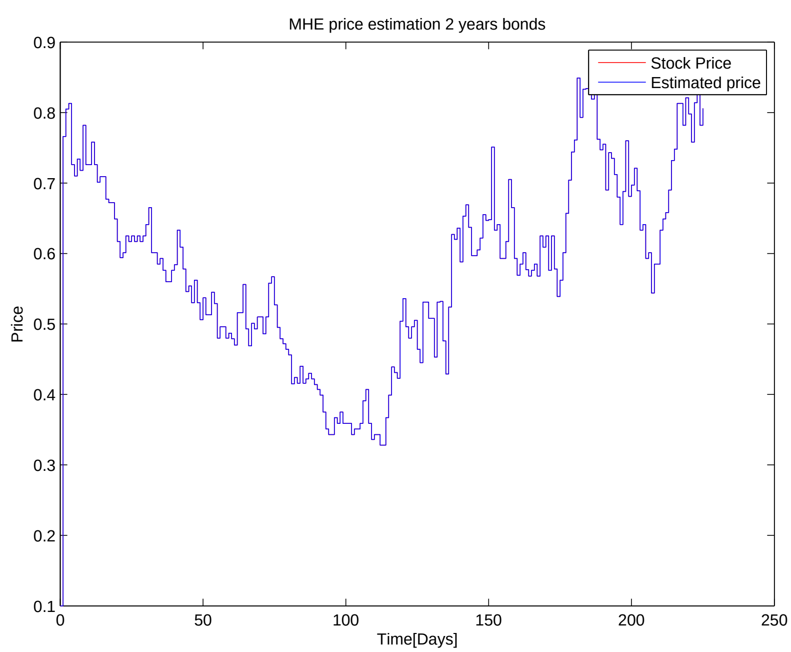

Figure **4-2**.: MHE price estimation fot the 2 years bonds asset

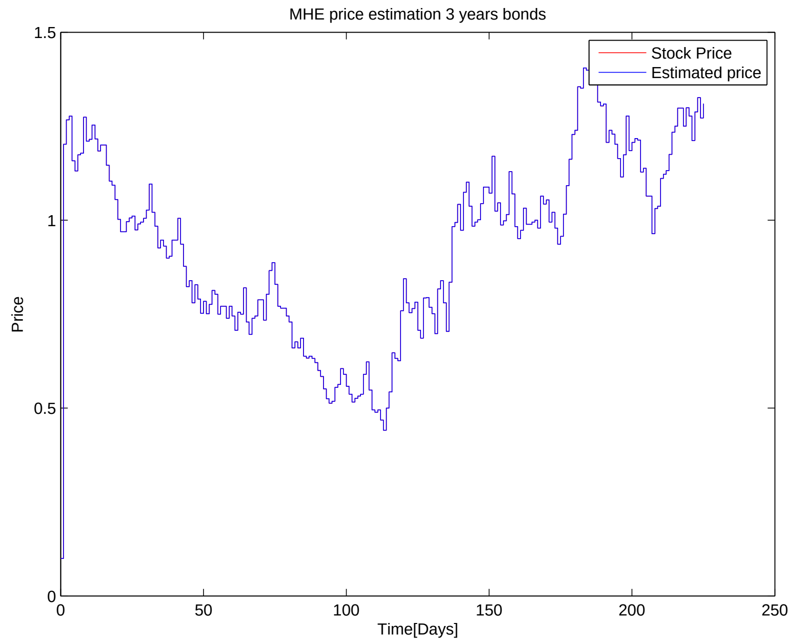

Figure **4-3**.: MHE price estimation for the 3 years bonds asset

The estimation error of each MHE is presented in the next figure

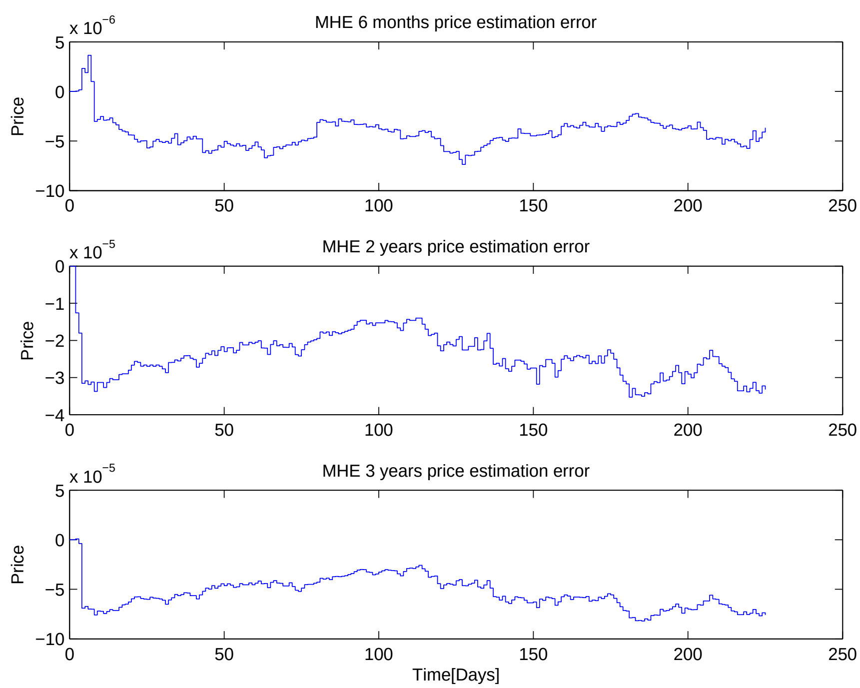

Figure **4-4**.: MHE price estimation error

The estimation error for each price can be considered as zero. This small error was achieved making the empirical off line tuning of the MHE.

Variable	Value
Q_1	10
Q_2	100
Q_3	1000
$x_i(0)$, $i = 1, 2, 3$	0.1

Table **4-1**.: MHE Variable definitions for 6 months, 2 years and 3 years assets

5. Model predictive control

This chapter explains the basic methodology of the Model predictive control (MPC) controller, and its application to the portfolio management. This chapter joins the application of the chapters 2 and 4 in an optimal solution to the management problem, constrained in the transaction cost and the amount of the short sales.

In the paper "A Brief Overview Of Model Predictive Control" [33] three main ideas are defined:

1. Explicit use of a model to predict the process output along a future time horizon.
2. Calculation of a control sequence to optimize a performance index.
3. A receding horizon strategy, so that at each instant the horizon is moved towards the future, which involves the application of the first control signal of the sequence calculated at each step.

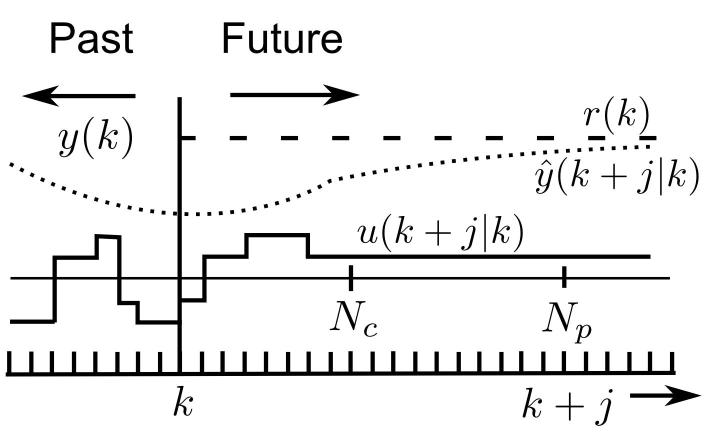

Figure **5-1**.: Receding horizon strategy

The predicted outputs $\hat{y}(k+j|k), j = 1, 2, ..., N_p$ in the prediction horizon N_p are calculated at each instant of time k using the process model. These depend upon the values up to instance k (past inputs and outputs), including the current output (initial condition) $y(k)$

and on the future control signals $u(k+j|k)$, $k = 0..N-1$, must to be calculated.

The sequence of future control signals is computed to optimize a performance criterion, often to minimize the error between a reference trajectory $r(k)$ and the predicted process output $\hat{y}$. Usually the control effort is included in the performance criterion.

Only the current control signal $u(k)$ is transmitted to the process. At the next sampling instant $y(k+1)$ is measured and the process is repeated.

Advantages and Disadvantages of MPC .
Some of the main advantages are:

- Can be used to control a great variety of processes, including those with non- minimum phase, long time delay or open-loop unstable characteristics.
- Can deal with multivariable, multi-input multi-output as well as single-input single-output processes.
- Process constraints can readily be treated within the optimization process.

A disadvantage is that it requires: **Requirement an appropriate model of the process.**

5.1. Elements of the model predictive control

5.1.1. Prediction model

The prediction model, can be linear or nonlinear, normally the MPC works with linear models. As a main base of the prediction for the MPC, the model can be written in a discrete state space equations, a general form of a state space is defined as:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{5-1}$$

where $x(k)$ are the states of the system, the A matrix describes the states evolution, B establishs the relationship between the inputs and the state, $u(k)$ the inputs of the system, $y(k)$ the output of the system, C is the relations between the states and the output and $D(k)$ a direct relation between the inputs and the outputs sometimes called the feedtrhough term.

5.1.2. Performance index

Also called cost criterion, penalizes the reference tracking errors and the amplitude of control actions used to minimize the function.

$$J(u(k), e(k)) = \sum_{t=1}^{N_p} e(k+j)^T e(k+j) + \sum_{t=1}^{N_c} u(k+j)^T u(k+j) \tag{5-2}$$

A more general cost criterion will be used. This formulation, includes Q and R matrices that allows to weight the states and control values respectively. This matrices are assumed to be symmetric positive definite.

$$J(u(k), e(k)) = \sum_{t=1}^{N_p} e^T(k+j) Q e(k+j) + \sum_{t=1}^{N_c} u^T(k+j) R u(k+j) \tag{5-3}$$

where $e(k+j) = \hat{y}(k) - r(k)$, N_p is the prediction horizon and N_c is the control horizon, $u(k+j)$ denotes the control input u at time step $k+j$, $e(k+j)$ denotes the error value e at time step $k+j$.

5.1.3. MPC formulation

Given the prediction model by the equation 5-1:

$x(k+1) = Ax(k) + Bu(k)$

$y(k) = Cx(k)$

a typical formulation for the MPC problem is proposed as follows:

$$J(\hat{u}(k), \hat{e}(k)) = \sum_{t=1}^{Np} e^T(k+t|k) Q e(k+t) + \sum_{t=1}^{Nu} u^T(k+t) R u(k+t|k) \tag{5-4}$$

If the prediction model is linear the substitution of the equation 5-1 in 5-4 gives to the cost function the form:

$$J(\hat{u}(k)) = \hat{u}^T(k) H \hat{u}(k) + 2f\hat{u}(k) \tag{5-5}$$

$$H = \hat{B}^T \hat{Q} \hat{B} + \hat{R} \tag{5-6}$$

$$f = (x(k)^T \hat{A}^T - \hat{y}^T)\hat{Q}\hat{B} \tag{5-7}$$

where $\hat{u}(k) = \begin{bmatrix} u^T(k) & u^T(k+1) & \ldots & u^T(N_p-1) \end{bmatrix}^T$, $u(k+t) = u(k+N_c)$, $\forall N_c \leq t \leq N_p$, $\hat{Q}$ and $\hat{R}$ are block diagonal matrices with the adequate dimensions, being Q and R the blocks of $\hat{Q}$ and $\hat{R}$ respectively, and

$$\hat{B} = \begin{bmatrix} CB & 0 & ... & 0 \\ CAB & CB & ... & 0 \\ \vdots & & \ddots & 0 \\ CA^{N_p-1}B & & ... & CB \end{bmatrix} \tag{5-8}$$

$$\hat{A} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{N_p} \end{bmatrix} \tag{5-9}$$

$$\hat{Q} = \begin{bmatrix} Q & 0 & ... & 0 \\ 0 & \ddots & ... & \\ \vdots & & \ddots & 0 \\ 0 & & ... & Q \end{bmatrix} \tag{5-10}$$

$$\hat{R} = \begin{bmatrix} R & 0 & ... & 0 \\ 0 & \ddots & ... & \\ \vdots & & \ddots & 0 \\ 0 & & ... & R \end{bmatrix} \tag{5-11}$$

where the objective is to: $\underset{u(k)}{\text{minimize}} \quad J(u(k))$
The optimization can be constrained. The constrains are explained in the next subsection

5.1.4. Constraints

In real applications, the processes have constraints related to the application of each problem, industrial processes have limits given by: temperature, operation limits and physical restrictions, in financial systems, the constraints can be related to the transactions cost, amount of stocks that can be negotiated. The MPC is a strong technique thanks to the constraints handling capabilities. If the problem does not have constraints, it has a explicit least squares solution.
Normally, the constraints are applied to the states, input or output signals, along all the time k

$$\begin{aligned} u_{min} &\leq u(k) \leq u_{max} \\ y_{min} &\leq y(k) \leq y_{max} \\ x_{min} &\leq x(k) \leq x_{max} \\ \Delta u_{min} &\leq \Delta u_(k) \leq \Delta u_{max} \end{aligned} \tag{5-12}$$

The constraints must be met. This is made by forcing to the control inputs to be modified to guarantee the constraints conditions. There also exist the equality constraints, normally used to keep the control signal constant beyond certain time. For instance:

$$\begin{aligned} \Delta u(k+t|t) &= 0 \\ t &\geq N_c \end{aligned} \tag{5-13}$$

Regarding about the time duration of the problem, in order to obtain a problem with an amount of data or size tractable, it is used a control horizon. In other words, the full problem is not solved in a single step, as shown in the receding horizon strategy. When the input signal is calculated, it is assumed to be constant beyond a certain moment in the future. This horizon is called "control horizon" denoted by N_c and the formulation is:

$$u(k+t|t) = u(k+N_c-1|k) \text{ for } t \geq N_c \tag{5-14}$$

5.2. MPC applied to portfolio management

The essence of MPC is to optimize, over the manipulated inputs, forecasts of process behavior. The forecasting is accomplished with a process model and therefore, the model is the essential element of an MPC controller [32].

Portfolio management try to distribute the capital owned in a back account or another "safe" place, where the yield of the money typically is small, between the assets owned in the portfolio. The capital assignment can be performed by multiple criteria, among others: minimum variance, maximize the expected return or combination of them. Also sometimes it is desirable to track a reference portfolio. In all these cases the MPC allows to add restrictions over the variables of the portfolio.

5.2.1. Prediction state space model

As seen previously in this work, the portfolio management problem have a series of properties that make the problem a natural candidate to be solved with MPC theory, and this method highly depends on system model. For the MPC implementation, the management model that was explained in chapter 2 is now taken to the space state representation. So, the idea is to represent equation 2-9 of the management model in the space state defined in equation 5-1:

The evolution of the states is described by:

$$x(k+1) = \begin{bmatrix} x_1(k+1) \\ \vdots \\ x_n(k+1) \\ x_{n+1}(k+1) \\ x_{n+2}(k+1) \end{bmatrix} \tag{5-15}$$

The index $i = 1...n$ represents the n stocks that build the investment portfolio described in the equation 2-9 and the expression for x_{n+1} and x_{n+2} are defined by equations 2-11 and 2-12 respectively.

The A matrix describes the states evolution, each time that the time window changes of value, the matrix will change. As can be seen the A matrix is linear but time variant.

$$A = \begin{bmatrix} 1+\eta_1(k) & & ... & & 0 \\ 0 & \ddots & & & \\ \vdots & & 1+\eta_i(k) & & \vdots \\ & & & 1+r_1(k) & 0 \\ 0 & & ... & 0 & 1+r_2(k) \end{bmatrix} \tag{5-16}$$

In this work, the algorithm will estimate the η_i value with an MHE estimator (see section 4.1)

$$B = \begin{bmatrix} 1+\eta_1(k) & 0 & 1+\eta_1(k) & ... & 0 \\ 0 & \ddots & 0 & \ddots & 0 \\ & & 1+\eta_i(k) & 0 & 1+\eta_i(k) \\ (1+r_1)(-1-\alpha) & ... & (1+r_1)(1-\beta) & ... & 1+r_1(k) \\ 0 & 0 & 0 & 0 & 1+r_2(k) \end{bmatrix} \tag{5-17}$$

$$u(k) = \begin{bmatrix} p_1(k) \\ \vdots \\ q_1(k) \\ \vdots \\ v(k) \end{bmatrix} \tag{5-18}$$

$$C = \begin{bmatrix} 1 & ... & 1 & -1 \end{bmatrix} \tag{5-19}$$

$$y(k) = \begin{bmatrix} 1 & ... & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ \vdots \\ x_i(k) \\ x_{n+1}(k) \\ x_{n+2}(k) \end{bmatrix} \tag{5-20}$$

And finally the space state system can be expressed as:

$x(k+1) = Ax(k) + Bu(k)$

$y(k) = Cx(k)$

5.2.2. Constraints

As mentioned in 5.1.4, a characteristic of the MPC is its ability to handle constraints. These constraints help to give the solution a desired behavior. In this work, the constraints considered taken from [11] are defined by:

$$
\begin{aligned}
&p_i(k) \geq 0\\
&q_i(k) \geq 0\\
&u_{min} \leq u_i(k) \leq u_{max}
\end{aligned}
\tag{5-21}
$$

the x_{n+1} state, is the risk free capital asset of the portfolio, given, that the asset can not take negative values, that will mean loans in the account, can be defined that:

$$x_{n+1}(k) - v(k) - (1+\alpha)\sum_{i=1}^{n} p_i(k) + (1-\beta)\sum_{i=1}^{n} q_i(k) \geq 0 \tag{5-22}$$

The values considered in this constrain: $\alpha \geq 0, \beta \geq 0$ and from the constrain 5-21 are guarantee to be always positive, but the $1+\alpha$ count the cost of purchase assets and can decrease the amount of money in the risk free asset. Also, $v(k)$ can take positive and negative values variable represents, as mentioned, the transfer between the risk free asset and the the credit account x_{n+2}; this constrain will guarantee that the payments of credit and the transactions cost of the assets trade do not spent more money than the amount available in the risk free account.

The borrowing capital given by x_{n+2} give to the portfolio the ability to make loans to a credit account with a credit rate, if the amount of money given by $x_{n+1} = 0$ the MPC can take a loan, this loan must be constrained. The amount must be greater than zero and must have a top value $d_0(k)$. in other words the MPC can take an infinite loan to buy assets x_i to maximize the expected return $V(k)$ of the portfolio.

$$x_{n+2}(k) + v(k) \geq 0 \tag{5-23}$$

$$x_{n+2}(k) + v(k) \leq d_0(k) \tag{5-24}$$

Finally, the shorts sales in the portfolio can be constrained, this condition allows to the MPC, to sell stocks that really do not have, is a common operation used in the market operation, and will be limited by $d_i(k)$, as follows:

$$x_i(k) + p_(k) - q_i(k) \geq -d_i(k),\ i = 1...n \tag{5-25}$$

5.2.3. Results

Now the time series used to the case of study on this work are presented

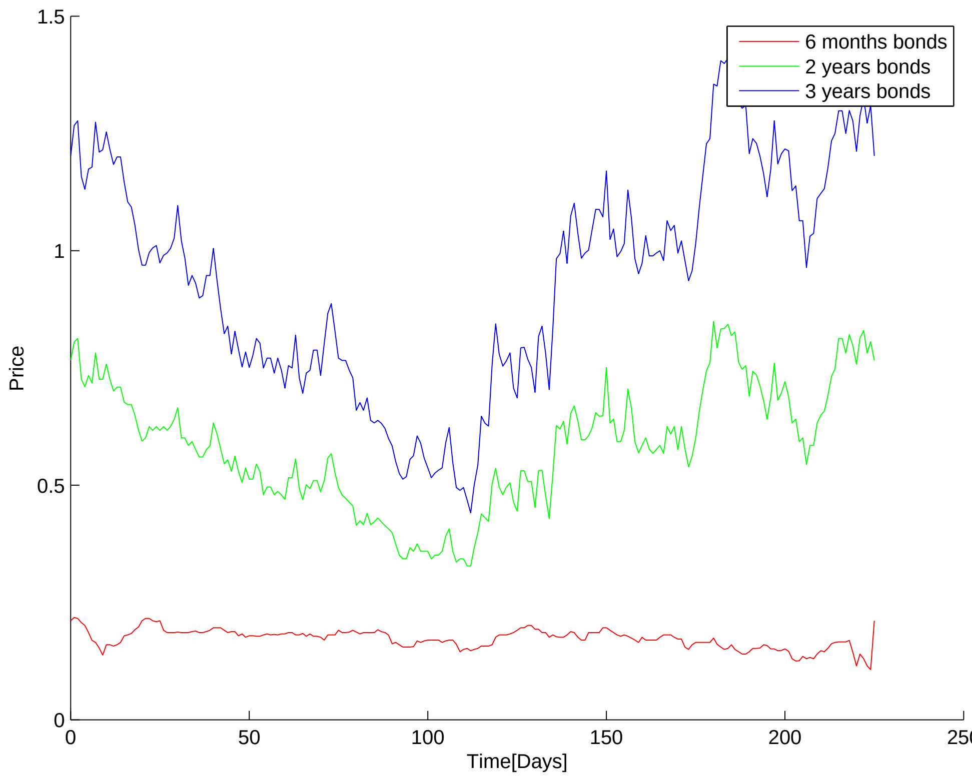

Figure **5-2**.: Colombia treasures bonds prices

The stocks selected are the national Colombia treasure bonds:the red line is the 6 months stocks, green represents the 2 years bonds and the blue line is the 3 years bonds. The time series represent the data between June 1st of 2010 and April 11th of 2011 to have a total of 225 days, of trade in the market.

The values used for the system are given in table 5.2.3.

Variable	Value
α	0.01
β	0.01
r_1	0.001
$r2$	0.04
$d_0(k)$	3
$d_i(k)$	0
$x(0)$	[0 0 1 1 0]
N_p	7
Nc	1
Q gain	1
R gain	0.1
Reference $r(k)$	100
p_i max	0.9
q_i max	0.9
$v(k)$ min	-0.9
$v(k)$ max	0.9
u_{max}	0.9

Table **5-1**.: Variable values given to the MPC and model variables

In this case Q and R are identity matrices, modified by the gain listed in table 5.2.3. These values are selected performing a offline tuning of the MPC controller α, β, r_1 and r_2 were selected based on the reference [11]. The initial conditions of $x(0)$ were determined such that an initial amount of asset in the lowest yield stock is placed in order to begin with low risk investment. The v value also is selected equal to 1 to guarantee the payment of the transactions cost of the first transactions, the $d_0(k)$ is selected equal to 3, allowing to the MPC the possibility to take a loan from the credit account when it is necessary and $d_i(k) = 0$ to avoid the short sales in the portfolio. Finally the maximum value of bought and sold assets can be greater than 0.9.

Analysis of the MPC with constraints

Now the MPC controller is tested with real prices. A "infinite" reference value is selected, in order to achieve the best performance of the controller. This value is theoretically infinite, forcing the MPC controller to perform the control actions within the constraints and try to get the highest return on each investment.

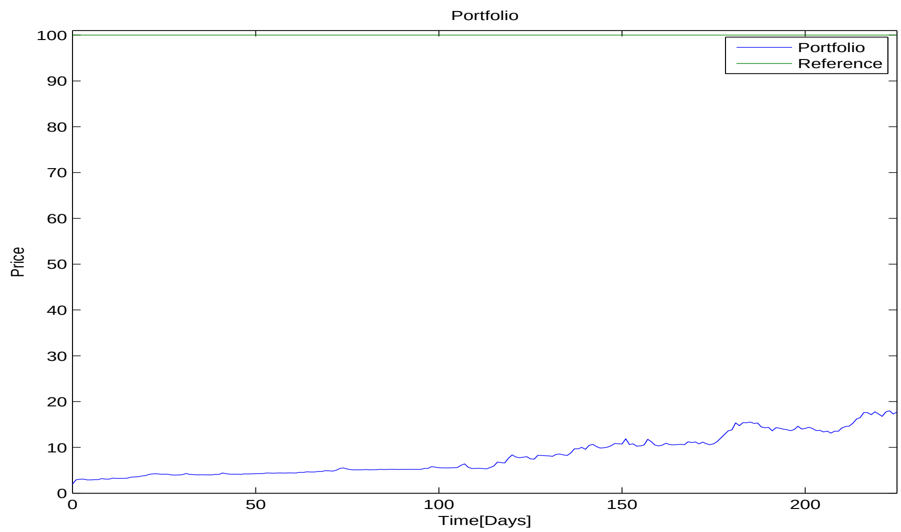

Figure **5-3**.: Portfolio evolution

Figure **5-3** shows how the portfolio value increases, trying to reach the infinite reference. The behavior of the portfolio have a higher trend to increase when the stocks exhibits a price increment, from the day 175 on, in the 3 years bond and 2 years bond, and small loses when the stocks have a price decrease in the same stocks around the 111th day (see figure **5-2**).

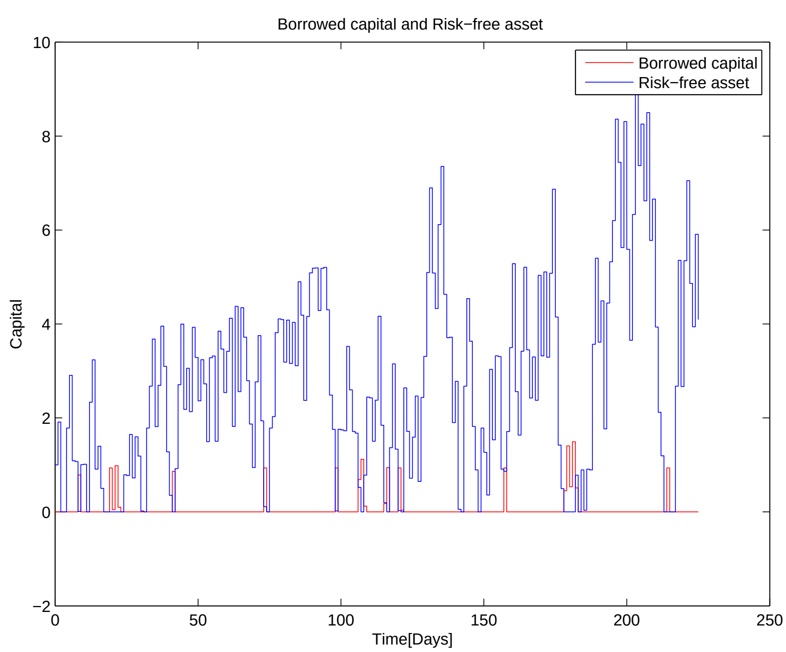

Figure **5-4**.: Borrowed capital and Risk-free asset

The evolution of the risk free asset x_{n+1} and the borrowed capital x_{n+2} have a high correlation. The MPC controller decides when it is necessary to take capital from the credit account an explicit case is presented when the risk free asset is zero, and it is necessary to cover the transactions cost and the price of the assets for a new investment. this effect can be seen in the time elapsed between days 17 to 31 in figure **5-4**.
Figure **5-5** shows the $v(k)$ variable, that represents the transfer to do between the risk free asset and the borrowing capital calculated by the MPC.

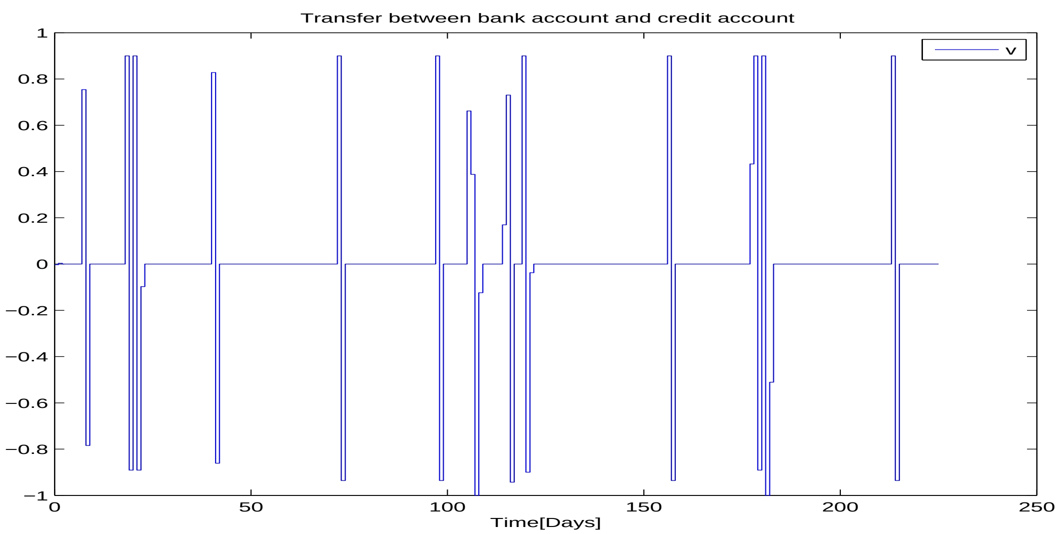

Figure **5-5**.: Transfer between bank account and credit account

Finally the actions of the controller are reflected over the amount of stocks of the portfolio assets, the amount of shares each day is show in figure **5-6** (bottom). The 6 months stock get the top amount of shares between the 15th day and the 23th day. This fact is due to high low trend of the 2 years and 3 years bonds and in addition the 6 months stock have a growing trend. Due to this fact, the MPC decides to buy a big participation in the 6 months stock, and to sell the remaining stocks of the other two portfolio assets. After that, the 3 years bond have a growing trend from the 25th till the 31th day and the shares of the stock begin to be purchased from the 24th day and begin to be sold from the 32th day. Meanwhile the 2 year bond have a similar behavior, the 6 months share try have a stable but decreasing trend, so the MPC sell all the shares of the stock.

From the day 88 to 94 when all the share prices down, the MPC decide to sell all the shares of the portfolio anticipating a big loss in the portfolio return. In a similar way from the 100th day and forward, the MPC with the growth of prices in the 2 years and 3 years bonds increase the shares of the stocks, keeping the 6 months shares with a low participation and some times on zero, due to the trend of the prices.

Finally, due to a big fall on the prices of the 3 years bonds the MPC sells a big amount of the stocks, and increases the amount of 2 years bonds that have a more stable price. It is important to remember, that the MPC also takes into account on the decisions the transactions costs, trying to minimize the loses for the negotiation actions.

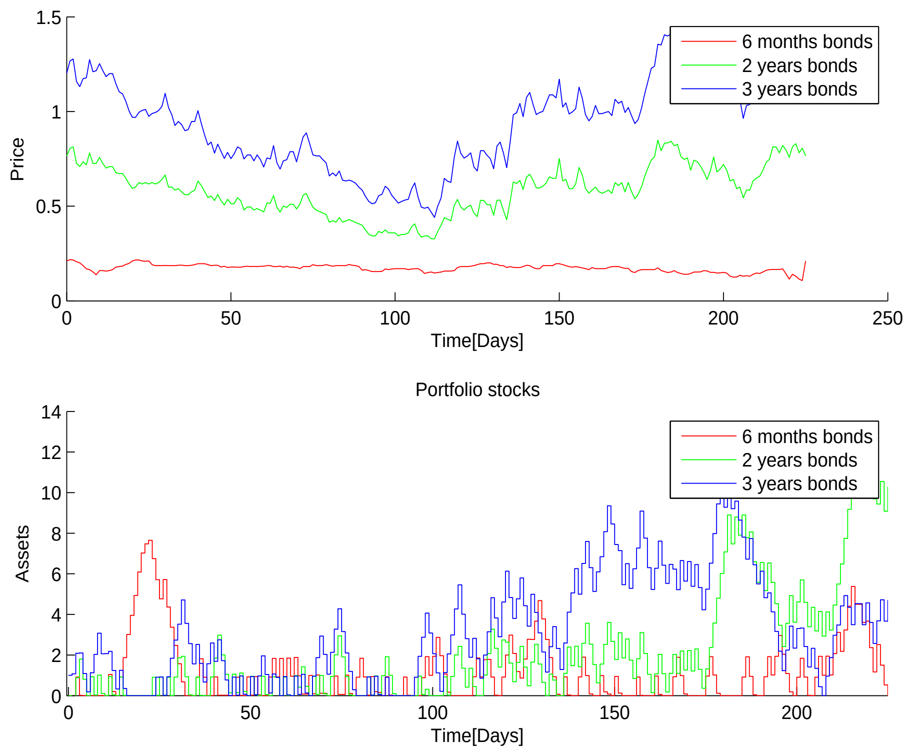

Figure **5-6**.: Evolution of the stock prices and the amount of assets of the portfolio

The control actions that determines the MPC actions explained before are presented in figure **5-7**. As can be seen, the control actions along the simulated time are within the constrains value. This means that the controller is trying to obtain the best performance of the system. The infinite reference value assigned is responsible of this effect.

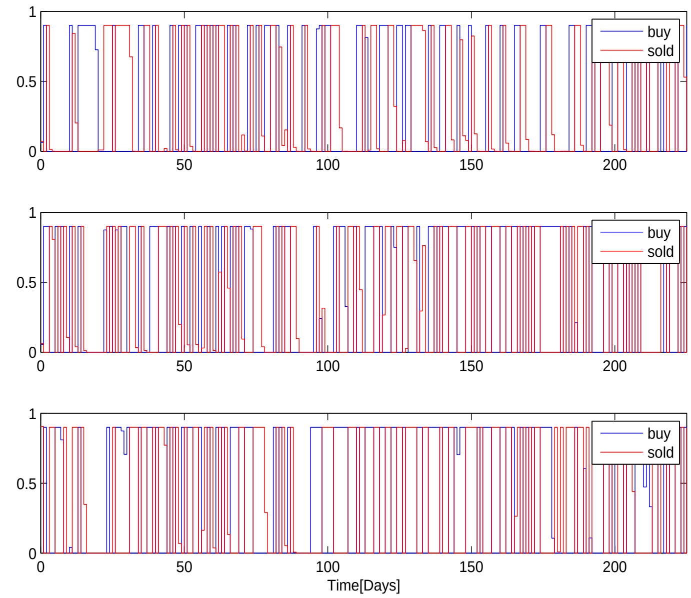

Figure **5-7**.: Control actions for the portfolio assets. (Top) 6 months bonds. (Middle) 2 years bonds. (Bottom) 3 years bonds

In figure **5-8**, one can see the trend values calculated for the MHE for each stock with a time window of 7 days the prices of the stocks and the portfolio assets. The lines corresponding are: Blue line 6 months bonds, Green line 2 years bonds, red line 3 years bonds.

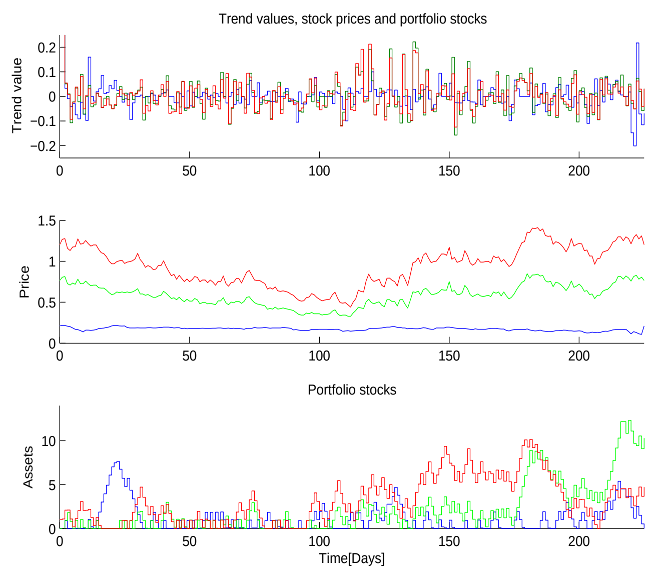

Figure **5-8**.: Trend values stock prices and portfolio stocks

5.2.4. Additional test

As additional test to the MPC strategy a series of modifications are presented, in order to illustrate another possible scenarios of the portfolio management.

Short sales allowed

Allowing the short sales in the system making $d_i(k) = -1$ the MPC can perform short sales in the market, avoid lending in the credit account, obtaining liquidity of the short sales.

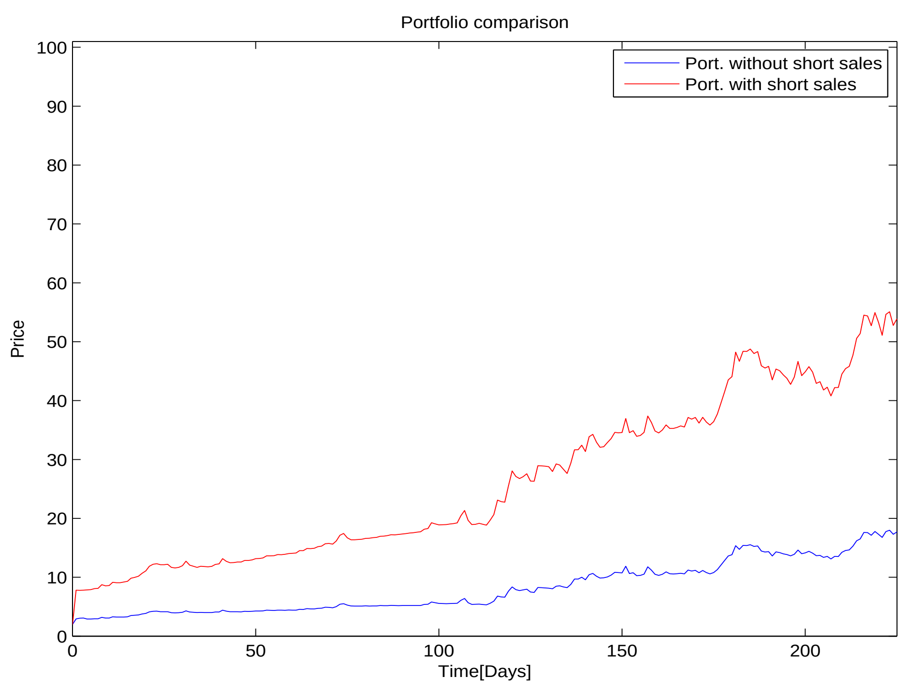

Figure **5-9**.: Comparison between portfolios with and without short sales

The portfolio value with the short sales allowed has more freedom degrees to operate in the market, and the use of the credit account x_{n+2} is decreased almost to zero using the short sales as loan method.

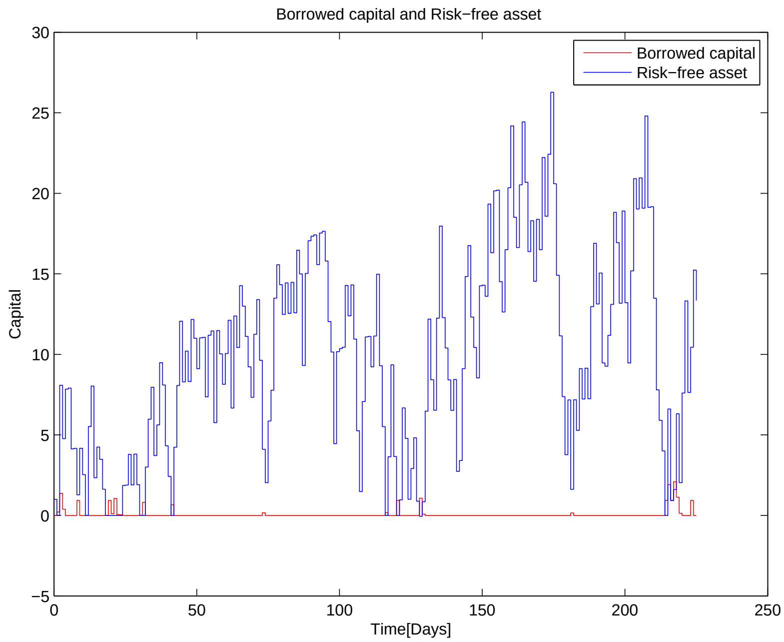

Figure **5-10**.: Borrowed capital and Risk-free asset with short sales

Finally the assets amount of each stock increases, allowing the increase of yield possible to obtain in each transaction.

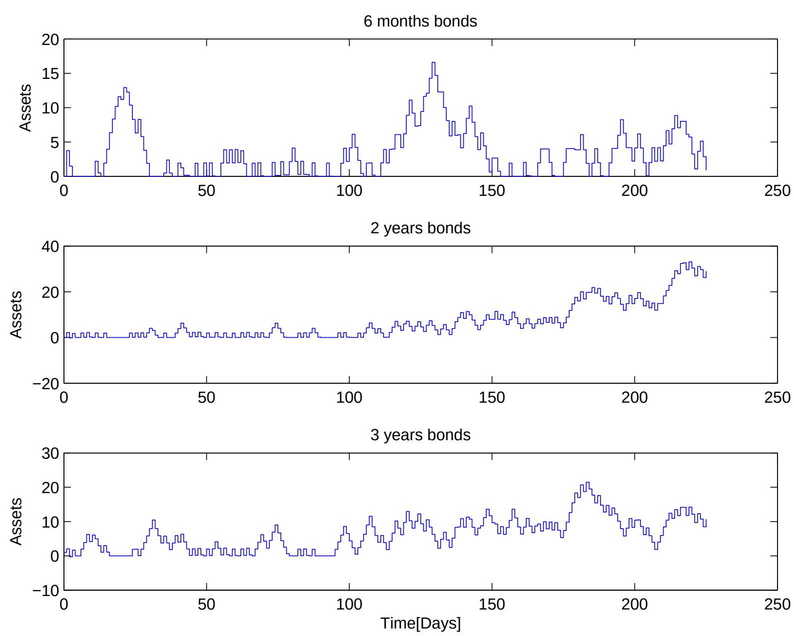

Figure **5-11**.: Portfolio assets with short sales allowed

No loans and no short sales allowed

Restricting all the loans and short sales, the MPC must try to work only with the risk free asset to perform the portfolio management. This fact decreases the freedom of the portfolio making more difficult to get a bigger return.

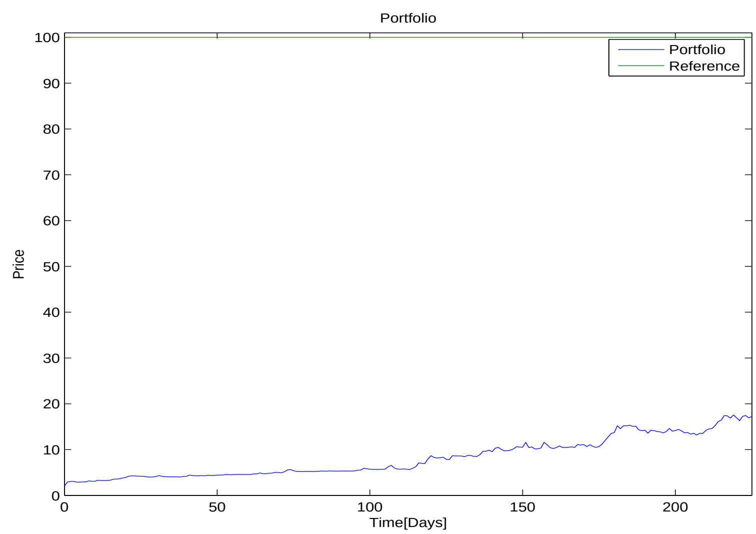

Figure **5-12**.: Portfolio evolution with short sales and loans restricted

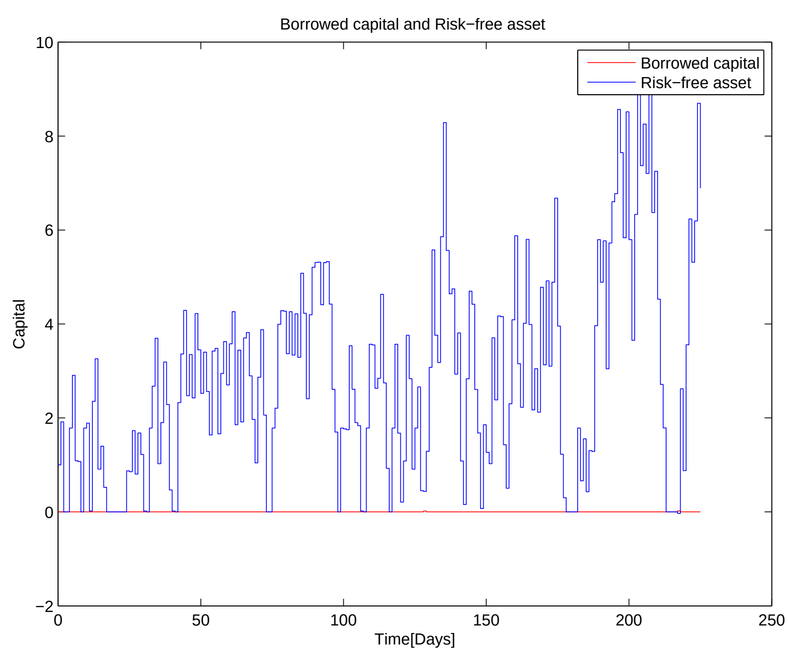

Figure **5-13**.: Borrowed capital and Risk-free asset

Lastly due to the hard constraints selected, the assets amount of each stock decreases, restricting the increase of yield possible to obtain in each transaction.

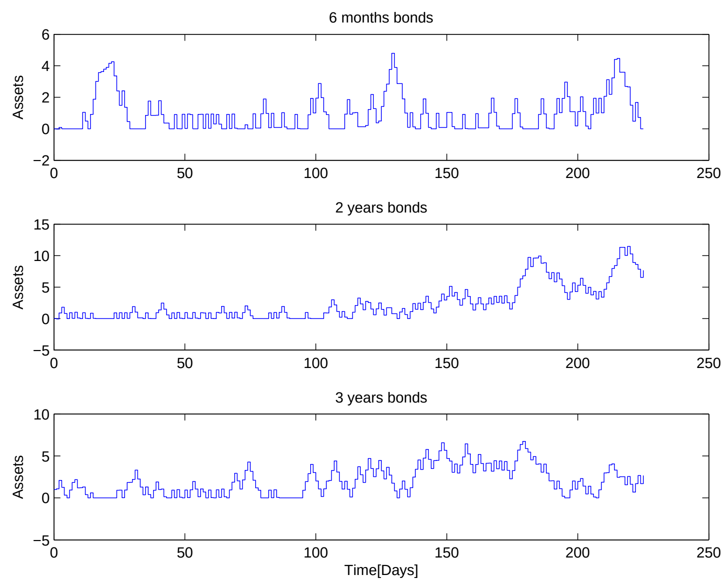

Figure **5-14**.: Portfolio assets with short sales and loans not allowed

As a final remark, the portfolio values of the three test to the MPC are presented.

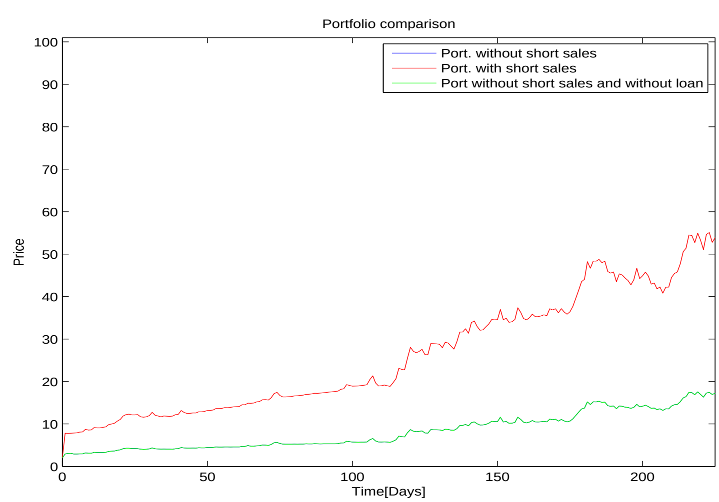

Figure **5-15**.: Comparison between portfolios with and without short sales

The behavior of the portfolio with loan and no short sales and the portfolio without short sales and without loan are very similar, this fact is concluded by the low loan level in the portfolio with no short sales.

6. Conclusions and future work

6.1. Conclusions

The implementation of the portfolio management strategy based on a model predictive control solution is successful. The performance of the MHE estimator shows an efficient strategy to make the assets prediction based on the trend of the stock prices. This estimation allows the MPC to take optimal actions over the system based on a multi-period strategy. Multi-period strategy allows to take actions similar to the classic strategy adopted by the traders in the markets over the world, selling the assets when the prices go down, and buying when they have an ascending trend. If a moving time window is taken to perform the prediction, the solution contains significant information of the past data, and the MPC takes the actions based on future predictions. The constrained solutions of the MPC are successful, where other solution strategies fail. This can be explained by the fact that it considers transaction cost and limits the amount of assets to be trade , giving to the strategy more chances to make the management strategy more suitable to fit the needs of investors, and move closer to the reality of actual market transactions.

The stock estimation requires a trend value based on the past of the history data. The MHE adjus the η value adjust, for this estimation. Previously an off line tuning procedure is required. The tuning procedure adjust the values of the time window, and the gain value of the MHE. This algorithm has not been used in financial applications, and shows a good performance in the estimation presented. As a Kalman filter modification, with constrains included the MHE have a promissory future in the finance research area.

The portfolio stock selection, implies the management strategy. In the management, stocks with high volatility will need investment actions more often implying more transactions cost and a bigger chance of losing or winning money. The stocks selected on this work are more stable with a low volatility, exhibiting a more predictable behavior, so the MPC and the estimator can perform better based on the time series data and the future estimation based on the prediction model. Also, the values of the time series implies the values related with variables like interest rates and cost of transaction, normally the values are between the 0.1 and 0.001 of the stock prices, or given by the market regulator.
The MPC strategy provides an optimal solution based on the cost function and the constrains of the problem. The performance index which describes the optimization objective requires

the tuning of the Q and R gains. Actually these variables do not have tuning methods, and a trial and error procedure was performed, until a desired behavior of the controller was reached. The particular conditions of each problem made the MPC problem a particular case the tolerances values of the optimization algorithm must be carefully selected. The initial conditions of the states of the MPC must be chosen in order to avoid not feasible solutions that are given by the constrains of the MPC, an example is to guarantee the conditions to cover the transaction cost of the operation, if the risk free asset is empty and there is not chance to make a loan to finance the stock trade. The MPC can not perform any action over the portfolio stocks, leading the solution to a trivial solution or to a mathematical error.
Finally, the solutions given by the MPC controller are consistent with the stocks prices behavior, and given the mathematical formulation of the problem, the solution is optimal in terms of the constrains and cost function each time window. This fact immediately gives to this technique an advantage over all the single period solutions. Also it is important to remark that other optimal solutions can give another solutions to the same problem, depending on the cost function, the constrains values and the reference given to the portfolio.

6.2. Future work

As future work, the solution proposed can be complemented with other conditions such: Consider some risk value reference as VaR in the cost function or as constrain, to give to the portfolio solution a statistical measure of the risk of loss in the investment actions. Some solutions to the portfolio management are based on the risk value, but most of them do not consider the transaction cost and other variables included in this work. Also, include as reference to the constraints a variable like the "Chicago Board Options Exchange Market Volatility Index" (VIX), that represents the market volatility, in order to make the decision based in the overall market behavior.
Based on the positive results of this work, adapt other optimal control techniques to the management problem, in order to solve some problems as: the windows size of the estimator and the MPC according to the stock prices.

A. Appendix A: Kalman filter

In the 1960s R.E. Kalman published a paper entitled A new Approach to Linear Filtering and Prediction Problems [22]. In this work the author presented a novel filtering method to process data for solving problems that deal with measurements hampered by random biases, with the goal of providing a recursive solution for estimation of linear discrete-time dynamic systems. A Kalman Filter is an optimal recursive data processing algorithm; each of these words composes the formula that makes one aware of what this filter is:

- "**data processing algorithms**", this simply means that the Kalman Filter is a set of mathematical formulas implemented in software, and the filtering process is therefore digital;
- "**recursive**", that is, the filter repeats at each iteration (i.e. at each measurement done) the same operations;
- "**optimal**", namely, the Kalman filter achieves the best performance if we assume the observed process is linear and the measurement noise is white and gaussian.

The purpose of the filter is to estimate a quantity by both the measurement and the *a priori* knowledge about the observed phenomenon. The principle followed is to collect and wisely process the available information. The data process is made up of two stages:

- **Prediction**: a function estimates the state of the Kalman Filter, projecting the quantity for future estimates. During this stage the measurement value of the quantity is also collected; these steps translate to equations as follows

$$\begin{aligned} m_k^- &= A_{k-1} m_{k-1} \\ P_k^- &= A_{k-1} P_{k-1} A_{k-1}^T + Q_{k-1}. \end{aligned} \tag{A-1}$$

- **Update**: the data collected at the previous stage is weighted by an appropriate coefficient called the *Kalman Gain*, determining the estimation for the observed value. The equations for this stage are:

$$\begin{aligned} v_k &= y_k - H_k m_k^- \\ S_k &= H_k P_k^- H_k^T + R_k \\ K_k &= P_k^- H_k^T S_k^{-1} \\ m_k &= m_k^- + K_k v_k \\ P_k &= P_k^- - K_k S_k K_k^T, \end{aligned} \tag{A-2}$$

where

- m_k^- and P_k^- are, respectively, the predicted mean and covariance of the state on the time step k before taking into account the current measurement.
- m_k and P_k are, respectively, the estimated mean and covariance of the state on time step k after taking into account the current measurement.
- v_k is the innovation or the measurement residual on time step k.
- S_k is the measurement prediction covariance on the time step k.
- K_k is the filter gain, an indicator of how much the predictions should be corrected on time step k.

The measurement function is responsible for projecting the state of the filter into the future and contains information about the observed process. This function describes a mathematical model of the measurable monitored phenomenon. In this way we can interpret the measurement obtained dynamically and coherently. The Kalman Gain is composed of quantities that are related to uncertainty both of the measurements and of the model used. We are assuming an environment impaired by white gaussian noise; therefore we can take into account both measurement and projection uncertainties referring to the noise and the covariances of the model respectively

In certain cases a Gaussian assumption is too primitive, but a Gaussian process only requires the first and second order statistics (i.e. mean and covariance) to be statistically described. The Gaussian assumption often allows complex problems to become more tractable. Furthermore, because the real data are influenced by many variables (e.g., scatter noise, multi-path channel, measurement noise, used device bias, etc.) we can utilize the Central Limit Theorem, which is at the core of probability theory. This theorem , proves that when a number of independent random variables are added together, their overall effect can be described well using a Gaussian probability density.

The assumption of white noise leads to some contradictions. This supposition implies noise values are not correlated in time, e.g. a channel can change from one state to another one abruptly. The assumption of whiteness also implies that all frequencies have the some power, i.e. a noise with infinite power. Despite this, the white-noise model is still useful both for treatment simplicity and because we can overcome the above problems using some "tricks". For instance, any physical system of interest has a bandpass frequency response; we can just disregard the frequencies outside this band to get away from the requirement to consider infinite noise power.

Note that in this case the predicted and estimated state covariances on different time steps do not depend on any measurement, so that they could be calculated off-line before making any measurements provided that the matrices A, Q, R and H are known on those particular time steps. It is also possible to predict the state of system as many steps ahead as wanted just by looping the predict step of Kalman filter, but naturally the accuracy of the estimate decreases with every step.

A.1. Kalman Filters for Non-linear State Estimation

In many cases interesting dynamic systems are not linear by nature, so the traditional Kalman filter cannot be applied in the estimation of the state of such systems. In these kind of systems, one or both the dynamics and the measurement processes can be non-linear. In this section we describe two extensions to the traditional Kalman filter, which can be applied for estimating non-linear dynamical systems by forming Gaussian approximations to the joint distribution of the state x and measurement y. First we present the Extended Kalman filter (EKF), which is based on Taylor series approximation of the joint distribution, and then the Unscented Kalman filter (UKF), which is based on the unscented transformation of the joint distribution.

A.1.1. Extended Kalman Filter (EKF)

The extended Kalman filter extends the scope of Kalman filter to non-linear optimal filtering problems by forming a Gaussian approximation to the joint distribution of state x and measurements y using a Taylor series based transformation. A first order EKF is presented using linear approximation; higher order filters are also possible, but not presented here.
The filtering model used in the EKF is

$$\begin{aligned} x_k &= f(x_{k-1}, k-1) + q_{k-1} \\ y_k &= h(x_k, k) + r_k, \end{aligned} \tag{A-3}$$

where $x_k \in \Re^n$ is the state, $y_k \in \Re^m$ is the measurement and $q_{k-1} \sim N(0, Q_{k-1})$ is the process noise, $r_k \sim N(0, R_k)$ is the measurement noise, f is the (possibly non-linear) dynamic model function and h is the (again possibly non-linear) measurement model function. The first order extended Kalman filter approximate the distribution of state x_k given the observations $y_{1:k}$ with a Gaussian:

$$p(x_k \mid y_{1:k-1}) \approx N(x_k \mid m_k, P_k). \tag{A-4}$$

Extended Kalman Filter Procedure

Like Kalman filter, also the extended Kalman filter is separated into two steps. The steps for the first order EKF are:

- **Prediction**:

$$\begin{aligned} m_k^- &= f\left(m_{k-1}, k-1\right) \\ P_k^- &= F_x\left(m_{k-1}, k-1\right) P_{k-1} F_x\left(m_{k-1}, k-1\right) + Q_{k-1}. \end{aligned} \tag{A-5}$$

- **Update**:

$$\begin{aligned} v_k &= y_k - h(m_k^-, k) \\ S_k &= H_x(m_k^-, k) P_k^- H_x^T(m_k^-, k) + R_k \\ K_k &= P_k^- H_x^T(m_k^-, k) S_k^{-1} \\ m_k &= m_k^- + K_k v_k \\ P_k &= P_k^- - K_k S_k K_k^T, \end{aligned} \tag{A-6}$$

where the matrices $F_x(m, k-1)$ and $H_x(m, k)$ are the Jacobians of f and h, with elements

$$\left[F_x(m, k-1)\right]_{j,j'} = \frac{\partial f_j(x, k-1)}{\partial x_{j'}} \mid_{x=m} \tag{A-7}$$

$$\left[H_x(m, k)\right]_{j,j'} = \frac{\partial h_j(x, k)}{\partial x_{j'}} \mid_{x=m} . \tag{A-8}$$

Note that the difference between first order EKF and KF is that the matrices A_k and H_k in KF are replaced with Jacobian matrices $F_x(m_{k-1}, k-1)$ and $H_x(m_k^-, k))$ in EKF. Predicted mean m_k^- and residual of prediction v_k are also calculated differently in the EKF.

The Limitations of EKF

The EKF has some serious drawbacks, which should be kept in mind when it is used:

1. The linear transformation produces reliable results only when the error propagation can be well approximated by a linear function. If this condition is not achieved the performance of the filter can be extremely poor. At worst, its estimates can diverge altogether.

2. The Jacobian matrices (and Hessian matrices with second order filters) need to exist so that the transformation can be applied. However, there are cases where this is not true. For example, the system might be jump-linear and the parameters can change abruptly.

3. In many cases the calculation of Jacobian and Hessian matrices can be a very difficult process, and its also prone to human errors (both derivation and programming). These errors are usually very hard to debug, because it its hard to see which parts of the system produces the errors by looking at the estimates, especially as usually we do not know which kind of performance we should expect.

A.2. Kalman filter applications in finance

Some applications of the Kalman filter in finance has been made, in [3] an interesting application of neural networks and Kalman filter is used to Model stock returns sensitivity, they compare the Kalman filter and Neural network to predict the stock sensitivity. The results concluded that the Kalman filter is robust in the sensitivity. This fact is because the Kalman filter achieves the best performance if we assume the observed process is linear and the measurement noise is white and Gaussian. Therefore in this order the stock prices can be handled as a system with this characteristics if the price evolution is considered like the equation 2-8 or other stochastic structure; the estimation is also performed with a neural network, which gives a better estimation with the influence of external variables, also called generality. These results are partial, and the authors left an open case of study where the solution can be the combination of both techniques. In the paper "A comparison of Extended Kalman Filter and Levenberg-Marquardt methods for neural network training" [9] a possible solution to this problem is show. The paper shows the implementation of a neural network trained with a Extended Kalman Filter and it is compared with the Levenberg-Marquardt training, this comparison is performed with a real data set.

In "Prediccion de betas y VaR de portafolios de acciones mediante el filtro de Kalman y los modelos GARCH" [19], the paper developes the study of the performance of the Kalam filter as estimator of the β coefficientes or the VaR associated to an investment portfolio and compare the results with the RiskMetrics [1] estimator. The paper also include a literature review about the Kalman filter applied to betas estimation, which can be useful for a more detailed explanation, but this work did not focus on the VaR problem.

[1]http://www.msci.com/

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